Code-Aided Interference Suppression for DS/CDMA Overlay Systems

STEFANO BUZZI, MEMBER, IEEE, MARCO LOPS, SENIOR MEMBER, IEEE, AND
H. VINCENT POOR, FELLOW, IEEE

Encouraged Paper

The push toward more efficient and flexible use of the radio spectrum has led to the consideration of the overlay of spread-spectrum (SS) communication networks on preexisting narrow-band networks. Such systems are already in widespread and rapidly expanding use in the lightly regulated spectrum where personal networking is largely centered and they are of increasing interest in the more tightly regulated spectrum due to a dearth of spectrum for new services, the desire to incorporate multi-rate (e.g., multimedia) traffic in the same network and the survival of legacy systems. Even though SS signals are inherently robust to the effects of narrower bandwidth cochannel signals, it has been shown that the use of additional processing aimed at interference suppression can result in substantial performance improvement. Motivated by this consideration, the past quarter century has seen the development of a very large body of techniques for improving the performance of SS communication systems in the presence of narrow-band interference (NBI). Early techniques (up to late 1980s) have been reviewed in the survey by Milstein (1988). Since that time, more sophisticated strategies have been developed, making use of advances from the fields of beamforming, multiuser detection (MUD), and adaptive filtering. Also, the focus of interest has shifted from techniques aimed primarily at the suppression of NBI from single-user SS systems to systems in which the SS signaling is being used to implement a code-division multiple-access (CDMA) protocol. This paper provides a tutorial overview of the progress made in this area over the past 15 years. The focus of the paper is on direct-sequence CDMA (DS/CDMA) systems and on the so-called "code-aided" techniques for NBI suppression, a term recently coined to indicate those strategies in which knowledge of the spreading code of a SS signal of interest is explicitly exploited in suppressing NBI. Particular attention is devoted to the case in which the CDMA signals are subject to frequency-selective fading and to the issue of blind adaptive MUD in the presence of external NBI. In particular, with regard to the former issue, the effects and implications of channel-state information on system design and performance are discussed. With regard to the latter issue, it is observed that the external NBI may introduce the need for a periodically time-varying detection rule, which has significant implications in the design of blind adaptive MUD algorithms for overlaid DS/CDMA systems. The performance of the techniques discussed is compared through analysis and simulation, as well as through considerations of their relative computational complexity and required prior information. Finally, the paper is concluded by a discussion of several challenging open problems in this area.

Keywords—Blind adaptive algorithms, CDMA, code-aided techniques, multiuser detection, narrow-band interference, overlay applications, spread spectrum communications.

I. INTRODUCTION

The idea of transmitting information via spread-spectrum (SS) signaling, i.e., by using a transmission bandwidth much greater than the source information rate, dates back to the 1940s, when it was developed as a means to prevent interception of military communications from a third party by reducing the peak transmitted power and spreading the information over a large bandwidth. Since these beginnings, SS signaling has been of increasing interest, as its unique features have proven to be useful in a number of military and, more recently, commercial applications. Today, SS signals are currently employed in the most common current and emerging wireless services. Such services include both second generation (IS-95) [181] and third-generation [wide-band code-division multiple access (WCDMA), code-division multiple access (CDMA) 2000] cellular telephony, both terrestrial and satellite-based [3], [40], [179], digital broadcasting (orthogonal frequency division multiplexing), global positioning systems, and, more generally, personal communication services, i.e., piconets (Bluetooth), wireless local area networks (LANs) (IEEE 802.11a/b, Hiperlan), and wireless local loop.

A number of interesting properties of SS signals have been discovered and exploited since the 1940s, both in military and in civilian applications. Of particular interest is the inherent antijamming capability of SS signals, which was an

early motivator for their adoption in tactical communications. Also, their intrinsic resistance to any type of narrower bandwidth cochannel interference [i.e., narrow-band interference (NBI)] is of crucial importance for all systems operating in unregulated bands, such as the industrial, scientific, and medical (ISM) bands, where wireless LANs, cordless phones, and Bluetooth piconets operate as SS systems overlaid on narrower bandwidth networks [147]. However, despite this natural resistance to NBI, the performance of SS systems can be enhanced significantly in the presence of NBI through the use of active interference suppression methods. This problem of active NBI suppression and, more generally, that of cochannel interference suppression in SS communication systems has received considerable attention over the past several decades, attracting the interest of workers across many disciplines of electrical engineering, such as communication systems, signal processing, aerospace and electronic systems, information theory, neural networks, and automatic control. Early attention to this problem was focused on the enhancement of the natural antijam capabilities of single-user SS systems. More recent research in SS systems has shifted to the consideration of multiple-access communication systems and in particular to direct-sequence CDMA (DS/CDMA) systems, due to their leading role in the realization of the air interface of current and emerging wireless networks.

The push toward more bandwidth-efficient systems exploiting the overlay concept is witnessed by the large body of literature that has appeared on this topic in the past 20 years [21], [86]–[88], [126], [127], [135], [138], [157], [158], [177], [190]–[192], [194]–[197], [211], [213]. Although neither IS-95 nor third-generation cellular standards require overlaying a CDMA multiplex on other services, it is foreseen that the shortage of available bandwidth in the radio spectrum, in conjunction with the explosive growth of new applications such as wireless Internet access and mobile computing, will motivate the development of overlay architectures for mobile telephony as well. For example, multirate systems [25], [26], [128] have already been standardized to meet the increasing demand for mobile multimedia communications and this naturally leads to the concept of overlaying several CDMA networks with different bandwidths within a common portion of the radio spectrum. Also, it is anticipated that WCDMA networks will be allocated in frequency bands that in many parts of the world are not yet vacated by existing narrow-band communication systems and, further, it is reasonable to expect that the deployment of third-generation wireless networks will occur gradually, whereby there will be a transition period in which the new systems will be *overlaid* on preexisting narrow-band cellular telephony systems and will have to coexist with them. Moreover, in the lightly regulated ISM bands, the projected proliferation of (frequency hopping) Bluetooth devices and DS wireless LANs has raised a considerable degree of concern about interference in both industry and regulatory organizations. Leaving aside the influence of a CDMA network on a coexisting NBI system, which can be neglected only for lightly loaded systems

[156]–[158], the dual problem, i.e., the cancellation of the NBI when demodulating CDMA users, is a key issue that has been investigated extensively in the past decade.

In principle, separation of a CDMA multiplex and external NBI can be achieved by simple matched-filtering, i.e., despreading and low-pass filtering of the received signal. Indeed, despreading a desired user amounts to further spreading the other users' signals and to spreading the NBI, whereby low-pass filtering of the despread signal allows the receiver to pick up all of the energy of the desired user's signal while reducing the other users' and the NBI energies by the bandwidth expansion factor (or processing gain). Leaving aside the problem of unequal received CDMA users' powers, it is known that simple matched filtering may not be sufficient for NBI suppression if the ratio of the NBI power to the SS power is comparable to the square of the processing gain, especially in the presence of stringent reliability constraints. Furthermore, even in situations where the NBI power is not very large so that it may be compensated for by the SS signal processing gain, intelligent signal processing techniques for active suppression of NBI may substantially improve the performance of such systems.

Work in this area until the late 1980s, as reviewed in the excellent survey by Milstein [122], was mainly concerned with NBI suppression in the presence of a single SS signal and thus was oriented to either single-user systems or to military applications. Since this time, further steps have been taken in at least two directions. First, as a natural development of the work discussed by Milstein, a number of increasingly elaborate NBI suppression techniques for SS systems have been developed, getting to a point where the communication reliability is almost completely restored to that of an NBI-free environment upon suitable processing. On a parallel track, a number of researchers have been regarding the NBI suppression problem as an aspect of the more general cochannel interference cancellation issue, wherein a CDMA network is to coexist with an external system and reliably detecting each user requires joint suppression of multiple access interference (MAI) and NBI.

In this paper, we focus on this latter track and provide a survey of recently developed techniques aimed at joint suppression of NBI and MAI in overlay CDMA networks. Since the main results in this area have been concerned primarily with DS SS systems, we will restrict attention to such systems throughout the paper. In the following paragraphs, we will give a brief classification of the research results concerning NBI suppression methods that have appeared in the literature since Milstein's survey and then we will describe the organization of this paper.

A. NBI Suppression in SS Communications

Existing active NBI suppression methods used in SS communications can be effectively grouped into three main classes: linear predictive (LP), nonlinear predictive (NLP), and multiuser detection (MUD) techniques.

All predictive methods, whether linear or not, are based on the simple idea that the NBI is much more predictable

than the wide-band SS, once the processing length is fixed. It, thus, becomes possible to form a replica of the NBI by applying a predictor to the received signal and then to subtract it from the received signal prior to despreading, low-pass filtering, and decision circuitry. Of course, the nature of the predictor to be used is strongly tied to the modeled behavior of the NBI. A basic technique used in many studies (e.g., [9]–[12], [74], [76], [77], [83], [85], [89], [100], [101], [116], [118], [122], [131], [137], [172], and [180]; see [2], [94], [122], and [127] for reviews) is to use a simple tapped-delay-line (TDL) filter at the sampled output of a chip-matched filter to implement the predictor. In a certain context, such filters can be considered to be finite-impulse response (FIR) approximations to the infinite-impulse response filter arising from the design of an optimal predictor based on the Kalman–Bucy approach. In particular, on modeling the NBI as an autoregressive (AR) Gaussian process, the suppression problem can be given a state-variable form, which henceforth can be solved through standard linear estimation tools. The suboptimality of TDL-based systems is reflected in an irreducible floor induced in the bit-error rate (BER) for increasingly large NBI power; indeed, the subtractor inevitably induces self-noise into the observables, whereby reliable communication can be achieved only at the price of heavy coding and, ultimately, reduced efficiency. This effect is worsened by the presence of multipath, which, roughly speaking, adds more interference in the form of intersymbol interference (ISI). However, TDL-based methods do have several useful properties, such as their applicability irrespective of the NBI model, their amenability to adaptive least-mean squares (LMS) implementations (e.g., [175]), the very limited amount of required hardware equipment, and, overall, their high degree of modularity. Recently, improved versions of this approach have been developed in [59] for the suppression of a digital NBI.

Related to the LP methods are the so-called transform-domain methods, wherein the contrast between the wide-band nature of the SS signal and the narrow-band nature of the interference is exploited to notch out the latter in the frequency-domain prior to making decisions [106], [166]. Further performance improvements can be also obtained by coupling the frequency-domain excision with coding, so as to compensate, with appropriate redundancy, for the small information loss induced by notching [185]. The concept of notching out the NBI, albeit not necessarily in the frequency domain, is also at the basis of the so-called projection on convex set (POCS) techniques, proposed in [46] for SS signal restoration in overlay applications. Interestingly, POCS-based receivers lend themselves to adaptive implementation quite straightforwardly, as shown in [45].

Further performance enhancement can be obtained by using NLP methods, wherein the non-Gaussian nature of the SS signal is explicitly accounted for at the design level. Indeed, starting upon the obvious consideration that the desired signal at the output of the chip matched filter is a binary random sequence, even though the NBI and the ambient noise are assumed to be Gaussian, the *optimum* [in the minimum mean square error (MMSE) sense] prediction

filter will be nonlinear. The use of nonlinear predictors for NBI suppression was proposed first in [189]. In this context, the starting point is the work by Masreliez, who in [115] proposed suboptimum approximate conditional mean (ACM) filters for estimating the state of a linear system with Gaussian state noise and non-Gaussian measurement noise. In [189], the ACM filter was applied to the problem of AR-type NBI suppression and it was shown that NLP techniques are able to achieve substantial performance gains compared to LP techniques. Since then, many other studies have considered the use of nonlinear techniques for NBI suppression in SS systems, including, e.g., [58], [143], [150], [164], [198]–[200], and [201]. However, it should be noted that, even though these methods exploit the non-Gaussian nature of the SS signal, they do not exploit the explicit structure of the spreading codes used by the SS signal.

A major advance in the development of NBI suppression techniques has been made by exploiting the spreading codes through the application of MUD techniques, designed to suppress MAI, to NBI cancellation [148], [165]. The underlying idea of these works is to split a digital NBI into a set of nonoverlapping virtual users, so that the overlay of an SS signal on digital NBI can be regarded as a CDMA system and the MUD tools can, therefore, be applied. As explained later, such an approach has been further developed in [151], wherein MUD techniques are shown to be applicable to suppress a much broader class of NBIs, including AR processes, sinusoidal tones, etc.

B. NBI Suppression in CDMA Systems

If the SS signal of interest consists of multiplexed SS signals arranged in a CDMA transmission format, then the problem of detecting each user's symbol against a mixture of MAI and NBI arises. Relevant, in this case, is also the situation in which the CDMA signals undergo multipath distortion, which, as mentioned above, adds further to the interference to be suppressed in the form of ISI. In such situations, we are faced with the problem of joint equalization and cochannel interference suppression.

Early studies of this problem focused on a receiver structure in which NBI is handled through a dedicated linear suppression block, whereas MAI suppression is performed through conventional matched filtering, possibly coupled with maximal ratio-combining (MRC) in the case of fading channels [194]. Such an approach, which leads to fairly simple structures and allows the use of simple FIR LP methods, may suffer from strong NBIs and even more so, from a combined effect of NBI, MAI and multipath distortion, which typically gives rise to a lower bound to the attainable BER. The conclusion is that these systems are trading hardware complexity for bandwidth efficiency due to the attendant need for heavy coding and for software complexity due to the need for heavy bit interleaving [194]–[196].

Nonlinear filtering techniques have not proven as successful in CDMA systems as in single-user SS communication systems. Indeed, the results reported in [164] reveal

that the effectiveness of ACM filters in NBI prediction against multiplexed SS users decreases for an increasing number of users.

Overall, one of the fundamental reasons why NLP techniques yield relatively poor performance in CDMA situations is that, by performing a chip-by-chip processing of the observables, they do not take into account the code structure of the SS signals, as noted previously. Under this point of view, the so-called *code-aided* techniques (a term coined in [151]) are clearly superior. The application of MUD techniques to the multiuser case is by far the most promising such approach, descending directly from the concepts first developed in [165] with reference to SS signals. A methodological approach to the system design and analysis is presented in [151], wherein it was first recognized that MMSE MUD can cope with any interference source, whether NBI or MAI, with few restrictions on the nature of the NBI. In [152] and [153], the issue of adaptive implementation in such systems is also tackled, showing that MAI and NBI can be jointly rejected based upon the knowledge of the timing, phase offset and the spreading code of only the user of interest, whence the name of *code-aided* techniques. Further developments on MUD techniques are contained in [105] and [108], where the issue of time-varying processing for joint suppression of MAI and NBI is also introduced in a zero-forcing (ZF) optimization framework.

More recently, much attention has been devoted to the case in which the CDMA signals undergo multipath distortion and the two main linear MUDs, the decorrelator and the MMSE detector, have been adapted to this context [18], [21], [23], [26]. In this case, time-varying processing may be needed as well and the implications of this fact on adaptive system implementations have been investigated in [16], [17], [23]–[26].

C. Paper Organization

Generally speaking, this paper provides a review of MUD techniques for NBI suppression in CDMA channels. Implicit in this choice is that we will be concerned with code-aided techniques only (in the sense specified in [147] and [151]). Since code-aided techniques mainly differentiate from the other techniques by taking into account the structure of the signal to be demodulated, they require batch rather than chip-by-chip processing. Each processing batch can be made of the samples from one or more signaling intervals, the latter situation being typical of block detection. On the other hand, the choice of the processing interval may have significant impact on the receiver structure for some types of NBI, whereby the issue of proper signal representation is a key one in MUD NBI suppression techniques. The problems of signal modeling and representation are, thus, accorded a dedicated section, Section II, wherein a number of points of relevant theoretical and practical interest are discussed with reference to both nonfading and fading channels.

The problem of MUD NBI suppression techniques in nonfading channels is examined in Section III, wherein some key issues, such as the optimization strategy to be adopted and the tradeoffs between system complexity and achievable performance, are discussed in depth. In a novel framework, we connect these issues to the definition of proper cost functions and, ultimately, to the attainability of global minima thereof.

The impact of linear channel distortion on receiver structure and performance is the subject of Section IV in which MUD techniques for NBI suppression in fading dispersive channels are discussed. As in the nonfading case, we frame the discussion in a more general context in which the cost functions to be optimized depend on the availability of channel state information (CSI) at the receiver end. We consider the relevant cases of: 1) complete CSI (CCSI) on the user of interest, but not on the other users, and 2) CCSI as to the user of interest and to the other users, which may represent the situation at the base station in a cellular overlay CDMA system. Ideas on how to proceed in cases of no CSI at all are also given as pointers for future developments.

Section V focuses on the issue of *adaptive* NBI and MAI suppression. The joint influence of the fading model and of the NBI model on the ultimate system performance is also discussed and a number of tradeoffs between complexity and performance are considered in depth. Special attention is devoted to the connection between the channel coherence time, the average convergence time of the adaptive algorithms, and the steady-state system performance.

Finally, Section VI contains a discussion of some key open research issues in this area, while Section VII contains concluding remarks.

II. SIGNAL MODEL

In an overlay architecture, the received signal can be modeled as the superposition of the CDMA multiplex, NBI, and white noise, whereby its complex baseband equivalent admits the general expression

$$r(t) = S(t) + I(t) + N(t) \qquad (1)$$

where $S(t)$ is the said multiplex, $N(t)$ is a sample function from a complex white Gaussian noise with power spectral density $2\mathcal{N}_0$, and $I(t)$ is the NBI. The CDMA multiplex can in turn be decomposed into the contributions from the active users and admits diverse expressions based upon the channel nature.

A. Models of the CDMA Multiplex

For a single-path asynchronous channel, the signal $S(t)$ can be written as

$$S(t) = \sum_{k=0}^{K-1} \sqrt{\mathcal{E}_k} \sum_{n=-\infty}^{\infty} b_k(n)\alpha_k s_k\left(t - nT_b - \tau_k\right) \quad (2)$$

where

K	number of active users;
$\mathcal{E}_k$	energy transmitted in each signaling interval by the kth user;
$\{b_k(n)\}_{n=-\infty}^{+\infty}$	symbol stream from the kth user;
α_k	complex gain accounting for channel effects;
T_b	symbol interval;

$s_k(t)$ signature of the kth user, which, assuming DS spreading, is written as

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} c_{k,i} \psi(t - iT_c) \quad (3)$$

where $\{c_{k,i} : i = 0, \ldots, N-1\}$ is the spreading code assigned to the kth user, $\psi(\cdot)$ is the unit-energy chip waveform, $N = T_b/T_c$ is the processing gain, and $1/T_c \simeq W$ is the chipping rate. Notice that the spreading code is independent of the symbol epoch, which means that short codes are being considered here[1] ;

$\tau_k \in [0, T_b]$ propagation delay of the kth user's channel.

If, instead, the channel is a fading dispersive one, i.e., if the signal bandwidth W exceeds the channel coherence bandwidth B_c, so that $L \simeq W/B_c$ nearly independent paths are generated [155], the signal $S(t)$ is alternatively expressed as

$$S(t) = \sum_{k=0}^{K-1} \sqrt{\mathcal{E}_k} \sum_{n=-\infty}^{\infty} b_k(n) \sum_{l=0}^{L-1} \alpha_{k,l}(n) s_k(t - nT_b - \tau_{k,l}) \quad (4)$$

where

$\alpha_{k,l}(n)$ random variable representing the lth tap weight of the kth user's channel in the nth signaling interval. Implicit in this model is the assumption that T_b is small with respect to the channel coherence time $(\Delta t)_c$, so that the weights remain constant during the signaling interval;

$\tau_{k,l}$ delay of the lth path of the kth user's channel.

Subsumed in models (2) and (4) are a number of cases of relevant practical interest. For example, if the tap weights and the delays of the different users are assumed to be coincident, the model may represent the received signal in the downlink of a mobile network, while different weights and delays correspond to an uplink communication channel. Moreover, (2) and (4) may also model the received signal in a multirate DS/CDMA system implemented with the multicode access technique [128], wherein high-rate users are assigned multiple signature waveforms so that they can transmit, in each symbol-interval, more than one symbol by modulating as many signatures. In this case and assuming that the signatures $s_0(t), \ldots, s_{G-1}(t)$ are assigned to one user transmitting at a rate G times larger than the network basic rate, then we have, for $k = 0, \ldots, G-1$, the same delays and channel tap-weights in both the forward and reverse links of the network.

B. Models of the NBI

The NBI models that have been considered so far and that we adopt here can be grouped into two classes: 1) quasi-

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[1]The use of a short-code model is not critical to any of the techniques discussed in this paper, except for those in Section V involving adaptivity. Adaptivity in long-code systems will be discussed briefly in Section VI.

deterministic signals and 2) entropic narrow-band stochastic processes.

Members of class 1 are all those signals subsumed in the so-called digitally-modulated NBI

$$I(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{K_I - 1} \sqrt{\mathcal{E}_{Ik}} \alpha_{Ik} d_k(n) v_k(t - \tau_{Ik} - nT_I) e^{j2\pi f_{Ik} t} \quad (5)$$

where K_I is the number of external (isochronous) digital NBIs, $\mathcal{E}_{Ik}$ is the energy of the kth NBI, α_{Ik} is a complex gain accounting for channel propagation effects, $d_k(n)$ is its symbol stream, $v_k(\cdot)$ is the corresponding signature, τ_{Ik} is the propagation delay, while $T_I \gg T_c$ and f_{Ik} are the values of the (common) NBI symbol interval and kth interferer's frequency offset, respectively. For the sake of simplicity, we assume that all of the NBIs share the same symbol interval, but the extension to anisochronous NBIs is straightforward. Likewise, in writing expression (5), we have considered a single-path communication channel. Otherwise stated, for the sake of simplicity, we assume here that the bandwidth of the NBI $I(t)$ is smaller than the channel coherence bandwidth B_c, so as to neglect the multipath distortion. However, what follows can be easily generalized to the case where this assumption does not hold.

Subsumed in the general model (5) are a number of special cases of some practical interest. Indeed, if $d_k(n) = d_k$ and $v_k(\cdot)$ is a rectangular pulse of duration T_I, the NBI is the superposition of K_I sinusoidal tones. This model has been considered, for instance, in [151]–[153]. Likewise, if the signatures $v_k(\cdot)$ are themselves SS signals, the NBI is itself a CDMA multiplex and the general model (1) represents the received signal in a dual-rate (multicarrier) CDMA system. Of course, if the frequency offsets of the NBI coincide, the situation reduces to a simple dual-rate CDMA network, as considered, e.g., in [25]. Also, the case of a single digitally modulated NBI, which has been considered in, e.g., [18], [21], [23], [24], [26], [106], [148], [151]–[153], and [165], is obtained by letting $K_I = 1$ and assuming that the signature $v_0(\cdot)$ is any narrow-band waveform.

Alternatively, class 2 models the situation in which the current outcome of the NBI process consists of a contribution predictable based upon a number of past observations plus an independent unpredictable part—the innovation. This model has been used in several studies (see, e.g., [122], [123], and [151]–[153]) with the additional constraint that the predictable part is linearly predictable, i.e., that the NBI can be approximated through a Gaussian AR process. Further details on such models will be given after we discuss the signal discretization process.

As a final remark, we note that recently a novel NBI model has been proposed based on a hidden Markov model (HMM) [34]. This model finds application in those circumstances wherein narrow-band intereferers enter and leave the channel at random epochs and at arbitrary frequencies within the bandwidth of the SS signal, whereby a HMM may be adopted for the random process controlling the exit and the entry of NBIs in the channel. An active interference

suppression method for this scenario has been developed in [34], while other works related to NBI and HMM include [79], [91]. These models will be discussed briefly in Section VI.

C. Signal Representation—Windowing and Sampling

As already noted, code-aided techniques exploit the knowledge of the spreading code of the user to be detected, whereby they are based on batch rather than chip-by-chip processing of the observables. Thus, the received signal can be windowed so as to include an integer number of symbol intervals T_b, if block detection is used, or a single signaling interval, if symbol-by-symbol (i.e., one shot) detection is adopted. It should be noted that, due to network asynchrony and the presence of NBI, sequence detection would in principle be needed for globally optimal detection [146]. However, we will not be concerned with this approach in what follows due to its typical high complexity.

Without loss of generality, let us assume that we are interested in demodulating the information symbol stream from user zero and that $\tau_0 = 0$, if the nonfading model (2) is in force or, alternatively, that $\tau_{0,0} = 0$, if the multipath case (4) is considered. As a consequence, in a symbol-by-symbol detector, the processing interval to detect the bit $b_0(p)$ is $[pT_b, (p+1)T_b]$,[2] whereby the reduced observables can be conveniently recast as

$$
\begin{aligned}
r_p(t) = & r(t)\mathrm{rect}_{T_b}\left(t - pT_b\right) = \sqrt{\mathcal{E}_0}b_0(p)\alpha_0 s_{0,p}(t) \\
& + \sum_{k=1}^{K-1} \sqrt{\mathcal{E}_k} \sum_{i\in\{-1,0\}} b_k(p+i)\alpha_k s_{k,p+i}(t) \\
& + (I(t) + N(t))\mathrm{rect}_{T_b}\left(t - pT_b\right)
\end{aligned}
\tag{6}
$$

and as

$$
\begin{aligned}
r_p(t) = & r(t)\mathrm{rect}_{T_b}\left(t - pT_b\right) = \sqrt{\mathcal{E}_0}b_0(p)\sum_{l=0}^{L-1}\alpha_{0,l}(p)s_{0,p}^l(t) \\
& + \sum_{k=0}^{K-1} \sqrt{\mathcal{E}_k} \sum_{i\in\Omega_k} b_k(p+i)\sum_{l=0}^{L-1}\alpha_{k,l}(p+i)s_{k,p+i}^l(t) \\
& + (I(t) + N(t))\mathrm{rect}_{T_b}\left(t - pT_b\right)
\end{aligned}
\tag{7}
$$

for the case of nonfaded single-path channels and fading dispersive channels, respectively. In the above equations, $\mathrm{rect}_{T_b}(\cdot)$ is a unit-height rectangular signal supported on the interval $[0, T_b]$, the sets Ω_k are defined as

$$
\Omega_k =
\begin{cases}
\left\{-\left\lceil\frac{\tau_{0,L-1}}{T_b}\right\rceil, \ldots, -1, \right\}, & \text{if } k = 0 \\
\left\{-\left\lceil\frac{\tau_{k,L-1}}{T_b}\right\rceil, \ldots, -1, 0\right\}, & \text{if } k \neq 0
\end{cases}
\tag{8}
$$

while

$$
\begin{aligned}
s_{k,p+i}(t) = & s_k\left(t - \tau_k - (p+i)T_b\right) \\
& \cdot \mathrm{rect}_{T_b}\left(t - pT_b\right)
\end{aligned}
\tag{9}
$$

[2]Notice that we are here deliberately neglecting the residual energy from the bit $b_0(p)$ that channel dispersivity may spread outside the observation interval.

and

$$
\begin{aligned}
s_{k,p+i}^l(t) = & s_k\left(t - (p+i)T_b - \tau_{k,l}\right) \\
& \cdot \mathrm{rect}_{T_b}\left(t - pT_b\right).
\end{aligned}
\tag{10}
$$

Note that (6) and (7) are just regroupings of the received signal into its several components. In particular, the first term represents the "desired signal," i.e., it includes all of the received energy that can be relied upon in detecting the bit $b_0(p)$. The second term represents the interference due to both the multiaccess nature of the channel and to its (possible) dispersivity, while the last two terms represent the contribution from the NBI and the thermal noise, respectively.

A major issue in dealing with the detection problem is the representation via sampling of $r_p(t)$ as a vector in a continuous finite-dimensional linear space. First notice that, if no NBI were present and if the delays were perfectly known to the receiver, then such a representation could be easily obtained by projecting $r_p(t)$ onto a linearly independent subset of the waveforms of the set

$$
\left\{ s_{0,p}(t) \bigcup \left\{ \{s_{k,p+i}(t)\}_{i=-1,0} \right\}_{k=1}^{K-1} \right\}
$$

for the model (6) and of the set

$$
\left\{ s_{0,p}^m(t) \bigcup \left\{ \{s_{k,p+i}^m(t)\}_{i\in\Omega_k} \right\}_{k=0}^{K-1} \right\}_{m=0}^{L-1}
$$

for the model (7). Since such an assumption is not realistic and would also prevent any adaptive implementation of the receiver, it is customary to resort to chip-matched filtering. If band-limited chip waveforms are employed, such as the raised-cosine waveforms, this operation consists of (antialiasing) low-pass filtering followed by sampling at the chip rate. If, instead, time-limited rectangular (infinite-bandwidth) chip pulses are employed, this operation is equivalent to projecting $r_p(t)$ onto the N-dimensional system

$$
\left\{ \frac{1}{\sqrt{T_c}} u_{T_c}\left(t - pT_b - jT_c\right) : j = 0, \ldots, N-1 \right\}
\tag{11}
$$

with $u_{T_c}(\cdot)$ a unit-height rectangular pulse supported on the interval $[0, T_c]$. Of course, (11) provides an errorless representation of the CDMA multiplex only if the delays are all integer multiples of T_c. If this is not the case, using the system (11) may result in CDMA signals leaking outside their original subspace in a phenomenon that is conceptually similar to aliasing in conventional sampling. As will be discussed later, this could lead to signal space saturation even with underloaded networks. A remedy that has been proposed and that is borrowed from the literature on fractionally spaced equalization is signal-space oversampling, which consists of using an NM-dimensional system (with *oversampling factor M* an integer greater than or equal to one) to represent SS signals with processing gain N. Of course, the simplest way to oversample is to make the oversampling uniform, which corresponds to adopting the representation system

$$
\left\{ u_{T_{OS}}\left(t - pT_b - jT_{OS}\right) : j = 0, \ldots, NM-1 \right\}
\tag{12}
$$

with $T_{OS} = T_c/M$ and $u_{T_{OS}}(\cdot)$ a unit-height rectangular pulse supported on the interval $[0, T_{OS}]$. Notice that over-
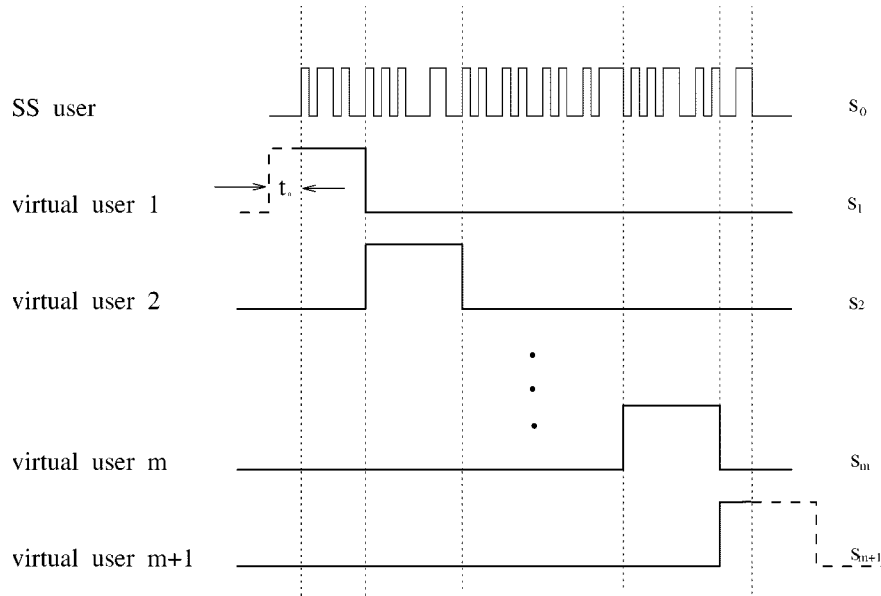
Fig. 1. System of one SS user and one digital narrow-band interferer with integer T_b/T_I; can be viewed as a virtual CDMA system. In this system, the digital NBI signal can be regarded as $m+1$ virtual users, each with its virtual signature sequence.

sampling is even more necessary in the presence of external NBI. Indeed, since the structure of such a signal may not be known to the receiver, its representation would necessarily call for an infinite-dimensional system, whereby oversampling represents a reasonable compromise between the conflicting requirements of errorless representation and acceptable computational complexity. Notice that oversampling is beneficial also in systems with band-limited chip pulses in that the presence of the NBI and the windowing inherent in the batch processing inevitably destroys bandlimitedness.

In the remainder of this paper, we consider the case of rectangular chip pulses, even though the majority of the following derivations apply to the case of band-limited chip pulses as well. Projecting the observables (7) onto (12)$\forall p$ is equivalent to considering the discrete-time sequence $r(n)$ defined as

$$r(n) = \frac{1}{T_{OS}} \int_{nT_{OS}}^{(n+1)T_{OS}} r(t)dt = S_n + I_n + N_n,$$
$$n = \ldots, -1, 0, 1, \ldots \tag{13}$$

and then retaining the NM samples $r(NMp), \ldots, r(NMp + NM - 1)$ corresponding to the pth signaling interval $[pT_b, (p+1)T_b]$. Upon stacking these samples in a vector $\boldsymbol{r}(p)$, we obtain the following expression for the observables:

$$\boldsymbol{r}(p) = \boldsymbol{s}(p) + \boldsymbol{i}(p) + \boldsymbol{n}(p) \tag{14}$$

in which the three terms on the right-hand-side represent the discrete-time versions of the CDMA multiplex, NBI, and thermal noise, respectively. Deferring to the next section, a more thorough characterization of the structure of the NBI $\boldsymbol{i}(p)$, we recall here that the vector $\boldsymbol{n}(p)$ in (14) is a proper complex Gaussian vector with covariance matrix $2\mathcal{N}_0\boldsymbol{I}_{NM}$, $\boldsymbol{I}_{NM}$ being the identity matrix of order NM. With regard to

$\boldsymbol{s}(p)$, it is easily shown that, after some algebraic manipulations, this vector can be expressed as

$$\boldsymbol{s}(p) = \sqrt{\mathcal{E}_0}\alpha_0 b_0(p)\boldsymbol{s}_0 + \sum_{k=1}^{K-1} \sqrt{\mathcal{E}_k}\alpha_k \sum_{i=-1,0} b_k(p+i)\boldsymbol{s}_{k,i} \tag{15}$$

for the case of nonfaded single-path channels and as

$$\boldsymbol{s}(p) = \sqrt{\mathcal{E}_0}b_0(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0(p)$$
$$+ \sum_{k=0}^{K-1} \sqrt{\mathcal{E}_k} \sum_{i\in\Omega_k} b_k(p+i)\boldsymbol{S}_{k,i}\boldsymbol{\alpha}_k(p+i) \tag{16}$$

for the case of fading dispersive channels, respectively. In (15), $\boldsymbol{s}_0$ is the vector arising from projection of the waveform $s_{0,p}(t) = s_0(t - pT_b)$ onto the set (12) and, similarly, the vectors $\boldsymbol{s}_{k,i}$ are the projections of the waveforms $s_{k,p+i}(t)$. Alternatively, in (16), the matrices $\boldsymbol{S}_{k,i}$'s are $NM \times L$-dimensional matrices containing in their columns the projection vectors of the signals $s_{k,p+i}^0(t), \ldots, s_{k,p+i}^{L-1}(t)$ and $\boldsymbol{\alpha}_k(p+i)$ is the vector of the L tap weights of the kth user at epoch $p+i$.

D. Discrete-Time Model of the NBI

In order to further specialize the model (14), we now look closer at the expression of the NBI vector $\boldsymbol{i}(p)$ for the several previously considered models. Indeed, the effect of windowing and sampling on the NBI depends very much on its structure. Let us first assume that the NBI is a single digitally modulated signal whose signaling interval T_I is an integer submultiple of T_b. In this situation (see Fig. 1), the number of NBI symbols falling within the reference window is at most $(T_b/T_I + 1)$ and the digital interferer may be split up into $(T_b/T_I + 1)$ independent users with as many different signatures: the net effect, as discussed in [151] and [165], is a set of independent "virtual" users that can be incorporated
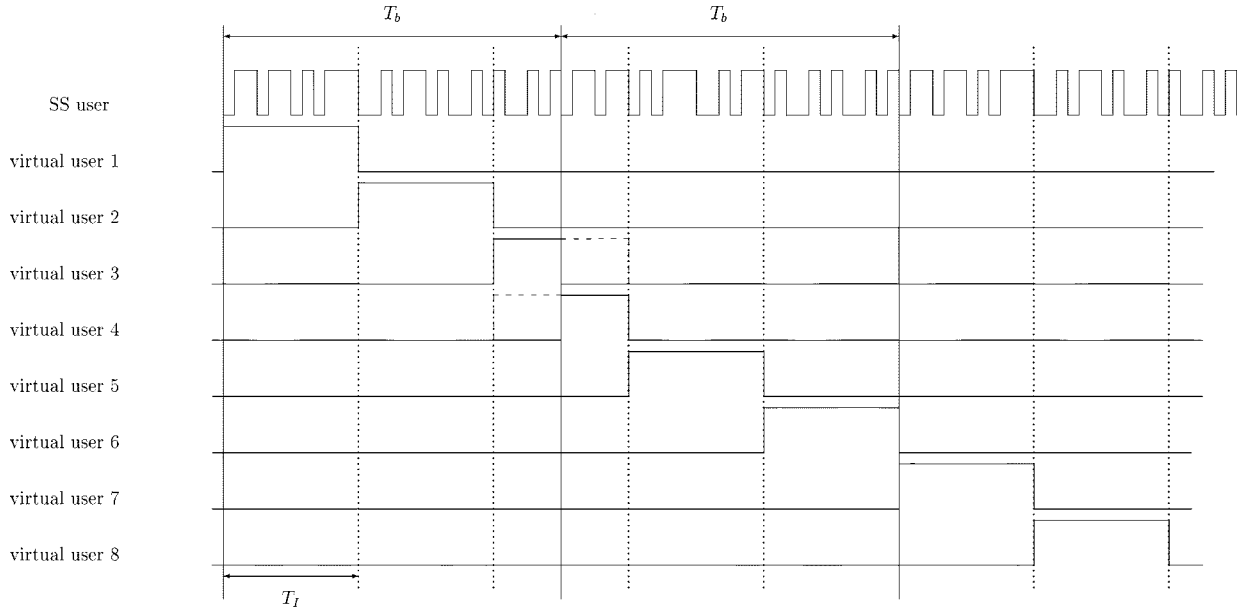
Fig. 2. System of one SS user and one digital narrow-band interferer with T_b/T_I; a rational number can be viewed as a virtual CDMA system. In this system, the digital NBI signal can be regarded as $m+1$ virtual users, each with its virtual time-dependent signature sequence. In this case, $T_b/T_I = 5/2$. Signatures corresponding to the virtual users are PTV with period 2. Note that virtual user 1 has the same signature as virtual user 7.

into the MAI. In particular, it is seen from Fig. 1 that the digital NBI is decomposed into $m+1$ virtual users (with $m = T_b/T_I$) having a signature whose support has a duration equal to T_I. It is also seen that, for proper values of the NBI delay τ_{I0}, the number of virtual users may decrease to m. In the more general situation that T_I is in a rational ratio with T_b, i.e., $T_b/T_I = q/r$, the situation is as illustrated in Fig. 2. The digital interferer can again be split into as many independent users as there are independent symbols falling within the reference window, but the corresponding virtual signatures are not the same and indeed change with p. This is a consequence of the anisochrony between the CDMA network and the narrow-band interferer, i.e., of the fact that a noninteger number of NBI periods fits into T_b. Thus, the same fictitious NBI signatures will occur after m epochs, where m is the smallest integer such that mT_b is an integer multiple of T_I. In particular, Fig. 2 refers to the case that $T_b/T_I = 5/2$ and that $\tau_{I0} = 0$. It is seen that, in each bit interval, the digital NBI is split into three virtual users. However, unlike the previous situation, the signatures of the virtual users differ in consecutive bit intervals, i.e., they depend on the temporal index p. Since $2T_b$ is an integer multiple of T_I, in this situation the signatures repeat every $m = 2$ intervals and this is indeed confirmed by the figure, wherein it is seen that the virtual user 7 has the same signature as the virtual user 1.

The above arguments hold true in the case of multiple anisochronous digitally modulated NBIs as well, where the virtual signatures' repetition period is dictated by the common period of the sets of the NBI and CDMA signaling intervals. The case that T_b/T_I is not rational will not be considered any further, although, at the analysis stage, it might deserve some attention if the impact of timing jitter on system performance is to be assessed. In particular, since

signaling rates (in bits per second) are almost always integer valued, the case in which T_b/T_I is irrational is of limited interest.

For the class of quasi-deterministic NBI, a fairly general model for $\boldsymbol{i}(p)$ is, thus, given by [24]

$$\boldsymbol{i}(p) = \sum_{k=0}^{K_I-1} \sqrt{\mathcal{E}_{Ik}} \alpha_{Ik} \sum_{i \in \Omega'_k(p)} b_{Ik}(p+i) \boldsymbol{v}_k^i(p) e^{j2\pi\nu_{Ik}pNM}$$

(17)

where

$$\Omega'_k(p) = \left\{ -\left(1 + \left\lceil \frac{-T_b + \tau_{Ik}(p)}{T_I} \right\rceil \right), \ldots, \right.$$
$$\left. -1, 0, 1, \ldots, \left\lceil \frac{T_b - \tau_{Ik}(p)}{T_I} \right\rceil \right\}$$

with

$$\tau_{Ik}(p) = T_I - \left((pT_b - \tau_{Ik}) - \left\lfloor \frac{pT_b - \tau_{Ik}}{T_I} \right\rfloor T_I \right)$$

and $\nu_{Ik} = f_{Ik}T_{OS}$.

Note that this model essentially reproduces the one adopted to model asynchronous MAI [see (2)]. Note also that, for sinusoidal tones, the above model simplifies to

$$\boldsymbol{i}(p) = \sum_{k=0}^{K_I-1} \sqrt{\mathcal{E}_{Ik}} \alpha_{Ik} e^{j\phi_{Ik}} T_{OS} \text{sinc}\,(\nu_{Ik}) \boldsymbol{g}_{Ik}(p) \quad (18)$$

with ϕ_{Ik} a proper phase term and[3]

$$\boldsymbol{g}_{Ik}(p) = \left[e^{j2\pi\nu_{Ik}(NMp)}, e^{j2\pi\nu_{Ik}(NMp+1)}, \ldots, \right.$$
$$\left. e^{j2\pi\nu_{Ik}(NMp+NM-1)} \right]^T. \quad (19)$$

---

[3]Throughout the paper, $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote transposition, conjugation and conjugate transposition, respectively.

If $f_{Ik}T_b$ is an integer for any $k = 0, \ldots, K_I - 1$, the above model reduces to [151], [152]

$$\boldsymbol{i}(p) = \sum_{k=0}^{K_I-1} \sqrt{\mathcal{E}_{Ik}} \alpha_{Ik} e^{j\phi_{Ik}} T_{OS} \text{sinc}\,(\nu_{Ik})\, \boldsymbol{g}_{Ik} \qquad (20)$$

with

$$\boldsymbol{g}_{Ik} = \left[1, e^{j2\pi\nu_{Ik}}, \ldots, e^{j2\pi(NM-1)\nu_{Ik}} \right]. \qquad (21)$$

Finally, let us consider the case that the NBI is an AR process. In this case, the NBI vector $\boldsymbol{i}(p)$ is obtained by stacking NM samples from epoch NMp to epoch $NM(p+1) - 1$ from the AR discrete-time process [151]

$$I_n = -\sum_{j=1}^{q} a_j I_{n-j} + \epsilon_n \qquad (22)$$

where q is the order of the AR process and ϵ_n is a white Gaussian sequence with variance σ_ϵ^2. The constants $\{a_j\}_{j=1}^{q}$ are the coefficients of the AR filter generating the process I_n.

E. Signal Representation—Second-Order Measures

Since the current paper is primarily concerned with linear receivers, it is convenient to exploit a subspace approach to signal representation. As a consequence, global second-order measures such as signal covariances are of primary concern. Strictly speaking, the covariance properties of the complex-valued random process $\boldsymbol{r}(p)$ should be given through the matrix pair

$$\begin{aligned} \boldsymbol{M_{rr}}(p) &= E\left\{ \boldsymbol{r}(p)\boldsymbol{r}^H(p) \right\} \\ &= \boldsymbol{M_{ss}}(p) + \boldsymbol{M_{ii}}(p) + 2\mathcal{N}_0 \boldsymbol{I}_{NM} \qquad (23) \\ \boldsymbol{M'_{rr}}(p) &= E\left\{ \boldsymbol{r}(p)\boldsymbol{r}^T(p) \right\} = \boldsymbol{M'_{ss}}(p) + \boldsymbol{M'_{ii}}(p) \qquad (24) \end{aligned}$$

where $\boldsymbol{M_{ii}}(p) = E\{\boldsymbol{i}(p)\boldsymbol{i}^H(p)\}$, $\boldsymbol{M_{ss}}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^H(p)\}$, $\boldsymbol{M'_{ss}}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^T(p)\}$, and, finally, $\boldsymbol{M'_{ii}}(p) = E\{\boldsymbol{i}(p)\boldsymbol{i}^T(p)\}$. Here and throughout the paper, $E\{\cdot\}$ denotes statistical expectation. The quantity (24) is known as the *pseudocovariance* of the complex-valued random process $\boldsymbol{r}(p)$ and, if it is identically zero, $\boldsymbol{r}(p)$ is said to be a *proper* process [133]. We will not give here further details on the conditions to be fulfilled for a process to be proper, i.e., to be characterized by just the first of the above covariances and refer the reader to [133] for a thorough discussion. What matters here is to recall that properness of the complex-valued baseband equivalent of a bandpass random process is ensured if and only if the original bandpass process is wide-sense stationary (WSS), in which case the pseudocovariance is identically zero. Deferring to subsequent sections the implications of a full (i.e., entailing both matrices) or a partial (i.e., entailing just the conventional covariance) exploitation of the covariance properties of the observables, it is worth noticing here that the two matrices $\boldsymbol{M_{ii}}(p)$ and $\boldsymbol{M'_{ii}}(p)$ may be time varying under certain NBI models. In particular, if the NBI consists of a digitally modulated signal, then, from (17), we have

$$\begin{aligned} \boldsymbol{M_{ii}}(p) &= \sum_{k=0}^{K_I-1} \mathcal{E}_{Ik} |\alpha_{Ik}|^2 \sum_{i \in \Omega'_k(p)} \boldsymbol{v}_k^i(p)\boldsymbol{v}_k^{iH}(p) \\ \boldsymbol{M'_{ii}}(p) &= \sum_{k=0}^{K_I-1} \mathcal{E}_{Ik}\alpha_{Ik}^2 \\ &\quad \cdot \sum_{i \in \Omega'_k(p)} \boldsymbol{v}_k^i(p)\boldsymbol{v}_k^{iT}(p) e^{j4\pi NM\nu_{Ik}p} \qquad (25) \end{aligned}$$

from which the following is readily seen.

1) If the digital NBI symbol interval is an integer sub-multiple of T_b *and* the frequency offset is zero, then the covariance and the pseudocovariance are time invariant. Moreover, they coincide if the signatures and the channel gains α_{Ik} are real.

2) If the digital NBI symbol interval is in a rational ratio with T_b, $\boldsymbol{M_{ii}}(p)$ is periodic in p with period m, while $\boldsymbol{M'_{ii}}(p)$ is periodic with period m, if $\nu_{Ik} = 0$. If the numerical frequencies ν_{Ik} are rational numbers, the sequence $e^{j4\pi\nu_{Ik}NMp}$ is itself periodic with period $r_{*k} = q_{3k}/(2\nu_{Ik}NM)$, where q_{3k} is the smallest integer such that r_{*k} is integer. As a consequence, the matrix sequence $\boldsymbol{M'_{ii}}(p)$ can also be regarded as being periodic with period $Q = \text{l.c.m.}(m, r_{*0}, \ldots, r_{*K_I-1})$, with l.c.m.$(\cdot, \cdot)$ denoting least common multiple. The case in which some of the ν_{Ik} are irrational, which, as noted previously, will not be considered further, gives rise to a polyperiodic matrix sequence $\boldsymbol{M_{ii}}(p)$ [53], [55].

Similar considerations also hold for the case in which the NBI consists of the sum of sinusoidal tones. In particular, under the model (18), the covariance matrices can be written as

$$\begin{aligned} \boldsymbol{M_{ii}}(p) &= \sum_{k=0}^{K_I-1} \mathcal{E}_{Ik} |\alpha_{Ik}|^2 \\ &\quad \cdot (T_{OS}\, \text{sinc}\,(\nu_{Ik}))^2\, \boldsymbol{g}_{Ik}(p)\boldsymbol{g}_{Ik}^H(p) \qquad (26) \\ \boldsymbol{M'_{ii}}(p) &= \sum_{k=0}^{K_I-1} \mathcal{E}_{Ik}\alpha_{Ik}^2 e^{j2\phi_{Ik}} \\ &\quad \cdot (T_{OS}\, \text{sinc}\,(\nu_{Ik}))^2\, \boldsymbol{g}_{Ik}(p)\boldsymbol{g}_{Ik}^T(p). \qquad (27) \end{aligned}$$

It is readily seen that, since the outer matrix product $\boldsymbol{g}_{Ik}(p)\boldsymbol{g}_{Ik}^H(p)$ is independent of p, the covariance matrix $\boldsymbol{M_{ii}}(p)$ is actually independent of the temporal index p, while, on the contrary, the pseudocovariance $\boldsymbol{M'_{ii}}(p)$ is again periodic with period $R = \text{l.c.m.}(r_{*0}, \ldots, r_{*K_I-1})$. If, instead, the simplified model (20) is adopted, both the covariance and the pseudocovariance are independent of the index p.

Finally, our discussion on the second-order moments of the NBI is concluded by considering the entropic AR model (22) of order q. In this case, due to the properness of Gaussian AR processes, the pseudocovariance is identically zero. As to the covariance matrix, a nice closed-form formula can

be given for its inverse rather than for the covariance matrix itself [151]. Indeed, upon defining the following $NM \times NM$-dimensional matrix:

$$
\boldsymbol{Q} = \begin{bmatrix}
1 & a_1 & a_2 & \dots & a_q & 0 & \dots & 0 \\
0 & 1 & a_1 & \dots & \dots & a_q & \dots & 0 \\
\vdots & 0 & \ddots & \dots & \dots & \dots & \ddots & 0 \\
\vdots & \vdots & \vdots & 1 & a_1 & \dots & \dots & a_q \\
\vdots & \vdots & \vdots & \vdots & 1 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \dots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \dots & \ddots & \vdots \\
0 & 0 & \dots & \dots & \dots & \dots & \dots & 1
\end{bmatrix}
$$
$$
= \begin{bmatrix} \boldsymbol{Q}_{1,1} & \boldsymbol{Q}_{1,2} \\ \boldsymbol{0} & \boldsymbol{I}_q \end{bmatrix} \tag{28}
$$

where $\boldsymbol{Q}_{1,1}$ and $\boldsymbol{Q}_{1,2}$ are a square matrix of order $NM - q$ and an $(NM - q) \times q$-dimensional matrix, respectively, and $\boldsymbol{I}_q$ is the identity matrix of order q, the inverse of the covariance matrix of the NBI $\boldsymbol{i}(p)$ can be written as

$$
\boldsymbol{M}_{\boldsymbol{ii}}^{-1}(p) = \frac{1}{\sigma_\epsilon^2} \begin{bmatrix} \boldsymbol{Q}_{1,1}^H \boldsymbol{Q}_{1,1} & \boldsymbol{Q}_{1,1}^H \boldsymbol{Q}_{1,2} \\ \boldsymbol{Q}_{1,2}^H \boldsymbol{Q}_{1,1} & \boldsymbol{Q}_{1,2}^H \boldsymbol{Q}_{1,2} + \sigma_\epsilon^2 \boldsymbol{R}_{\boldsymbol{i},q} \end{bmatrix} \tag{29}
$$

with $\boldsymbol{R}_{\boldsymbol{i},q}$ a $q \times q$-dimensional matrix depending on the correlation properties of the NBI [151]. However, since the covariance matrix $\boldsymbol{M}_{\boldsymbol{ii}}(p) = E\{\boldsymbol{i}(p)\boldsymbol{i}^H(p)\}$ is, for AR processes, a circulant matrix, its inverse is *persymmetric*, i.e., it is symmetric along its northeast-southwest diagonal [66], whence the southeast $q \times q$-dimensional block is uniquely determined by the northwest $q \times q$-dimensional block in (29). As a final remark on AR-type NBI, we notice that, unlike a digital NBI and/or sinusoidal tones, it always has a p-independent full-rank covariance matrix. As will be discussed later, this latter fact has consequences for the receiver properties in that it prevents any hope of coming up with a linear near–far resistant receiver.

Let us now move to the characterization of the second-order statistical properties of the CDMA multiplex. In the case of nonfading channels, the covariance matrix pair $\boldsymbol{M}_{\boldsymbol{ss}}(p)$ and $\boldsymbol{M}'_{\boldsymbol{ss}}(p)$ is actually independent of p, since

$$
\boldsymbol{M}_{\boldsymbol{ss}}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^H(p)\} = \mathcal{E}_0 |\alpha_0|^2 \boldsymbol{s}_0 \boldsymbol{s}_0^H
$$
$$
+ \sum_{k=1}^{K-1} \sum_{i \in \{-1,0\}} \mathcal{E}_k |\alpha_k|^2 \boldsymbol{s}_{k,i} \boldsymbol{s}_{k,i}^H, \tag{30}
$$

$$
\boldsymbol{M}'_{\boldsymbol{ss}}(p) = \mathcal{E}_0 \alpha_0^2 \boldsymbol{s}_0 \boldsymbol{s}_0^T + \sum_{k=1}^{K-1} \sum_{i \in \{-1,0\}} \mathcal{E}_k \alpha_k^2 \boldsymbol{s}_{k,i} \boldsymbol{s}_{k,i}^T. \tag{31}
$$

Alternatively, for fading channels, it is important to distinguish between the situation in which CCSI is not available, so that the covariance properties of the CDMA multiplex are known just in the *ensemble* of channel state realizations and that in which CCSI is available. In the former situation, the

knowledge that can be relied upon is the unconditional multiplex covariance matrix pair, i.e.,

$$
\boldsymbol{M}_{\boldsymbol{ss}}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^H(p)\} = \mathcal{E}_0 \boldsymbol{S}_{0,0} \boldsymbol{\Sigma}_0 \boldsymbol{S}_{0,0}^H
$$
$$
+ \sum_{k=0}^{K-1} \mathcal{E}_k \sum_{i \in \Omega_k} \boldsymbol{S}_{k,i} \boldsymbol{\Sigma}_k \boldsymbol{S}_{k,i}^H \tag{32}
$$
$$
\boldsymbol{M}'_{\boldsymbol{ss}}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^T(p)\} = 0. \tag{33}
$$

Implicit in the above equations is the customary assumption of modeling the fading channels' tap-weights vectors $\boldsymbol{\alpha}_k(\cdot)$ as realizations from proper WSS processes, i.e., such that the $L \times L$-dimensional matrices $\boldsymbol{\Sigma}_k = E\{\boldsymbol{\alpha}_k(p)\boldsymbol{\alpha}_k^H(p)\}$ are time-invariant matrices and $E\{\boldsymbol{\alpha}_k(p)\boldsymbol{\alpha}_k^T(p)\} = \boldsymbol{0}_{L,L}$. Accordingly, it readily follows that the matrix (32) is actually independent of p.

If CCSI is available, a more concise characterization of the multiplex covariance properties is given through the conditional quantities

$$
\boldsymbol{M}_{\boldsymbol{ss}|A}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^H(p)|A\}
$$
$$
= \mathcal{E}_0 \boldsymbol{S}_{0,0} \boldsymbol{\alpha}_0(p)\boldsymbol{\alpha}_0^H(p) \boldsymbol{S}_{0,0}^H
$$
$$
+ \sum_{k=0}^{K-1} \mathcal{E}_k \sum_{i \in \Omega_k} \boldsymbol{S}_{k,i} \boldsymbol{\alpha}_k(p+i)
$$
$$
\cdot \boldsymbol{\alpha}_k^H(p+i) \boldsymbol{S}_{k,i}^H \tag{34}
$$
$$
\boldsymbol{M}'_{\boldsymbol{ss}|A}(p) = E\{\boldsymbol{s}(p)\boldsymbol{s}^T(p)|A\}
$$
$$
= \mathcal{E}_0 \boldsymbol{S}_{0,0} \boldsymbol{\alpha}_0(p)\boldsymbol{\alpha}_0^T(p) \boldsymbol{S}_{0,0}^T
$$
$$
+ \sum_{k=0}^{K-1} \sum_{i \in \Omega_k} \mathcal{E}_k \boldsymbol{S}_{k,i} \boldsymbol{\alpha}_k(p+i)
$$
$$
\cdot \boldsymbol{\alpha}_k^T(p+i) \boldsymbol{S}_{k,i}^T \tag{35}
$$

where $A = [\boldsymbol{\alpha}_0(\cdot) \dots \boldsymbol{\alpha}_{K-1}(\cdot)]$. Notice that these quantities may be time varying in the case of time-varying fading realizations. Notice also that, conditioned upon these realizations, the multiplex becomes an improper process, whereas in the ensemble of the fading realizations it is proper. Moreover, it is worth pointing out that, assuming that both matrices (32) and (34) are rank deficient, the rank of the conditional covariance matrix (34) is approximately L times smaller than the rank of the unconditional covariance matrix (32). These facts, which are rather obvious, and, perhaps, of marginal theoretical interest, have some relevant practical implications, as will be explained in subsequent sections.

III. NBI SUPPRESSION IN NONFADING CHANNELS

We first consider the case in which the CDMA multiplex undergoes no channel distortion, so that it can be expressed as in (15), whereas the NBI vector is expressed through one of the models presented in Section II-D. Since the present paper is mainly concerned with *linear* receivers, the decision rule to detect the bit $b_0(p)$ will always be in the form

$$
\widehat{b}_0(p) = \text{sgn}\left[\Re\left\{\boldsymbol{d}^H(p)\boldsymbol{r}(p)\right\}\right] \tag{36}
$$

where $\Re\{\cdot\}$ denotes the real part and $\boldsymbol{d}(p)$ is a vector sequence to be suitably designed. Notice that implementing rule (36) requires NM complex multiplications and additions. Before proceeding to the receiver derivation in overlay channels, it is worth recalling briefly the basics of linear MUD with no NBI.

A. Linear MUD in the Absence of NBI

Let us consider the case where there is no NBI, in which the discrete-time version of the observables is expressed as in (14), with $\boldsymbol{i}(p) = 0$ and with the discrete-time version of the CDMA multiplex $\boldsymbol{s}(p)$ given by (15). The two most popular linear detectors, i.e., the MMSE and the ZF, or decorrelating, detector can be obtained by minimizing—possibly subject to some constraints—a risk, i.e., the average of a properly defined cost function. Indeed, an MMSE detector selects the vector $\boldsymbol{d}(p)$ so as to minimize the following risk [114]:

$$\boldsymbol{d}_{\mathrm{MMSE}}(p) = \arg\min_{\boldsymbol{d}} \mathcal{R}_{\mathrm{MMSE}}(\boldsymbol{d}, p)$$
$$= \arg\min_{\boldsymbol{d}} E\left\{\left|\boldsymbol{d}^H \boldsymbol{r}(p) - b_0(p)\right|^2\right\} \quad (37)$$

while, assuming a constant-envelope modulation, a ZF detector is obtained by solving (38) at the bottom of the page [205]. In order to solve problem (37), we notice that

$$\mathcal{R}_{\mathrm{MMSE}}(\boldsymbol{d}, p) = \boldsymbol{d}^H \boldsymbol{M}_{\boldsymbol{rr}} \boldsymbol{d} - 2\Re\left(\boldsymbol{d}^H E\left\{\boldsymbol{r}(p)b_0^*(p)\right\}\right) + 1$$

while the gradient of the above quantity with respect to $\boldsymbol{d}$ is written as

$$\nabla_{\boldsymbol{d}} \mathcal{R}_{\mathrm{MMSE}}(\boldsymbol{d}, p) = 2\boldsymbol{M}_{\boldsymbol{rr}}\boldsymbol{d} - 2E\left\{\boldsymbol{r}(p)b_0^*(p)\right\}$$
$$= 2\boldsymbol{M}_{\boldsymbol{rr}}\boldsymbol{d} - 2\sqrt{\mathcal{E}_0}\alpha_0 \boldsymbol{s}_0.$$

Setting to zero the above gradient and solving for $\boldsymbol{d}$ yields the MMSE solution

$$\boldsymbol{d}_{\mathrm{MMSE}} = \sqrt{\mathcal{E}_0}\alpha_0 \boldsymbol{M}_{\boldsymbol{rr}}^{-1} \boldsymbol{s}_0. \quad (39)$$

In order to solve the constrained optimization problem (38), we resort to Lagrangian techniques, i.e., we consider the functional

$$\mathcal{L}(\boldsymbol{d}, p, \xi) = E\left\{\left|\boldsymbol{d}^H(\boldsymbol{r}(p) - \boldsymbol{n}(p))\right|^2\right\} + \xi\left(\boldsymbol{d}^H \boldsymbol{s}_0 - 1\right)$$
$$= \boldsymbol{d}^H \boldsymbol{M}_1 \boldsymbol{d} + \xi\left(\boldsymbol{d}^H \boldsymbol{s}_0 - 1\right) \quad (40)$$

where

$$\boldsymbol{M}_1 = E\left\{(\boldsymbol{r}(p) - \boldsymbol{n}(p))(\boldsymbol{r}(p) - \boldsymbol{n}(p))^H\right\} = \boldsymbol{M}_{\boldsymbol{rr}} - 2\mathcal{N}_0 \boldsymbol{I}_{NM}$$

is the noiseless observable covariance matrix and ξ is the Lagrange multiplier. Taking the gradient of the functional $\mathcal{L}(\cdot, \cdot, \cdot)$ with respect to $\boldsymbol{d}$ and setting it to zero, we obtain

$$\nabla_{\boldsymbol{d}} \mathcal{L}(\boldsymbol{d}, p, \xi) = 2\boldsymbol{M}_1 \boldsymbol{d} + \xi \boldsymbol{s}_0 = \boldsymbol{0}.$$

Since $\boldsymbol{s}_0$ is in the span of the (rank-deficient) matrix $\boldsymbol{M}_1$, the above equation can be solved with respect to $\boldsymbol{d}$ yielding

$$\boldsymbol{d} = -\frac{\xi}{2}\boldsymbol{M}_1^\dagger \boldsymbol{s}_0$$

where $(\cdot)^\dagger$ denotes Moore–Penrose generalized inverse [73]. Finally, setting the Lagrange multiplier ξ so as to fulfill the constraint in (38), we obtain the solution for the ZF receiver

$$\boldsymbol{d}_{\mathrm{ZF}} = \frac{1}{\boldsymbol{s}_0^H \boldsymbol{M}_1^\dagger \boldsymbol{s}_0} \boldsymbol{M}_1^\dagger \boldsymbol{s}_0 \quad (41)$$

Notice that, if the useful signature $\boldsymbol{s}_0$ is not contained in the subspace spanned by the MAI, the ZF receiver (41) totally nullifies the MAI, whatever its power level may be. In contrast, the MMSE receiver (39) nullifies the MAI only in the limiting cases that the MAI power is increasingly large and/or the thermal noise floor $\mathcal{N}_0$ vanishes. In these circumstances, the vector $\boldsymbol{d}_{\mathrm{MMSE}}$ ends up proportional to that of the ZF receiver (41) [114]. These considerations do not imply that the ZF receiver exhibits improved BER performance with respect to the MMSE receiver. Indeed, the ZF receiver nullifies the projections of the received signal onto the subspace spanned by the MAI, independently of the MAI strength. Thus, when the levels of the useful signal and of the interferers are comparable, this strategy produces an unnecessary waste of useful energy or, equivalently, an unnecessary output-noise enhancement. The MMSE strategy, instead, seeks the least-interfered (in the mean square sense) direction, taking into account both the MAI and the noise. This can be interpreted as an attempt to use all of the useful signal energy by recovering it from both interfered and clear directions of the signal space. Interestingly, the MMSE strategy yields automatic nullification of the contributions from those directions where the interference strength is much larger than the useful signal strength, thus, precluding any hope of recovery. As a result, the MMSE receiver typically has better BER performance than the ZF receiver, except in the case of arbitrarily large MAI or vanishingly small noise in which the two receivers coincide (see [130]).

Notice also that the solutions (39) and (41) are also time invariant, a direct consequence of the fact that the cost functions in (37) and (38) depend on the observable covariance matrices, which have been previously shown to be independent of p when no NBI is present. This fact, in turn, implies

$$\begin{cases} \boldsymbol{d}_{\mathrm{ZF}}(p) = \arg\min_{\boldsymbol{d}} \mathcal{R}_{\mathrm{ZF}}(\boldsymbol{d}, p) = \arg\min_{\boldsymbol{d}} E\left\{\left|\boldsymbol{d}^H(\boldsymbol{r}(p) - \boldsymbol{n}(p))\right|^2\right\} \\ \text{subject to} \quad \boldsymbol{d}^H \boldsymbol{s}_0 = 1 \end{cases} \quad (38)$$
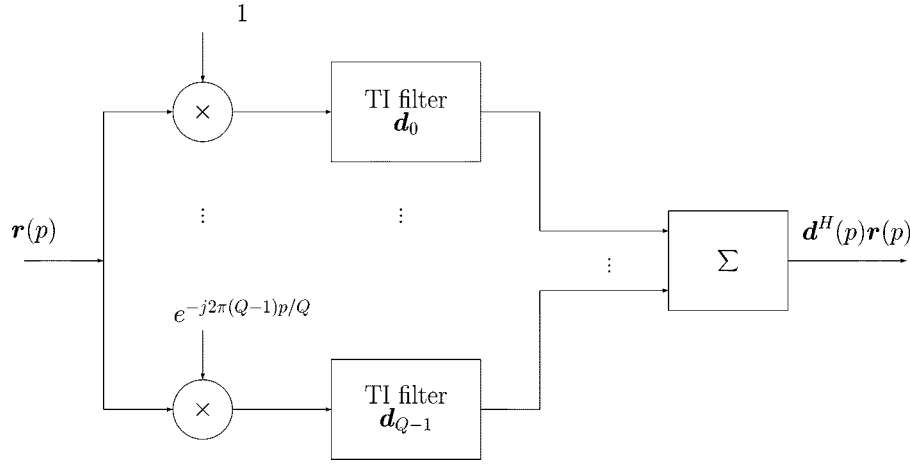
Fig. 3. FRESH implementation of a PTV filter.

that these vectors need to be computed only once at the beginning of the detection process.

It is also worth recalling here that, for any PSK modulation format, a receiver equivalent to the MMSE detector can be obtained by constrained minimization of the mean output energy (MOE), i.e., by solving (42) at the bottom of the page [71], [151], [205]. The acronym MMOE stands for minimum MOE. The solution to the above problem, which can be computed applying again Lagrangian techniques, is written as

$$d_{\text{MMOE}} = \frac{1}{s_0^H M_{\overline{rr}}^{-1} s_0} M_{\overline{rr}}^{-1} s_0. \tag{43}$$

Notice that d_{MMOE} is immediately seen to be proportional to (39) through a positive constant, which is inconsequential for data detection via (36).

B. Linear MUD Techniques in Overlay Channels

We now return to the issue of designing the vector $d(p)$ for the linear decision rule (36) in the presence of NBI. In such a situation, due to the possible dependence of the NBI covariance matrix on the temporal index p, the risks in (37) and (42) may be periodically time varying (PTV). Let us assume, indeed, that $M_{\overline{ii}}(p)$ [and $M_{\overline{rr}}(p)$] is PTV with period Q and let us focus our discussion on the MMSE case. Solving the problem (37) for each p leads to the following optimum vector sequence $d(p)$, which is PTV with period Q:

$$d_{\text{MMSE}}(p) = \alpha_0 \sqrt{\mathcal{E}_0} M_{\overline{rr}}^{-1}(p) s_0. \tag{44}$$

Obviously, if the NBI is stationary, i.e., if its covariance matrix is time invariant, then the solution (44) is time invariant, consistently with the results in [148], [151]–[153], and [165]. Notice also that, for PTV NBI, using (44) in the decision rule

(36) and exploiting the PTV nature of the solution yields the PTV decision rule

$$\widehat{b}_0(p) = \text{sgn}\left[\Re\left\{d^H(p \bmod Q) r(p)\right\}\right]. \tag{45}$$

A first conclusion that can be drawn from these result is that a nonstationary NBI induces a net increase in the system complexity. Indeed, (45) indicates that, unlike the time-invariant case, where just one NM-dimensional vector d is to be stored and used for all time intervals, in a more general scenario the receiver should store and cyclically employ Q such vectors, whereby the receiver memory requirements are Q times larger.[4] Notice also that, under certain values of the ratio T_b/T_I, the resulting value of the period Q could end up being unmanageably large. To cope with these situations, it appears necessary to simplify the solution (44), so as to be able to trade performance for complexity.

A first step toward devising suboptimum approximations to the PTV solution is to notice that the sequence $d_{\text{MMSE}}(p)$ can be expanded in a Fourier series as

$$d_{\text{MMSE}}(p) = \sum_{k=0}^{Q-1} d_k e^{j2\pi kp/Q}$$

$$\text{with } d_k = \frac{1}{Q}\sqrt{\mathcal{E}_0}\alpha_0 \sum_{p=0}^{Q-1} M_{\overline{rr}}^{-1}(p) e^{-j2\pi kp/Q} s_0. \tag{46}$$

As a consequence, the PTV MMSE filter admits the frequency shift (FRESH) implementation shown in Fig. 3 [23], [53], [55]. It is seen that a FRESH structure achieves a separation between the time-varying and the time-invariant sec-

---

[4]Conversely, the receiver computational burden is unaltered in this more general situation, since a single inner product, equivalent to NM complex multiplications and additions, is required in each symbol interval.

$$\begin{cases} d_{\text{MMOE}}(p) = \arg\min_d \mathcal{R}_{\text{MMOE}}(d,p) = \arg\min_d E\left\{\left|d^H r(p)\right|^2\right\} \\ \text{subject to } d^H(p) s_0 = 1 \end{cases} \tag{42}$$

tions of the PTV filters: the time-varying part is just a set of Q oscillators keyed to the Q harmonic frequencies, which form as many frequency-shifted versions of the observables; these frequency-shifted replicas finally undergo linear time-invariant filtering through the vectors $\boldsymbol{d}_k$, i.e., the Fourier-series expansion coefficients of the PTV sequence $\boldsymbol{d}_{\mathrm{MMSE}}(p)$. Such an implementation is advantageous under several points of view: first, it allows shedding some light in the operation of a PTV filter, which is thus reduced to a parallel bank of LTI filters and oscillators; next, it naturally suggests a viable means to come up with simplified structures, which can be obtained from the original FRESH filter by dropping some branches, chosen according to some criterion. Luckily enough such intuition matches quite well with theory, as will be discussed shortly, whereby the Fourier-expanded solution (46) and the consequent FRESH implementation of Fig. 3 will be the starting points for future discussions.

To illustrate further, it is convenient to reframe the optimization problem by considering, instead of a statistically averaged risk, the following time-averaged (TA) risk:

$$
\begin{aligned}
\mathcal{R}_{\mathrm{MMSE}}^{\mathrm{TA}} & (\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{Q-1}) \\
& = \left\langle E \left\{ \left| \boldsymbol{x}_P^H \boldsymbol{r}(p) - b_0(p) \right|^2 \right\} \right\rangle_Q \\
& = \frac{1}{Q} \sum_{i=0}^{Q-1} \left[\boldsymbol{x}_i^H \boldsymbol{M}_{\boldsymbol{rr}}(i) \boldsymbol{x}_i - 2\sqrt{\mathcal{E}_0} \Re \left\{ \alpha_0 \boldsymbol{x}_i^H \boldsymbol{s}_0 \right\} + 1 \right].
\end{aligned}
\tag{47}
$$

Indeed, since, for the case at hand, the risk $\mathcal{R}_{\mathrm{MMSE}}(\boldsymbol{d}, p)$ is PTV with respect to p and is always nonnegative, minimizing the mean-square error (37) for $p = 0, \ldots, Q-1$ is equivalent to minimizing the TA risk (47) with respect to $(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{Q-1}) \in \mathcal{C}^{QNM}$ ($\mathcal{C}$ denotes the complex field). Otherwise stated, the optimization problem is now reformulated in terms of determining a QNM-dimensional supervector obtained by stacking up the Q different NM-dimensional vectors $\boldsymbol{d}_{\mathrm{MMSE}}(0), \ldots, \boldsymbol{d}_{\mathrm{MMSE}}(Q-1)$ (i.e., an entire period of the PTV sequence $\boldsymbol{d}_{\mathrm{MMSE}}(p)$). In particular, through standard differentiation techniques, it can be shown that $\boldsymbol{d}_{\mathrm{MMSE}}(i) = \boldsymbol{x}_{*i}$, $i = 0, \ldots, Q-1$, if and only if $\mathcal{R}_{\mathrm{MMSE}}^{\mathrm{TA}}(\boldsymbol{x}_{*0}, \ldots, \boldsymbol{x}_{*Q-1})$ is the global minimum of the TA risk (47). Additionally, if we let $\boldsymbol{x}_0 = \cdots = \boldsymbol{x}_{Q-1}$, i.e., we constrain the solution to lie in a proper NM-dimensional subspace of $\mathcal{C}^{QNM}$, we obtain the so-called time-invariant solution

$$
\boldsymbol{d}_{\mathrm{MMSE}}^{\mathrm{TI}} = \sqrt{\mathcal{E}_0} \alpha_0 \left[\langle \boldsymbol{M}_{\boldsymbol{rr}}(p) \rangle_Q \right]^{-1} \boldsymbol{s}_0.
\tag{48}
$$

Likewise, any PTV solution containing a subset of the harmonic frequencies of (46) may be deemed as the result of the minimization of the TA risk when $(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{Q-1})$ is constrained to lie in a certain subset (which is a vector space) of $\mathcal{C}^{QNM}$. Notice that the time-invariant solution (48) only appears to be similar to the optimal (time-invariant) solution of the MMSE problem for stationary NBI. Indeed, while the latter represents a global minimum for the corresponding (p independent) risk function, the former is only a constrained minimum: it trades its simplicity for some loss, which, as

will be shown in the following, may be heavy—especially in overloaded networks or near–far scenarios—with respect to the optimum solution. A viable solution to cope with the system complexity for large Q appears at this point to be to take advantage of the modularity of the FRESH structure and to retain just a subset of P parallel branches, to be suitably chosen. As noted above, this still corresponds to substituting the point of unconstrained minimum with one of constrained minimum, but the corresponding MSE is progressively closer, for increasing P, to the MMSE. We will not dwell longer on this point; instead, we refer the interested reader to [24], where further details on how to derive suboptimal solutions are given.

Similar considerations apply now to the design of a ZF receiver, whose expression we give here without further comments as

$$
\boldsymbol{d}_{\mathrm{ZF}}(p) = \frac{1}{\boldsymbol{s}_0^H \boldsymbol{M}_1(p)^\dagger \boldsymbol{s}_0} \boldsymbol{M}_1^\dagger(p) \boldsymbol{s}_0
\tag{49}
$$

with $\boldsymbol{M}_1(p)$ the PTV covariance matrix of the noiseless observables. The time-invariant approximation of (49) is, thus, given as

$$
\boldsymbol{d}_{\mathrm{ZF}}^{\mathrm{TI}} = \frac{1}{\boldsymbol{s}_0^H \left[\langle \boldsymbol{M}_1(p) \rangle_Q \right]^\dagger \boldsymbol{s}_0} \left[\langle \boldsymbol{M}_1(p) \rangle_Q \right]^\dagger \boldsymbol{s}_0.
\tag{50}
$$

Finally, the vectors $\boldsymbol{d}(p)$ minimizing the MOE cost function (42) are written as

$$
\boldsymbol{d}_{\mathrm{MMOE}}(p) = \frac{1}{\boldsymbol{s}_0^H \boldsymbol{M}_{\boldsymbol{rr}}^{-1}(p) \boldsymbol{s}_0} \boldsymbol{M}_{\boldsymbol{rr}}^{-1}(p) \boldsymbol{s}_0
\tag{51}
$$

while the corresponding time-invariant approximation is given by

$$
\boldsymbol{d}_{\mathrm{MMOE}}^{\mathrm{TI}} = \frac{1}{\boldsymbol{s}_0^H \left[\langle \boldsymbol{M}_{\boldsymbol{rr}}(p) \rangle_Q \right]^{-1} \boldsymbol{s}_0} \left[\langle \boldsymbol{M}_{\boldsymbol{rr}}(p) \rangle_Q \right]^{-1} \boldsymbol{s}_0.
\tag{52}
$$

C. Modified Linear MUD Techniques in Overlay Channels

The detectors presented in the previous section depend on the covariance matrix of the observables, but ignore any additional information contained in the pseudocovariance. In other words, since the cost functions are, both for the ZF and for the MMSE, square moduli, the phase information contained in the matrix pair (23) and (24) is neglected. This fact does not have any consequences when the pseudocovariance is zero, i.e., when the received signal is a proper complex random process. However, in the situation of nonzero pseudocovariance, these receivers do not account for such additional information at all and are, thus, possibly suboptimal. Situations in which the baseband equivalent of the CDMA multiplex and/or the NBI are improper complex random processes are described in [22], [24], [27], [223]. To examine this issue, consider the decision rule (36) and let us focus on the MMSE first. Since the symbol to be estimated is a

real-valued quantity and indeed its estimate relies on hard limiting a real part, a better estimator than the conventional one, which minimizes the cost function (37), is obtained by solving the following minimization problem:

$$\min_{\boldsymbol{d}} \mathcal{R}'_{\text{MMSE}}(\boldsymbol{d}, p) = \min_{\boldsymbol{d}} E\left\{ \left(\Re\left\{ \boldsymbol{d}^H \boldsymbol{r}(p) \right\} - b_0(p) \right)^2 \right\} \quad (53)$$

which essentially forces the estimator to be real. Likewise, the problems to be solved for ZF and MMOE optimization criteria are defined as

$$\begin{cases} \min_{\boldsymbol{d}} \mathcal{R}'_{\text{ZF}}(\boldsymbol{d}, p) = \min_{\boldsymbol{d}} E\left\{ \left(\Re\left\{ \boldsymbol{d}^H (\boldsymbol{r}(p) - \boldsymbol{n}(p)) \right\} \right)^2 \right\} \\ \text{subject to} \quad \Re\left\{ \boldsymbol{d}^H \boldsymbol{s}_0 \right\} = 1 \end{cases} \quad (54)$$

and

$$\begin{cases} \min_{\boldsymbol{d}} \mathcal{R}'_{\text{MMOE}}(\boldsymbol{d}, p) = \min_{\boldsymbol{d}} E\left\{ \left(\Re\left\{ \boldsymbol{d}^H \boldsymbol{r}(p) \right\} \right)^2 \right\} \\ \text{subject to} \ \Re\left\{ \boldsymbol{d}^H(p) \boldsymbol{s}_0 \right\} = 1 \end{cases} \quad (55)$$

Deferring to [24] and [27] for the analytical details, the solutions to the above problems can be given by defining the following augmented vectors:

$$\boldsymbol{r}_a(p) = \begin{bmatrix} \boldsymbol{r}(p) \\ \boldsymbol{r}^*(p) \end{bmatrix}, \quad \boldsymbol{d}_a(p) = \frac{1}{2} \begin{bmatrix} \boldsymbol{d}(p) \\ \boldsymbol{d}^*(p) \end{bmatrix}$$

$$\text{and } \boldsymbol{s}_a = \begin{bmatrix} \boldsymbol{s}_0 \alpha_0 \\ \boldsymbol{s}_0 \alpha_0^* \end{bmatrix} \quad (56)$$

as

$$\boldsymbol{d}_{a,\text{MMSE}}(p) = \sqrt{\mathcal{E}_0} \boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}^{-1}(p) \boldsymbol{s}_a \quad (57)$$

for MMSE

$$\boldsymbol{d}_{a,\text{ZF}}(p) = \frac{1}{\boldsymbol{s}_a^H \boldsymbol{M}_{a,1}(p)^\dagger \boldsymbol{s}_a} \boldsymbol{M}_{a,1}^\dagger(p) \boldsymbol{s}_a \quad (58)$$

for ZF and

$$\boldsymbol{d}_{a,\text{MMOE}}(p) = \frac{1}{\boldsymbol{s}_a^H \left[\boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(p) \right]^{-1} \boldsymbol{s}_a} \left[\boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(p) \right]^{-1} \boldsymbol{s}_a \quad (59)$$

for MMOE, respectively. Notice that taking the first NM elements of these augmented vectors (possibly divided by two) yields the solutions to the respective optimization problems. Implementing the decision rule, thus, still requires NM complex multiplications and additions in each symbol interval. In the above formulas, we have also introduced the covariance matrix

$$\boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(p) = E\left\{ \boldsymbol{r}_a(p) \boldsymbol{r}_a^H(p) \right\} = \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{r}\boldsymbol{r}}(p) & \boldsymbol{M}'_{\boldsymbol{r}\boldsymbol{r}}(p) \\ \boldsymbol{M}'^*_{\boldsymbol{r}\boldsymbol{r}}(p) & \boldsymbol{M}^*_{\boldsymbol{r}\boldsymbol{r}}(p) \end{bmatrix} \quad (60)$$

as well as the matrix $\boldsymbol{M}_{a,1}(p)$, the covariance matrix of the noiseless augmented observables, which again can be easily expressed in terms of the covariance and pseudocovariance matrices of the observables.

The above solutions deserve some comment. First, notice that they do explicitly depend upon both covariance matrices of the observables: indeed, requiring that the cost functions assume the forms (53)–(55) implies exploiting the phase informations of the observables and this fact is mirrored by the structure of the solutions. Following along the same lines outlined in the previous section, it can be readily seen that, if the observable covariance matrix is time invariant, then the above solutions are time invariant as well. However, even in this situation full equivalence with the previous receivers is not achieved, unless the CDMA multiplexed signals *and* the NBI are proper processes. On the other hand, it has been demonstrated, although in the context of MUD with no NBI, that phase-asynchronous linear systems can take advantage of the phase information by exploiting the pseudocovariance of the observables [22], [27], [223]. The above equations also emphasize a relevant aspect when the NBI is a digitally modulated biphase shift keying (BPSK) signal with frequency offset. In particular, in this situation, all of the three solutions are still PTV, but now the period is the smallest common period between the covariance and the pseudocovariance of the NBI. This obviously represents a net complexity increase and indeed the modified solutions have a FRESH implementation, similar to that depicted in Fig. 3, with the relevant difference being that the period is now $Q = $ l.c.m.$(m, r_{*0}, \ldots, r_{*K_I-1})$, which may be intolerably large. In this case as well, however, upon definition of proper TA modified risks, it can be shown that:

1) the new cost functions admit a unique global minimum, i.e., the PTV solution with period Q and all of the nonzero harmonics thereof;
2) any PTV solution of period Q with a smaller number of harmonic frequencies represents a constrained minimum;
3) among constrained minima, we find the stationary (i.e., time-invariant) solution.

Thus, in this situation, the properties of the TA modified risks give more than a hint about how to trade complexity for performance, i.e., by retaining just a small subset of the branches of the FRESH structure. A thorough discussion and further details on this approach and on the issue of devising suboptimal reduced-complexity receiving structures are found in [24].

D. Performance Analysis

The usual measures employed to assess the performance of CDMA receivers are the BER, the output signal-to-interference ratio (SIR) and the near–far resistance [187].

With regard to the system BER, it is easy to show that, given the symbol $b_0(p)$ and the realization of the interference vector $\boldsymbol{z}(p)$, the test statistic in the decision rule (36) is a Gaussian random variable, whereby the conditional system BER in the pth signaling interval can be written as

$$P_p\left(e | \boldsymbol{z}(p)\right) = \frac{1}{2} \text{erfc} \left[\frac{\Re\left\{ \sqrt{\mathcal{E}_0} \alpha_0 \boldsymbol{d}_{(\cdot)}^H(p) \boldsymbol{s}_0 + \boldsymbol{d}_{(\cdot)}^H(p) \boldsymbol{z}(p) \right\}}{\sqrt{2\mathcal{N}_0 \left\| \boldsymbol{d}_{(\cdot)} \right\|^2}} \right] \quad (61)$$

with erfc$(\cdot)$ the complementary error function. Of course, the unconditional BER may be obtained by averaging the above
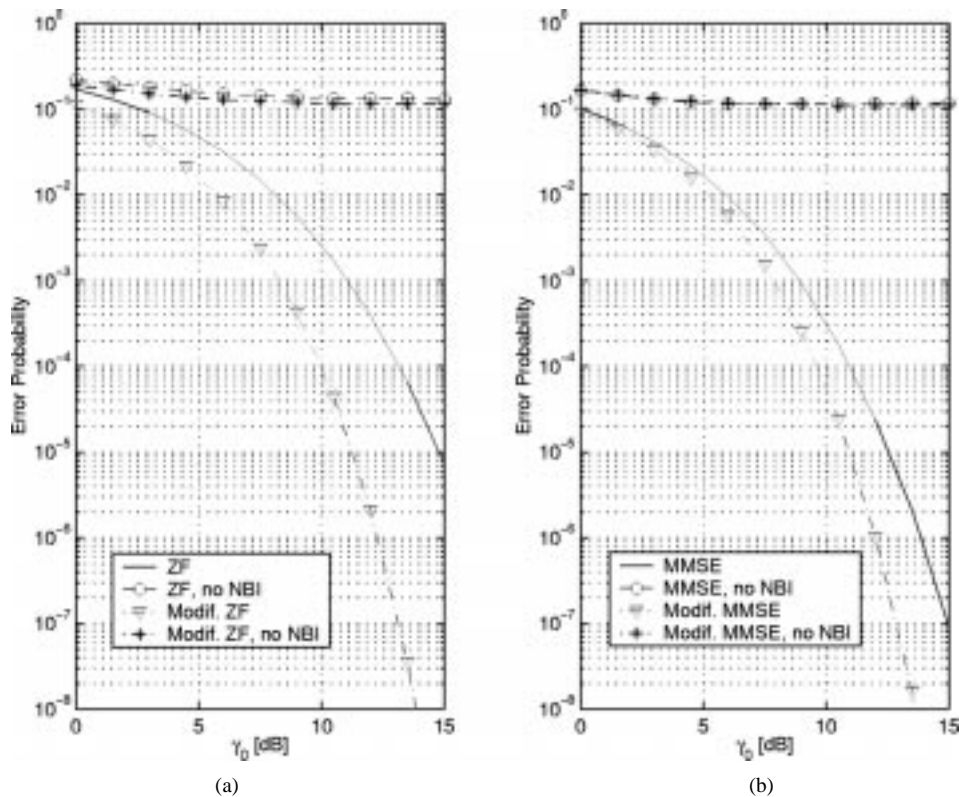
Fig. 4. Error probabilities for linear and modified multiuser receivers. (a) ZF receivers. (b) MMSE receivers. There are $K = 7$ equi-energy users and the NBI is a digital signal with $T_I = T_b/3$ having a 15-dB power advantage over the CDMA signals.

expression with respect to the interference and with respect to the possible periodicity of the decision rule, i.e.,

$$P(e) = \frac{1}{Q} \sum_{p=0}^{Q-1} E_{\boldsymbol{z}(p)} \{P_p(e|\boldsymbol{z}(p))\}. \tag{62}$$

In particular, evaluating the expectation over the interference $\boldsymbol{z}(p)$ requires averaging over the MAI information bits, as well as, in case of digital NBI, over the NBI information bits. Since such an average is usually computationally burdensome, the customary approach is to resort to a semianalytical computer-aided procedure, i.e., to average expression (61) with respect to only a (randomly generated) subset of all of the possible symbol realizations. If, instead, the NBI signal is a Gaussian AR process, then it is possible to determine an analytical expression for the conditional error probability given only the MAI realization, which of course also in this case must be averaged through the semianalytical procedure in order to estimate the unconditional system BER.

In Fig. 4, we show the BER for the MMSE and ZF multiuser detectors [(48) and (50)] and for their modified versions derived in the previous section versus the received energy contrast $\gamma_0 = (\mathcal{E}_0|\alpha_0|^2)/(2\mathcal{N}_0)$, expressed in decibels. Also, we show the performance of the same receivers when they do not account for the presence of the NBI, i.e., of the same receivers designed under the assumption that no NBI is present. The performance of these latter receivers, which suppress the MAI only, gives a measure of the BER

that conventional multiuser receivers may achieve when operating in an overlay architecture. The results shown correspond to an asynchronous DS/CDMA system with processing gain $N = 31$, $K = 7$ users and with no oversampling ($M = 1$). The NBI is a single digital NBI signal having a 15-dB power advantage over the SS signals, which are assumed to be equienergy. The ratio T_b/T_I is equal to three, so that the corresponding multiuser detectors are TI. In Fig. 5, the same scenario as in Fig. 4 is considered, with the exception that the NBI has now only 5-dB power advantage on the CDMA signals. The results of both figures confirm that conventional multiuser detectors, not accounting for the presence of the NBI, are not suited for operation in overlay architectures. As expected, comparing Figs. 4 and 5 confirms that the larger the NBI power, the poorer the performance of conventional systems. On the contrary, accounting for both MAI and NBI at the design level results in reliable communications and, ultimately, in satisfactory network capacity. Also, it is seen that the modified receivers, which explicitly take into account the phase information contained in the observable pseudocovariance, achieve sharp performance improvement over their conventional counterparts. Finally, results also confirm the well-known superiority of the MMSE approach with respect to the ZF one [187]. The merits of PTV versus TI receivers as the NBI exhibits PTV covariance properties are highlighted by Fig. 6 (taken from [24]), which corresponds to an asynchronous CDMA system with processing gain $N = 7$, $K = 5$ users and with a digital NBI interferer having $T_I = 5T_c$ and $f_I T_c = 3$, so that the receiver period is
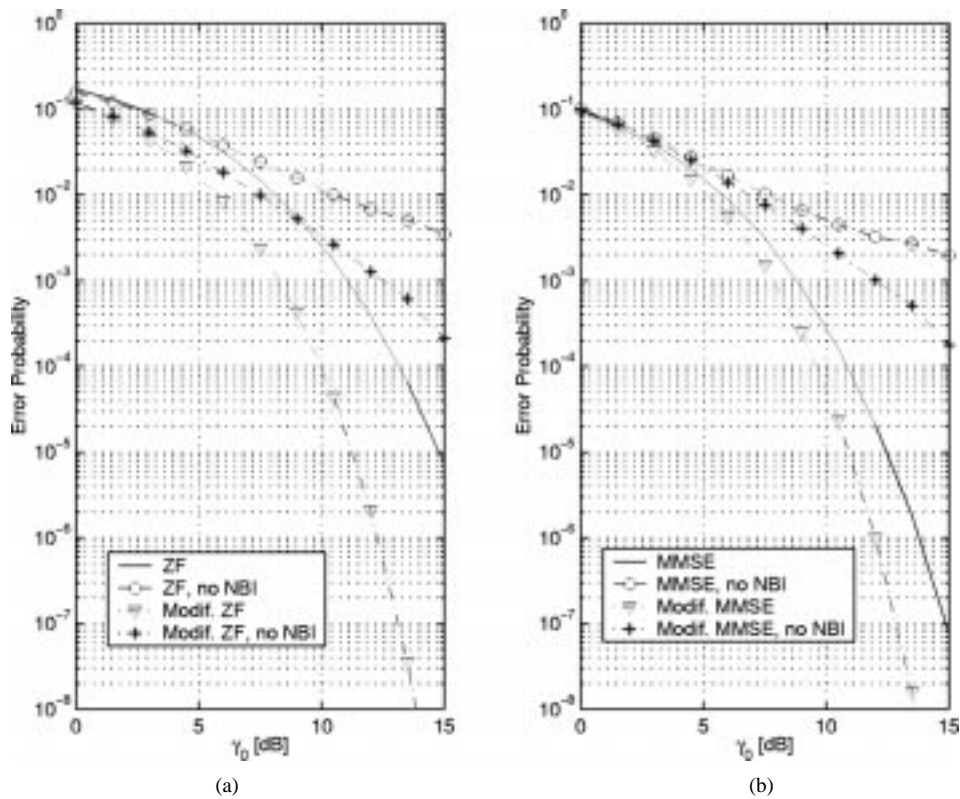
Fig. 5. Error probabilities for linear and modified multiuser receivers. (a) ZF receivers. (b) MMSE receivers. There are $K = 7$ equi-energy users and the NBI is a digital signal with $T_I = T_b/3$ having a 5-dB power advantage over the CDMA signals.
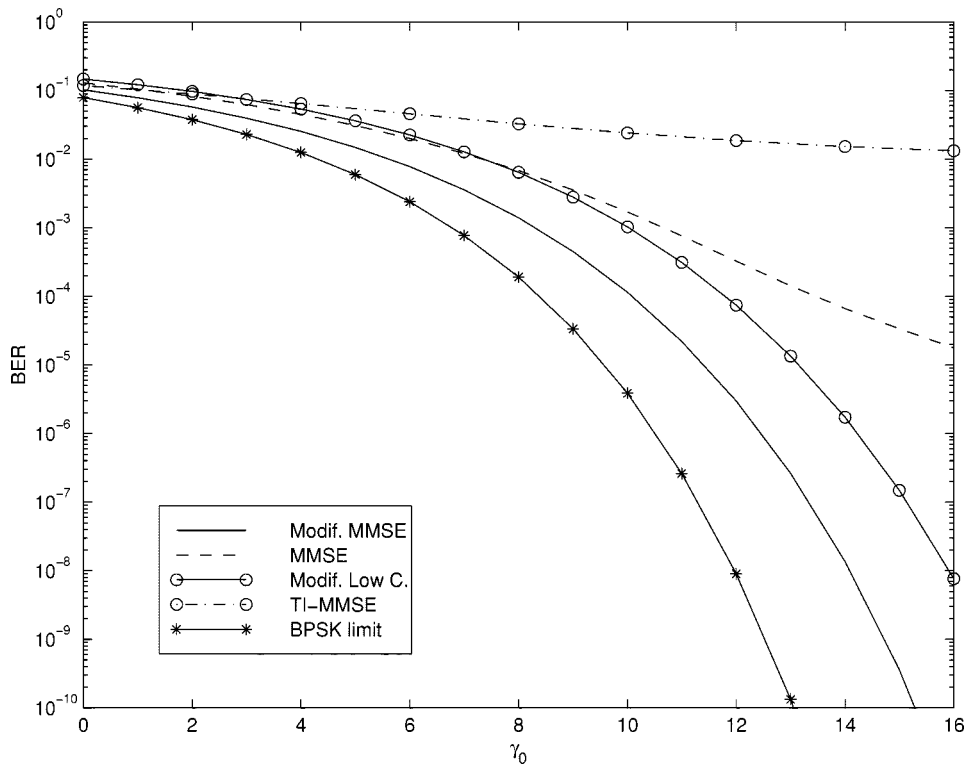

Fig. 6. Error probability for several multiuser detectors. Considered system has processing gain $N = 7$, $K = 5$ users and the interferer is a digital NBI having $T_I = 5T_c$ and $f_I T_c = 3$.

$Q = 15$. In Fig. 6, the zeroth user BER versus γ_0 is reported for the TI-MMSE receiver (48), the PTV MMSE receiver and its modified version, and a low-complexity approximation of the modified PTV MMSE receiver (for details on how such

a receiver is designed, we refer the reader to [24]). Also, for the sake of comparison, in the same figure the BER corresponding to an uncoded BPSK-modulated transmission over a single-user additive white Gaussian noise (AWGN) channel is reported. The results clearly show that, in PTV scenarios, adopting the TI solutions may lead to heavy performance degradation. Once again, it is seen that the modified receivers outperform their classical counterparts.

Besides the BER, another relevant performance measure for multiuser receivers is the output SIR, which we define here as [151]

$$\text{SIR}(p) = \frac{E^2\left\{\boldsymbol{r}^H(p)\boldsymbol{d}(p)|b_0(p)\right\}}{\text{var}\left\{\boldsymbol{r}^H(p)\boldsymbol{d}(p)\right\}}. \tag{63}$$

This quantity is also often referred to as the signal-to-interference-plus-noise ratio. Obviously, in the presence of PTV receivers, the above measure is actually dependent on the epoch p, whereby a more meaningful performance measure is its time average over the period. In (63), the vector $\boldsymbol{d}(p)$ has been purposely left unspecified and indeed it may be given by any of the solutions found in the previous sections. Note that MMOE and MMSE approaches, yielding proportional projection vectors, necessarily result in the same SIR value.

Leaving aside the mathematical details, which are again reported in [151], it can be shown that, for the MMSE and the MMOE receivers, the TA SIR is given by

$$\begin{aligned} \text{SIR}_{\text{MMSE}} =& \langle \text{SIR}(p) \rangle \\ =& \frac{1}{Q}\mathcal{E}_0\left|\alpha_0\right|^2 \\ &\cdot \sum_{p=0}^{Q-1} \boldsymbol{s}_0^H \left(\sum_{k=0}^{K-1} \sum_{i\in\{-1,0\}} \mathcal{E}_k\left|\alpha_k\right|^2 \boldsymbol{s}_{k,i}\boldsymbol{s}_{k,i}^H \right. \\ &\left. + \boldsymbol{M}_{\boldsymbol{i}\boldsymbol{i}}(p) + 2\mathcal{N}_0\boldsymbol{I}_{NM} \right)^{-1} \boldsymbol{s}_0 \end{aligned} \tag{64}$$

where, again, the PTV nature of the instantaneous SIR results from the possibly time-varying structure of the NBI covariance matrix.

Under the ZF design criterion, the instantaneous SIR is easily calculated provided that the useful signature $\boldsymbol{s}_0$ does not belong to the subspace spanned by the overall cochannel interference (i.e., the sum of the NBI, MAI, and ISI). Indeed, under these conditions, the ZF solution (49) can be shown, through standard MUD techniques, to be reduced to an orthogonal projector onto the orthogonal complement to the subspace spanned by the said cochannel interference. The SIR is, thus, given by

$$\text{SIR}_{\text{ZF}} = \left\langle \frac{\mathcal{E}_0\left|\alpha_0\right|^2 \left\|\boldsymbol{s}_0^\perp(p)\right\|^2}{2\mathcal{N}_0} \right\rangle_Q \tag{65}$$

where $\boldsymbol{s}_0^\perp(p)$ denotes the component of the signature of the desired user in the said orthogonal complement. More precisely, the vector $\boldsymbol{s}_0^\perp(p)$ is expressed as

$$\boldsymbol{s}_0^\perp(p) = \left(\boldsymbol{I}_{NM} - \boldsymbol{\Phi}(p)\boldsymbol{\Phi}^H(p)\right)\boldsymbol{s}_0 \tag{66}$$

where $\boldsymbol{\Phi}(p)$ is a matrix containing in its columns an orthonormal basis for the subspace spanned by the cochannel interference. Obviously, if the interference subspace happens to be PTV, then so are the matrices $\boldsymbol{\Phi}(p)$ and, hence, the sequence of the instantaneous projections $\boldsymbol{s}_0^\perp(p)$ onto the orthogonal complement thereof. In the alternative situation that either the cochannel interference spans the whole space C^{NM} or $\boldsymbol{s}_0$ belongs to such a subspace, then, upon denoting by $\boldsymbol{z}(p)$ the overall cochannel interference, i.e., the superposition of MAI and NBI, the SIR is written trivially as

$$\text{SIR}_{\text{ZF}} = \left\langle \frac{\mathcal{E}_0\left|\alpha_0\right|^2 \left\|\boldsymbol{d}_{\text{ZF}}^H(p)\boldsymbol{s}_0\right\|^2}{\boldsymbol{d}_{\text{ZF}}^H(p)\boldsymbol{M}_{\boldsymbol{z}+\boldsymbol{n},\boldsymbol{z}+\boldsymbol{n}}(p)\boldsymbol{d}_{\text{ZF}}(p)} \right\rangle_Q \tag{67}$$

where we have denoted by $\boldsymbol{M}_{\boldsymbol{z}+\boldsymbol{n},\boldsymbol{z}+\boldsymbol{n}}(p) = E\{(\boldsymbol{z}(p) + \boldsymbol{n}(p))(\boldsymbol{z}(p) + \boldsymbol{n}(p))^H\}$ the covariance matrix of the overall disturbance $\boldsymbol{z}(p) + \boldsymbol{n}(p)$, i.e., MAI plus thermal noise.

Let us now move to the case of the modified MMSE receivers. Formally, the SIR expressions for the MMSE and the ZF strategies parallel those for the conventional MMSE and ZF receivers. To illustrate this further, let us denote by $\boldsymbol{z}_a(p) = [\boldsymbol{z}^T(p)\boldsymbol{z}^H(p)]^T$ the augmented version of the MAI vector $\boldsymbol{z}(p)$. Notice that the overall interference is $\boldsymbol{z}(p)+\boldsymbol{n}(p)$ and its augmented version is $\boldsymbol{z}_a(p)+\boldsymbol{n}_a(p)$, where, due to the noise properness, the covariance matrix of $\boldsymbol{n}_a(p)$ is nonzero only in its northwestern and southeastern NM-dimensional square blocks, which are given by $2\mathcal{N}_0\boldsymbol{I}_{NM}$. Denoting by $\boldsymbol{M}_{\boldsymbol{z}_a+\boldsymbol{n}_a,\boldsymbol{z}_a+\boldsymbol{n}_a}(p)$ the covariance matrix of $\boldsymbol{z}_a(p) + \boldsymbol{n}_a(p)$ and by $\boldsymbol{s}_a^\perp(p)$ the component of $\boldsymbol{s}_a$ in the orthogonal complement of the range span of $\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a}$, i.e., the component of $\boldsymbol{s}_a$, which is free of cochannel interference, the SIRs for the modified MMSE and modified ZF receivers are given by

$$\begin{aligned} \text{SIR}_{\text{MMMSE}} ==& \frac{1}{Q}\mathcal{E}_0\left|\alpha_0\right|^2 \\ &\cdot \sum_{p=0}^{Q-1} \boldsymbol{s}_a^H \boldsymbol{M}_{\boldsymbol{z}_a+\boldsymbol{n}_a,\boldsymbol{z}_a+\boldsymbol{n}_a}^{-1}(p)\boldsymbol{s}_a \end{aligned} \tag{68}$$

$$\text{SIR}_{\text{MZF}} = \left\langle \frac{\mathcal{E}_0\left|\alpha_0\right|^2 \left\|\boldsymbol{s}_a^\perp(p)\right\|^2}{2\mathcal{N}_0} \right\rangle. \tag{69}$$

No simple analytical way to establish a relationship between these measures and the ones of the conventional receivers is available, but the results shown in Fig. 7 again confirm the superiority of the modified versions, as expected. In particular, this figure shows the SIR versus the number of users at $\gamma_0 = 14$ dB, with an NBI having a power advantage of 15 dB and for a system with processing gain $N = 31$ and oversampling factor $M = 2$. Also, notice that the period Q appearing in (68) and (69) may end up noticeably larger than that of (64) and (65), as Q represents now the common period of the NBI covariance and pseudocovariance.

The last relevant performance measure, the near–far resistance [187], characterizes a system's capability to suppress interference in the absence of power control. This concept can be extended to the NBI case as well [151]. Indeed, assuming that a ZF receiver is employed (the equivalence of MMSE and ZF receivers under near–far scenarios allows us
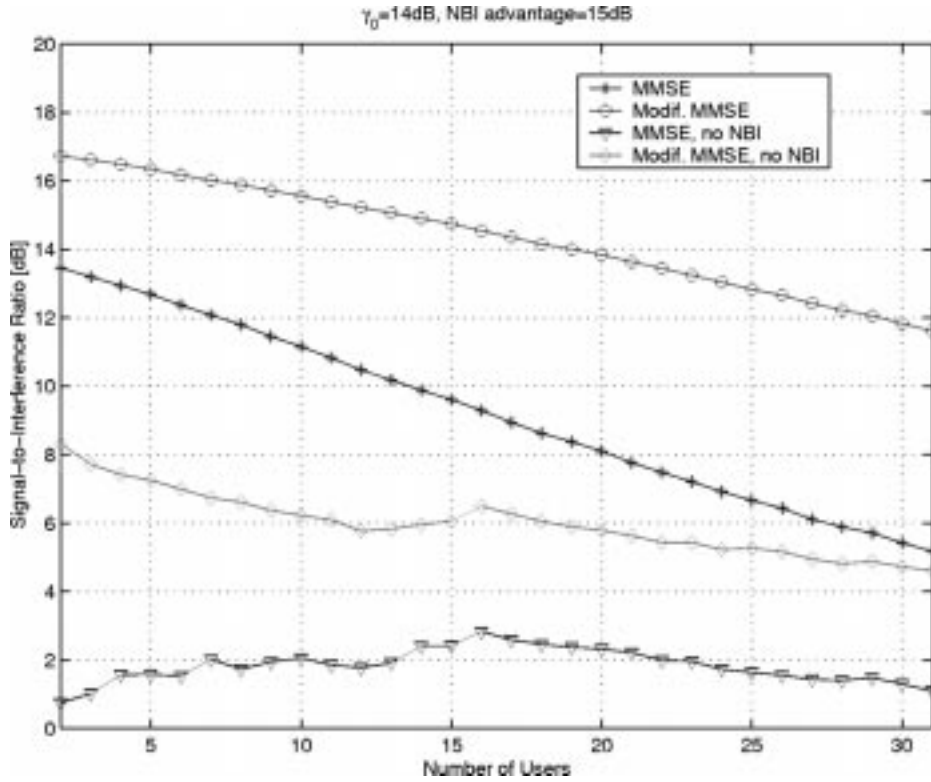
$\gamma_0 = 14\mathrm{dB}$, NBI advantage=15dB

Fig. 7. SIR versus the number of users at $\gamma_0 = 14$ dB. CDMA system has processing gain $N = 31$; the oversampling factor M is equal to 2. NBI has a power advantage of 15 dB over the CDMA signals.

to focus on either receiver), if, for all p, s_0 does not belong to the subspace spanned by the covariance matrix of $z(p)$,[5] then the inner product $d_{\mathrm{ZF}}^H(p)z(p)$ is zero $\forall p$ and the system is near–far resistant with respect to both the NBI and the MAI. If, instead, this is not the case, then the system is *not* near–far resistant. In particular, a general formula for the near–far resistance under PTV NBI and MAI is [24]

$$\eta = \min_{0 \leq p \leq Q-1} \frac{\left\| s_0^\perp(p) \right\|^2}{\left\| s_0 \right\|^2}. \tag{70}$$

At this point, the price to be paid for adopting suboptimal rather than optimal systems should clearly emerge. Indeed, let us assume, for a moment, that a TI solution is employed under a PTV scenario. In this situation, it can be shown that the near–far resistance is given by

$$\eta^{\mathrm{TI}} = \frac{\left\| s_0^\perp \right\|^2}{\left\| s_0 \right\|^2} \tag{71}$$

where now $s_0^\perp$ is the projection of s_0 onto the orthogonal complement of the subspace spanned by the TA cochannel interference covariance matrix $\langle M_{zz}(p) \rangle$. Since the average does not operate on the time-invariant covariance of the MAI, but only on the PTV matrix of the NBI, we can restrict our attention to this latter matrix. On the other hand, it is well known that, since $M_{ii}(p)$ is a sequence of nonnegative definite matrices, the TA matrix $\langle M_{ii}(p) \rangle$ has a rank not smaller than that of each summand and may end up with having full

---

[5]Notice that a necessary condition for this to happen is that the covariance matrix of $z(p)$ is singular for all p.

rank. As a consequence, adopting a TI solution could lead to a larger noise-enhancement if not, under certain circumstances, to complete signal space saturation and to the nullification of the near–far resistance.

Similar considerations can be given for the case of the modified ZF and MMSE receivers. Formally, the near–far resistance of the modified strategies is given by

$$\eta_M = \min_{0 \leq p \leq Q-1} \frac{\left\| s_a^\perp(p) \right\|^2}{\left\| s_a \right\|^2}. \tag{72}$$

Now, it can be shown that this measure cannot be smaller than (71), since the vectors in the null space of the matrix $M_{zz}(p)$ can be shown to result in augmented vectors $z_a(p)$ in the null space of the matrix $M_{z_a z_a}$, while the converse statement is not true. Indeed, this result is experimentally confirmed by Fig. 8, which reveals that the near–far resistance of the modified receivers is noticeably larger than that of the conventional ones. In particular, this figure shows the near–far resistance of the classical and modified receivers versus the number of users, for an asynchronous (the results are averaged over 500 random delay realizations) system with $N = 31$ and for several values of the oversampling ratio. The NBI is here again a digital signal with $T_b/T_I = 3$. It is also seen that moderate values of the oversampling factor may yield a remarkable increase in the system capacity.

Obviously, there is a price to be paid for adopting the modified strategies, which indeed are *less robust* than the conventional ones, namely, in keeping with a well-known principle of signal detection, they trade robustness for optimality. Indeed, for modified receivers too it is possible to use a time-in-
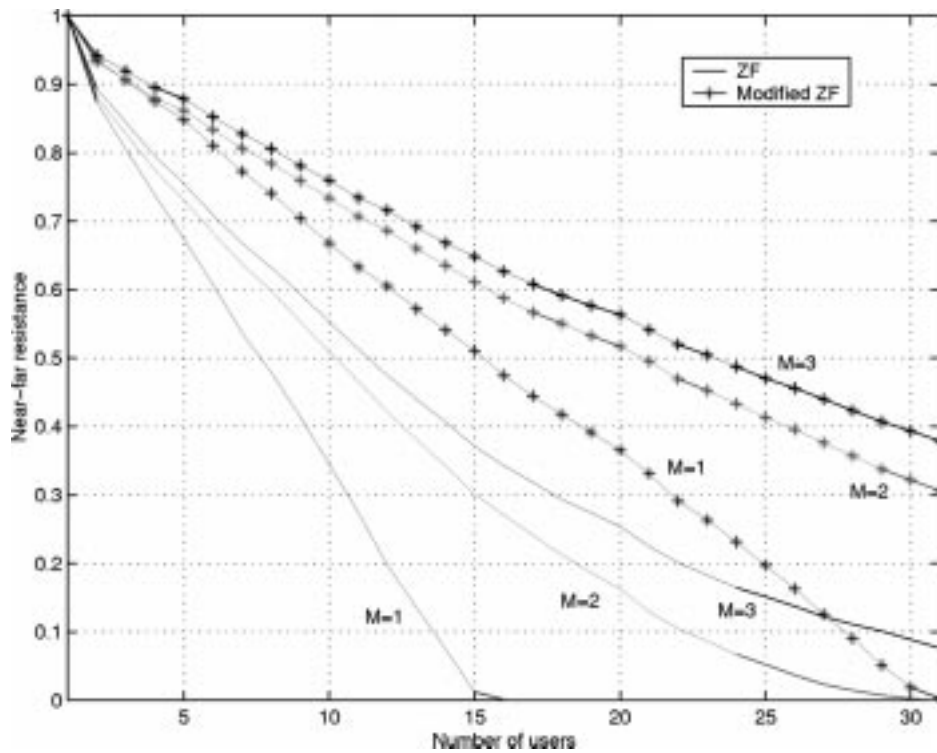
Fig. 8. Near–far resistance versus the number of users for the linear and the modified multiuser receivers. CDMA system has processing gain $N = 31$. Digital NBI has $T_I = T_b/3$.

variant approximation and the considerations already presented with reference to the TI approximations of the conventional receivers would hold for these as well. The point is that optimal modified PTV receivers typically have much longer periods than their conventional counterparts, whereby the time average of the covariance matrix of $z_a(p)$, albeit being $2NM$-dimensional, is typically more likely to reach full rank upon time-averaging over a whole period. Thus, adopting a TI solution may endanger near–far resistance and eventually will destroy it.

IV. NBI SUPPRESSION IN FADING DISPERSIVE CHANNELS

Let us now consider the case in which the channel introduces multipath distortion in the CDMA multiplex, which leads to a model for $s(p)$ as in (16). We first assume that the channel coherence time is much longer than the CDMA signaling interval, so that the dependence of the tap weights on p can be ignored. Also, we assume that an estimate of the tap weights of the user of interest is available, i.e., that the vector α_0 is already known at the receiver. Some discussion on how to obtain an estimate of this vector in overlay situations and on the effect on the system performance of possible estimation errors, will be given at the end of this section.

Furthermore, we also distinguish between the situation in which no further CSI is available at the receiver, i.e., in which the other-users' tap weight realizations are not known and the case that CCSI is available (see also Sections II-C and II-E). This distinction is based on the one proposed in, e.g., [75], [81], [95], [96], with reference to MUD in fading channel and on the issues exposed in [18] as to the availability of CSI and its impact on the performance of blind systems. In

particular, as discussed in [75], [95], [96], MUD can be accomplished in multipath fading channels following either of two approaches, corresponding to performing the cochannel interference rejection after of before multipath combining. In the former approach, the receiver needs knowledge of the channel realizations for all of the users, which is an obvious prerequisite for performing the multipath combining; such receivers are referred to as detectors with CCSI. Alternatively, in the latter approach the receiver structure, depicted in Fig. 9, resembles a two-stage RAKE receiver. In the first stage, the overall interference (i.e., MAI, ISI, NBI, and interpath interference) is suppressed on each branch, i.e., on each resolvable path. After this stage, the multipath combining takes place, whereby what the receiver needs is knowledge of the fading coefficients for the user to be demodulated. Accordingly, these receivers are referred to as incomplete CSI (ICSI) detectors. In the following, we further describe and compare these two different reception strategies.

A. NBI Suppression With CCSI

The case in which CCSI is available at the receiver, i.e., in which the realizations of all of the active users tap weights are known, is by far the more straightforward from the standpoint of receiver synthesis. Indeed, based on the model (16), it is readily seen that, if the fading is slow (i.e., the tap weights remain constant over several symbol intervals), the received signal appears at the receiver end as a CDMA multiplex with modified signatures in AWGN. To illustrate further, let us focus on user 0. In making a decision about $b_0(p)$, the useful signature is $S_{0,0}\alpha_0(p)$ and it is embedded in an overall interference consisting of the following elements:
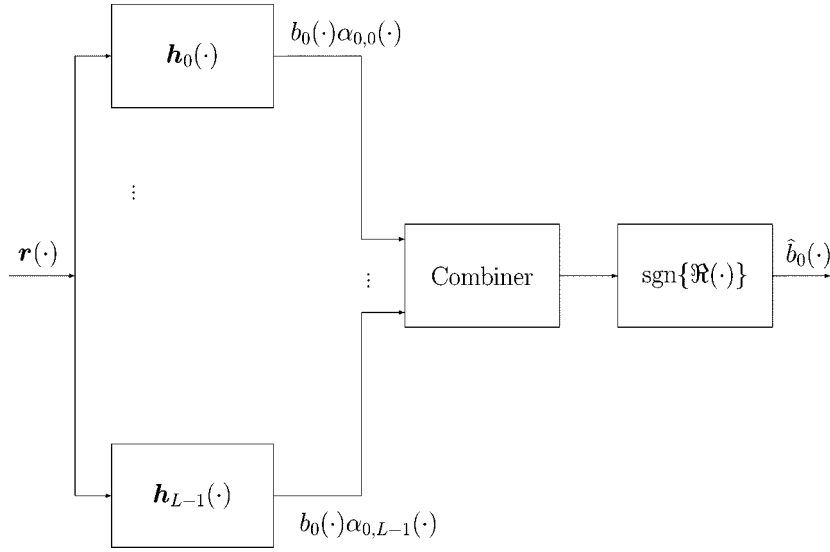
Fig. 9. Receiver structure for ICSI. Multipath combining takes place only after interference suppression has been accomplished. Fading coefficients for only the signal from the user to be demodulated are assumed known.

1) the neighboring symbols from user 0, i.e., the ISI, whose contribution is typically negligible due to the SS nature of the signals;
2) the interference from the other users, which can be dealt with by introducing new users with modified signatures $S_{k,i}\alpha_k$;
3) the NBI.

Thus, all of the techniques that have been illustrated in the previous section to handle NBI suppression for asynchronous nonfading system can be reapplied here quite straightforwardly.

Let us first focus our discussion on the MMSE optimization strategy. In the presence of CCSI, a convenient risk function for MMSE detection is the *conditional* MMSE

$$\mathcal{R}_{\mathrm{MMSE}}^{\mathrm{CCSI}}(\boldsymbol{d},p) = E\left\{\left|\boldsymbol{d}^H\boldsymbol{r}(p) - b_0(p)\right|^2 \Big| \boldsymbol{\alpha}_0,\ldots,\boldsymbol{\alpha}_{K-1}\right\}$$

$$= E\left\{\left|\boldsymbol{d}^H\boldsymbol{r}(p) - b_0(p)\right|^2 \Big| \boldsymbol{A}\right\} \qquad (73)$$

which is easily shown to achieve a minimum at

$$\boldsymbol{d}_{\mathrm{MMSE}}^{\mathrm{CCSI}}(p) = \sqrt{\mathcal{E}_0}\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{A}}^{-1}(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0. \qquad (74)$$

Likewise, a ZF detector is obtained by solving (75) at the bottom of the page, which yields the solution

$$\boldsymbol{d}_{\mathrm{ZF}}^{\mathrm{CCSI}}(p) = \frac{1}{\boldsymbol{\alpha}_0^H\boldsymbol{S}_{0,0}^H\boldsymbol{M}_{1|\boldsymbol{A}}^\dagger(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0}\boldsymbol{M}_{1|\boldsymbol{A}}^\dagger(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0$$

$$(76)$$

where $\boldsymbol{M}_{1|\boldsymbol{A}}(p) = \boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{A}}(p) - 2\mathcal{N}_0\boldsymbol{I}_{NM}$ is the noiseless observables' conditional covariance matrix. Finally, a MMOE solution can be found by solving (77) at the bottom of the page, which yields the solution

$$\boldsymbol{d}_{\mathrm{MMOE}}^{\mathrm{CCSI}}(p) = \frac{1}{\boldsymbol{\alpha}_0^H\boldsymbol{S}_{0,0}^H\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{A}}^{-1}(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0}\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{A}}^{-1}(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0$$

$$(78)$$

which is again proportional to the MMSE solution (74) through a positive constant.

Before proceeding with our discussion, it is worth noticing that all of the three solutions have the same structure: the observables first undergo a preliminary interference-suppression stage (depending, through the conditional covariance matrices of either the observables or their noiseless version, on the channel tap weights of all of

$$\begin{cases} \boldsymbol{d}_{\mathrm{ZF}}^{\mathrm{CCSI}}(p) = \arg\min_{\boldsymbol{d}}\mathcal{R}_{\mathrm{ZF}}^{\mathrm{CCSI}}(\boldsymbol{d},p) = \arg\min_{\boldsymbol{d}} E\left\{\left|\boldsymbol{d}^H(\boldsymbol{r}(p) - \boldsymbol{n}(p))\right|^2 \Big| \boldsymbol{A}\right\} \\ \text{subject to} \quad \boldsymbol{d}^H\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0 = 1 \end{cases} \qquad (75)$$

$$\begin{cases} \boldsymbol{d}_{\mathrm{MMOE}}^{\mathrm{CCSI}}(p) = \arg\min_{\boldsymbol{d}}\mathcal{R}_{\mathrm{MMOE}}^{\mathrm{CCSI}}(\boldsymbol{d},p) = \arg\min_{\boldsymbol{d}} E\left\{\left|\boldsymbol{d}^H\boldsymbol{r}(p)\right|^2 \Big| \boldsymbol{A}\right\} \\ \text{subject to} \quad \boldsymbol{d}^H(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0 = 1 \end{cases} \qquad (77)$$

the users) and then undergo a RAKE-type processing prior to being forwarded to the decision circuits. Note also that the interference-blocking transformation can be a reversible one (for MMOE and MMSE) or an irreversible one, as happens for ZF systems with singular $\boldsymbol{M}_{1|\boldsymbol{A}}(p)$. As long as the channel state remains constant, the above solutions remain constant as well, thus, implying that implementing the receiver still requires NM complex multiplications and additions. Obviously, when the channel state changes, the above solutions need to be recomputed from scratch and this brings a substantial complexity increase in the computational burden and memory requirements for the receiver.

Also, note that the interference-blocking transformation may end up either time invariant or time varying, depending on the nature of the NBI. Indeed, under the three optimization criteria, the blocking matrix contains the NBI covariance matrix as a summand, whereby time-invariant rules are optimal if and only if the NBI has a stationary covariance. In all other cases considered here, the decision rule is again PTV. We will not repeat here the detailed analysis of the previous section. It is, however, important to emphasize that, in the light of the time-varying nature of the interference, the above optimum solutions could be rederived as global minima of the time averages of the corresponding risk functions. As a consequence, any PTV solution containing only a subset of the harmonic frequencies of the optimum solution represents a constrained minimum of the TA risk and, among these minima, the time-invariant solutions are found.

Before moving on to the case of ICSI, it is worth mentioning here that the above considerations hold true only for the case of slowly fading channels, i.e., when the channel coherence time is sufficiently large. For small coherence times, indeed, further time variations in the solutions (74), (76), and (78) are induced and the filters' PTV structure is completely lost. In this situation, due to the fast fading variations, computation of the corresponding solutions require a matrix inversion at each symbol interval, a computational task that will usually be prohibitive. As will be shown in the following section, receivers based on ICSI or, equivalently, on precombining interference suppression overcome this problem.

B. NBI Suppression With ICSI

The unavailability of CCSI calls for a completely different detection strategy. As already proposed in [95] and [96], for MUD in the absence of NBI, this new receiver family consists of two blocks. The first block, aimed at interference suppression, may be viewed as a bank of filters, wherein each finger is aimed at interference cancellation for a given path of the useful signal, while the latter block is aimed at BER optimization. A schematic of such a receiver family is shown in Fig. 9. The interference-blocking stage consists of a linear estimator of the quantity $b_0(p)\boldsymbol{\alpha}_0(p)$, i.e.,

$$\widehat{b_0(p)\boldsymbol{\alpha}_0}(p) = \boldsymbol{H}^H(p)\boldsymbol{r}(p) \qquad (79)$$

where $\boldsymbol{H}(p)$ is an $NM \times L$ matrix to be suitably designed and we have denoted its ith column by $\boldsymbol{h}_i(p)$ in Fig. 9. A

number of design criteria can be adopted to determine $\boldsymbol{H}(p)$, but we consider here the three strategies illustrated in the previous section, i.e., the MMSE, the ZF, and the MMOE. In an MMSE context, the blocking matrix is obtained by minimizing the risk

$$\mathcal{R}_{\mathrm{MMSE}}^{\mathrm{ICSI}}(\boldsymbol{H}, p) = E\left\{\left\|\boldsymbol{H}(p)^H \boldsymbol{r}(p) - b_0(p)\boldsymbol{\alpha}_0(p)\right\|^2\right\} \quad (80)$$

yielding the solution

$$\boldsymbol{H}_{\mathrm{MMSE}}(p) = \sqrt{\mathcal{E}_0}\boldsymbol{M}_{\boldsymbol{r}\boldsymbol{r}}^{-1}(p)\boldsymbol{S}_{0,0}\boldsymbol{\Sigma}_0. \qquad (81)$$

In a ZF framework, instead, the blocking matrix is found as the solution to the constrained minimization

$$\begin{cases} \boldsymbol{H}_{\mathrm{ZF}}(p) = \arg\min_{\boldsymbol{H}} E\left\{\left\|\boldsymbol{H}^H(p)(\boldsymbol{r}(p) - \boldsymbol{n}(p))\right\|^2\right\} \\ \text{subject to } E\left\{\boldsymbol{\alpha}_0^H(p)\boldsymbol{H}^H(p)\boldsymbol{r}(p) \mid b_0(p)\right\} = b_0(p) \end{cases} \quad (82)$$

whose solution is written as

$$\boldsymbol{H}_{\mathrm{ZF}}(p) = \frac{\sqrt{\mathcal{E}_0}\boldsymbol{M}_1(p)^\dagger \boldsymbol{S}_{0,0}\boldsymbol{\Sigma}_0}{\mathrm{tr}\left(\boldsymbol{\Sigma}_0^2 \boldsymbol{S}_{0,0}^H \boldsymbol{M}_1(p)^\dagger \boldsymbol{S}_{0,0}\right)} \qquad (83)$$

where $\mathrm{tr}(\cdot)$ denotes the matrix trace. Finally, for the MMOE criterion, the interference-blocking matrix is defined as the solution to the problem:

$$\begin{cases} \boldsymbol{H}_{\mathrm{MMOE}}(p) = \arg\min_{\boldsymbol{H}} E\left\{\left\|\boldsymbol{H}^H(p)\boldsymbol{r}(p)\right\|^2\right\} \\ \text{subject to } E\left\{\boldsymbol{\alpha}_0^H(p)\boldsymbol{H}^H(p)\boldsymbol{r}(p) \mid b_0(p)\right\} = b_0(p) \end{cases} \quad (84)$$

which yields

$$\boldsymbol{H}_{\mathrm{MMOE}}(p) = \frac{\sqrt{\mathcal{E}_0}\boldsymbol{M}_{\boldsymbol{r}\boldsymbol{r}}(p)^{-1} \boldsymbol{S}_{0,0}\boldsymbol{\Sigma}_0}{\mathrm{tr}\left(\boldsymbol{\Sigma}_0^2 \boldsymbol{S}_{0,0}^H \boldsymbol{M}_{\boldsymbol{r}\boldsymbol{r}}(p)^{-1}\boldsymbol{S}_{0,0}\right)} \qquad (85)$$

which, once again, is proportional to the MMSE solution (81). Since the $\boldsymbol{H}(p)$ is $NM \times L$-dimensional, implementing the receiver first stage involves LNM complex multiplications and additions. Luckily, it is seen that, under the considered scenario, the matrix $\boldsymbol{H}(p)$ is independent of the fading channel realizations, so that there is no need to recompute it following variations in the propagation channel impulse response. Notice also that for all of the three criteria above, the interference-blocking matrix is again time invariant or time varying, depending on whether the NBI covariance matrix is stationary or not. Once again, if the NBI consists of a superposition of digitally modulated signals, the resulting blocking matrix may end up PTV. As before, reduced solutions may be devised in this case by approximating the optimal PTV solution through a PTV solution with a reduced number of harmonic frequencies and in particular through a stationary solution, which represents a constrained minimum for the corresponding risk.

Let us now move on to the second stage of Fig. 9. A first approach that can be followed is based on the assumption that, upon the transformation $\boldsymbol{H}(p)$, the residual interference, whether NBI or MAI, is negligible. This is true if the blocking matrix is designed according to a ZF criterion, while being only approximately true if MMSE or MMOE is adopted, although the latter two criteria can be shown to

be equivalent to a ZF in the limit of vanishingly small noise floor and/or increasingly large other-user amplitudes. This is to say that the output signal can be fairly well approximated as the superposition of the surviving useful signal and noise, i.e.,

$$
\begin{aligned}
\boldsymbol{y}(p) =& \boldsymbol{H}^H(p)\boldsymbol{r}(p) \\
\simeq& \sqrt{\mathcal{E}_0}b_0(p)\boldsymbol{H}^H(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0(p) + \boldsymbol{H}^H(p)\boldsymbol{n}(p) \quad (86)
\end{aligned}
$$

where now the vectors are L-dimensional. Detecting $b_0(p)$ from the observables (86) is a standard problem, which can be optimally solved by the cascade of a noise-whitening transformation and a matched filter. In particular, the whitening transformation is a possibly PTV $L \times L$ matrix $\boldsymbol{W}(p)$ given by

$$
\boldsymbol{W}(p) = \left(\boldsymbol{H}^H(p)\boldsymbol{H}(p)\right)^{-1/2}.
$$

It, thus, follows that the whitened observables can be written as

$$
\boldsymbol{y}_w(p) \simeq \sqrt{\mathcal{E}_0}b_0(p)\boldsymbol{W}(p)\boldsymbol{H}^H(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0(p) + \boldsymbol{n}'(p) \quad (87)
$$

with $\boldsymbol{n}'(p)$ a complex Gaussian random vector with covariance matrix $2\mathcal{N}_0\boldsymbol{I}_{NM}$, whereby the decision rule to detect the bit $b_0(p)$ is given by

$$
\widehat{b}_0(p) = \operatorname{sgn}\left[\Re\left\{\boldsymbol{\alpha}_0^H\boldsymbol{S}_{0,0}^H\boldsymbol{H}(p)\boldsymbol{W}(p)\boldsymbol{W}^H(p)\boldsymbol{H}^H(p)\boldsymbol{r}(p)\right\}\right]. \quad (88)
$$

Notice that the decision rule (88) is the optimal one (with respect to the transformed observables $\boldsymbol{y}(p)$) if $\boldsymbol{H}(p) = \boldsymbol{H}_{\text{ZF}}(p)$. The receiver implementing decision rule (88) will be termed a whitening filter (WF) detector.

A different strategy, which will be shown below to be very valuable in that it readily lends itself to reduced-complexity adaptive implementation, relies upon MRC of the reduced observables (86). That is, it uses multipath combining with no prior whitening, which leads to what we term the MRC detector, which implements the decision rule

$$
\widehat{b}_0(p) = \operatorname{sgn}\left[\Re\left\{\boldsymbol{\alpha}_0^H\boldsymbol{H}^H(p)\boldsymbol{r}(p)\right\}\right]. \quad (89)
$$

It is worth pointing out that, as long as the fading vector $\boldsymbol{\alpha}_0$ remains constant, both decision rules (88) and (89) can be implemented with just NM complex multiplications and additions in each symbol interval, since the vectors $\boldsymbol{\alpha}_0^H\boldsymbol{S}_{0,0}^H\boldsymbol{H}(p)\boldsymbol{W}(p)\boldsymbol{W}^H(p)\boldsymbol{H}^H(p)$ and $\boldsymbol{\alpha}_0^H\boldsymbol{H}^H(p)$ can be computed once and for all and stored in memory. Thus, receiver operation requires only an inner product in its symbol-rate processing. From a conceptual point of view, this is equivalent to letting the two stages of the receiver collapse into one. Notice also that, except that when $\boldsymbol{H}(p) = \boldsymbol{H}_{\text{ZF}}(p)$, nothing can be anticipated as to which one of the two rules (88) and (89) achieves better performance in that both of them are suboptimal receivers. What is interesting is that in both of these approaches, channel variations, e.g., due to new users from either the CDMA or narrow-band networks entering or leaving the scene, birth of new NBIs, etc., have an effect only on a part of the receiver, namely, the cascade of the blocking matrix and of the WF for the former rule and the blocking matrix only for the latter rule. Likewise, variations in the

channel tap weights for the interfering users do not affect the receiver structure at all, while variations in $\alpha_0(p)$ affect only the combining stage. The computational advantage in fast fading environments of this class of receivers with respect to those relying on CCSI now emerges clearly. Indeed, while, as already noted, implementing the MUD receivers with CCSI in fast fading channels requires a matrix inversion at each symbol interval, solutions (81), (83), and (85) do not depend on the channel fading realizations in that they rely only on the *unconditional* covariance matrix of the observables. As can be reasonably expected, the price to be paid for this computational advantage is degraded performance, especially in situations where the product KL is close to the processing gain and there is no power control.

C. Modified Linear MUD for Overlay Channels With CCSI

In this section, we hint at how one might fully exploit the CCSI to achieve better cochannel interference suppression in fading channels. The derivations parallel those of Section III-C in that, once again, the point is the full exploitation of the covariance properties of the CDMA multiplex. Indeed, in Section II, it has been established that, upon CCSI, the CDMA multiplex can be regarded as an improper process in keeping with the results established in [107].

Since we are in a CCSI context, the relevant cost functions for the three design criteria are (73), (75), and (77) for MMSE, ZF, and MMOE, respectively. For the MMSE criterion, the symbol estimator should now be sought in the form of a real-valued function of the observables and, thus, as the unique minimum of the modified risk

$$
\begin{aligned}
\min_{\boldsymbol{d}} &\mathcal{R}_{\text{MMSE}}'^{\,\text{CCSI}}(\boldsymbol{d}, p) \\
=& \min_{\boldsymbol{d}} E\left\{\left[\Re\left\{\boldsymbol{d}^H\boldsymbol{r}(p)\right\} - b_0(p)\right]^2 \Big| \boldsymbol{\alpha}_0, \ldots, \boldsymbol{\alpha}_{K-1}\right\} \\
=& E\left\{\left[\boldsymbol{d}_a^H\boldsymbol{r}_a(p) - b_0(p)\right]^2 \Big| \boldsymbol{A}\right\} \quad (90)
\end{aligned}
$$

where again the subscripts "a" denote the augmented versions of the corresponding vectors. Likewise, for ZF and MMOE strategies, we obtain (91) and (92), respectively, shown at the bottom of the next page.

The corresponding solutions are, thus, written in terms of the enlarged vectors as

$$
\boldsymbol{d}_{a,\text{MMSE}}(p) = \sqrt{\mathcal{E}_0}\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a|\boldsymbol{A}}^{-1}(p)\boldsymbol{s}_a\left(\boldsymbol{\alpha}_0\right) \quad (93)
$$

for MMSE

$$
\boldsymbol{d}_{a,\text{ZF}}(p) = \frac{1}{\boldsymbol{s}_a^H\left(\boldsymbol{\alpha}_0\right)\boldsymbol{M}_{a,1|\boldsymbol{A}}(p)^\dagger\boldsymbol{s}_a\left(\boldsymbol{\alpha}_0\right)}\boldsymbol{M}_{a,1|\boldsymbol{A}}^\dagger(p)\boldsymbol{s}_a\left(\boldsymbol{\alpha}_0\right) \quad (94)
$$

for ZF, and

$$
\begin{aligned}
\boldsymbol{d}_{a,\text{MMOE}}(p) =& \frac{1}{\boldsymbol{s}_a^H\left(\boldsymbol{\alpha}_0\right)\left[\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a|\boldsymbol{A}}(p)\right]^{-1}\boldsymbol{s}_a\left(\boldsymbol{\alpha}_0\right)} \\
&\cdot\left[\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}(p)|\boldsymbol{A}\right]^{-1}\boldsymbol{s}_a\left(\boldsymbol{\alpha}_0\right) \quad (95)
\end{aligned}
$$

for MMOE, respectively. Once again, the solution in terms of the original unknowns can be obtained by considering the

first NM entries of each of the above three vectors, and implementing the decision rule requires NM complex multiplications and additions in each symbol-interval. Notice that now the augmented version of the useful signal is

$$s_a\left(\boldsymbol{\alpha}_0\right) = \begin{pmatrix} \boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0 \\ \boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0^* \end{pmatrix}. \qquad (96)$$

It is also worth emphasizing again here that the phase information between the several users of the CDMA multiplex is now exploited in that the conditional covariance matrix of the augmented observables $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a|\boldsymbol{A}}^{-1}(p)$ as well as that of their noiseless counterparts, i.e.,

$$\boldsymbol{M}_{a,1|\boldsymbol{A}}(p) = E\left\{\left(\boldsymbol{r}_a(p) - \boldsymbol{n}_a(p)\right)\left(\boldsymbol{r}_a(p) - \boldsymbol{n}_a(p)\right)^H |\boldsymbol{A}\right\}$$

both contain the conditional covariances and the conditional pseudocovariances of the CDMA multiplex *and* of the NBI. Notice also that, as highlighted in [107], exploiting such information in the presence of fading dispersive channels is even more important than under nonfading channels in that we are now in the presence of L phase shifts for any active user of the multiplex.

The considerations that have already been presented with reference to modified receivers under single-path nonfading channels and NBI hold true here, too, and will not be repeated. In particular, recall that the solutions (93)–(95) may be time varying in the presence of nonstationary NBI and PTV if the NBI is a superposition of digitally modulated signals. In the latter situation, since the channel dispersivity does not alter the NBI covariance and pseudocovariance, the period is still the common period of the two. Moreover, any PTV solution containing a subset of the harmonic frequencies of the corresponding optimal solution would represent a constrained minimum for the TA risks and among these minima the stationary counterparts of (93)–(95) are found.

D. Performance Analysis

As for the case of nonfading channels, the usual performance indices adopted here are BER, SIR, and near–far resistance. Let us, thus, focus first on the system BER. The case of receivers with CCSI can be handled similarly to the case of nonfading channel by replacing the signatures of the real and virtual users $\boldsymbol{s}_{k,i}$ with their distorted versions $\boldsymbol{S}_{k,i}\boldsymbol{\alpha}_{k,i}$ and the observables' covariance matrix with the conditional covariance given the channel weights. The conditional error probability is thus written as in (61), with the understanding that the unconditional BER is obtained by averaging with respect to the fading channel realizations as well.

For the receivers with ICSI, some analytical formulas may be easily obtained. Indeed, we first observe that the decision rules (88) and (89) admit the following unified expression:

$$\widehat{b}_0(p) = \text{sgn}\left\{\Re\left[\boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{r}(p)\right]\right\} \qquad (97)$$

with the PTV matrix $\boldsymbol{Q}(p)$ given by (98) at the bottom of the page. Expression (97), thus, allows us to derive the system BER through a unified approach. Indeed, conditioned upon $b_0(p) = 1$, the cochannel interference vector $\boldsymbol{z}(p)$, and $\boldsymbol{\alpha}_0(p)$, the decision statistic appearing in (97) is a complex Gaussian random variable with mean value

$$\sqrt{\mathcal{E}_0}\boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0(p) + \boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{z}(p)$$

and variance

$$2\mathcal{N}_0\boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{Q}^H(p)\boldsymbol{\alpha}_0(p). \qquad (99)$$

Accordingly, the conditional system BER can be written as

$$P\left(e\,|\alpha_0(p), \boldsymbol{z}(p)\right) = \frac{1}{Q}\sum_{p=0}^{Q-1}\frac{1}{2}\text{erfc}$$
$$\cdot\left\{\frac{\sqrt{\mathcal{E}_0}\boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{S}_{0,0}^H\boldsymbol{\alpha}_0(p) + \boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{z}(p)}{\sqrt{2\mathcal{N}_0\boldsymbol{\alpha}_0^H(p)\boldsymbol{Q}(p)\boldsymbol{Q}^H(p)\boldsymbol{\alpha}_0(p)}}\right\}. \qquad (100)$$

Obviously, in order to obtain the unconditional system BER, this expression should be averaged with respect to the fading vectors, the NBI and MAI interfering bits and the NBI random parameters.

With regard to the SIR, the case of fading channel poses some difficulty in the definition of this quantity. Based upon definition (63), the averages in the numerator and in the denominator should be computed also in the *ensemble* of the fading realizations, which, for all of the receivers presented

$$\begin{cases} \min_{\boldsymbol{d}} \mathcal{R}_{\text{ZF}}'^{\text{CCSI}}(\boldsymbol{d}, p) = \min_{\boldsymbol{d}} E\left\{\Re^2\left\{\boldsymbol{d}^H(\boldsymbol{r}(p) - \boldsymbol{n}(p))\right\}|\boldsymbol{A}\right\} = \min \\ \text{subject to} \quad \Re\left\{\boldsymbol{d}^H\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0\right\} = 1 \end{cases} \qquad (91)$$

$$\begin{cases} \mathcal{R}_{\text{MMOE}}'^{\text{CCSI}}(\boldsymbol{d}, p) = E\left\{\Re^2\left\{\boldsymbol{d}^H\boldsymbol{r}(p)\right\}|\boldsymbol{A}\right\} = \min \\ \text{subject to} \quad \Re\left\{\boldsymbol{d}^H\boldsymbol{S}_{0,0}\boldsymbol{\alpha}_0\right\} = 1 \end{cases} \qquad (92)$$

$$\boldsymbol{Q}(p) = \begin{cases} \boldsymbol{S}_{0,0}^H\boldsymbol{H}(p)\left(\boldsymbol{H}^H(p)\boldsymbol{H}(p)\right)^{-1}\boldsymbol{H}^H(p) & \text{for the WF detector and} \\ \boldsymbol{H}^H(p) & \text{for the MRC detector.} \end{cases} \qquad (98)$$

in the previous sections, would be analytically unwieldy. On the other hand, it is questionable if such a measure would be a fair performance index. Indeed, we are assuming that a symbol-by-symbol decision is being made, whereby what matters is the SIR each decision is faced with for a fixed channels state realization. In order to account for the whole ensemble of such realizations, it is, thus, possible to average the SIRs and to take this average as a global performance measure, i.e.,

$$
\begin{aligned}
\mathrm{SIR}(p) &= E_{\boldsymbol{A}}\left\{\mathrm{SIR}_{\boldsymbol{A}}(p)\right\} \\
&= E_{\boldsymbol{A}}\left\{\frac{E^2\left[\boldsymbol{d}^H(p)\boldsymbol{r}(p)\,|b_0(p)\,,\boldsymbol{A}\right]}{E\left[\left|\boldsymbol{d}^H(p)\boldsymbol{z}(p)\right|^2\,|\boldsymbol{A}\right]}\right\}. \quad (101)
\end{aligned}
$$

Even so, however, no manageable closed-form expression can be found for the SIR. Indeed, even for the CCSI receivers, which, as already mentioned, are equivalent to those for non-fading channels provided that the signatures are properly redefined, it is not possible to go much further than the obvious conclusion that $\mathrm{SIR}_{\boldsymbol{A}}(p)$ is obtained from the case of nonfading channel by replacing the signatures of the real and virtual users with their distorted versions. Indeed, performing the final averaging operation appears to be analytically unwieldy and computer simulations are needed to numerically average (101).

With regard to the near–far resistance, a closed-form formula can be determined for systems with ICSI. In particular, adopting arguments similar to those adopted in [227] and [228], a bound can be found for the system that performs only MRC of the filtered observables $\boldsymbol{y}(p)$ and a closed-form formula for the system that performs whitening prior to MRC can be given. Assuming digitally modulated NBI, for the former systems, it can be shown that, conditioned upon the NBI signaling interval, phase, frequency offset, and delay, as well as on all of the delays of the CDMA multiplex, the following bounds hold, [23], as shown in (102) at the bottom of the page, where $\boldsymbol{G}_0(p)$ is defined as

$$
\boldsymbol{G}_0(p) = \lim_{\mathcal{N}_0 \to 0} \boldsymbol{M}_{\boldsymbol{r}\boldsymbol{r}}^{-1}(p)\boldsymbol{S}_{0,0} \quad (103)
$$

and $\lambda_{\min}(p)$ and $\lambda_{\max}(p)$ are, for each epoch p, the minimum and maximum eigenvalue of the matrix

$$
\left(\boldsymbol{B}_0^H(p)\right)^{-1}\boldsymbol{\Sigma}_0\boldsymbol{S}_{0,0}^H\boldsymbol{G}_0(p)\boldsymbol{B}_0^{-1}(p)
$$

where, in turn, the matrix $\boldsymbol{B}_0(p)$ is the $L \times L$-dimensional upper triangular nonsingular matrix obtained through

the Cholesky decomposition $\boldsymbol{\Sigma}_0\boldsymbol{G}_0^H(p)\boldsymbol{G}_0(p)\boldsymbol{\Sigma}_0 = \boldsymbol{B}_0^H(p)\boldsymbol{B}_0(p)$.

For the system performing both noise whitening and MRC, the near–far resistance in overlay channels admits a closed-form expression and is given by

$$
\eta_{\mathrm{WF}} = \min_{0 \leq p \leq Q-1}\left\{\left(\frac{\det\left(\boldsymbol{N}_0(p)\boldsymbol{S}_{0,0}\right)}{\det\left(\boldsymbol{S}_{0,0}^H\boldsymbol{S}_{0,0}\right)}\right)^{1/L}\right\} \quad (104)
$$

with $\boldsymbol{N}_0(p) = \boldsymbol{S}_{0,0}^H\boldsymbol{H}_{\mathrm{ZF}}(p)(\boldsymbol{H}_{\mathrm{ZF}}^H(p)\boldsymbol{H}_{\mathrm{ZF}}(p))^{-1}\boldsymbol{H}_{\mathrm{ZF}}^H(p)$. More illuminating than the cumbersome formulas above are the plots of Figs. 10 and 11, representing, for $K = 4$ and $K = 9$, respectively, the error probability of the above receivers versus the average received energy contrast, defined now as $\gamma_0 = (\mathcal{E}_0\mathrm{trace}(\boldsymbol{\Sigma}_0))/2\mathcal{N}_0$. The oversampling factor is $M = 2$ and the curves are obtained by averaging the BERs over 2500 random realizations of the fading coefficients and propagation delays. A multipath channel with $L = 3$ resolvable path has been considered. The NBI is a digital signal having 15-dB power advantage with respect to the SS users, whose average power are instead coincident (the instantaneous powers are random due to the fading realizations).

These results confirm that the receivers exploiting CCSI are superior to those based on ICSI. This is particularly true if a ZF design strategy is adopted, but holds for all criteria as either the network load increases or the noise floor vanishes (i.e., in the large signal-to-noise ratios region), since, under these circumstances, the interference-blocking matrices $\boldsymbol{H}_{\mathrm{MMSE}}(p)$ and $\boldsymbol{H}_{\mathrm{MMOE}}(p)$ approach $\boldsymbol{H}_{\mathrm{ZF}}(p)$. The somewhat disappointing asymptotic behavior of receivers designed under ICSI can be justified in the light of (103). Indeed, through standard MUD arguments, it can be shown that $\boldsymbol{G}_0(p) = \boldsymbol{S}_{0,0}^{\perp}(p)$, with $\boldsymbol{S}_{0,0}^{\perp}(p)$ containing in its ℓth column the vector $\boldsymbol{s}_{0,0}^{\perp\ell}(p)$. This quantity is the projection of the ℓth replica of the useful signal onto the orthogonal complement to the subspace spanned, *in the ensemble of all of the channel tap weight realizations*, by the overall interference, i.e., MAI, NBI, and ISI from the other users and the interpath interference induced by the replicas of the useful signal other than the ℓth one [23]. Thus, ICSI may result, in near–far scenarios, in noticeable noise enhancement (if not in signal-space saturation). Note also that adopting time-invariant approximations of the optimum PTV in the case of nonstationary NBI performs very poorly in this situation. In fact, due to ICSI, the near–far resistance is already endangered by the simultaneous presence of fading and NBI, whereby the issue of signal space dimension saving

$$
\left\{
\begin{aligned}
\eta_{\mathrm{MRC}} &\geq \min_{0 \leq p \leq Q-1}\left[\lambda_{\min}(p)\left(\frac{\det\left(\boldsymbol{\Sigma}_0\boldsymbol{S}_{0,0}^H\boldsymbol{G}_0(p)\right)}{\det\left(\boldsymbol{S}_{0,0}^H\boldsymbol{S}_{0,0}\right)}\right)^{1/L}\right] \\
\eta_{\mathrm{MRC}} &\leq \min\left\{\min_{0 \leq p \leq Q-1}\left[\lambda_{\max}(p)\left(\frac{\det\left(\boldsymbol{\Sigma}_0\boldsymbol{S}_{0,0}^H\boldsymbol{G}_0(p)\right)}{\det\left(\boldsymbol{S}_{0,0}^H\boldsymbol{S}_{0,0}\right)}\right)^{1/L}\right], 1\right\}
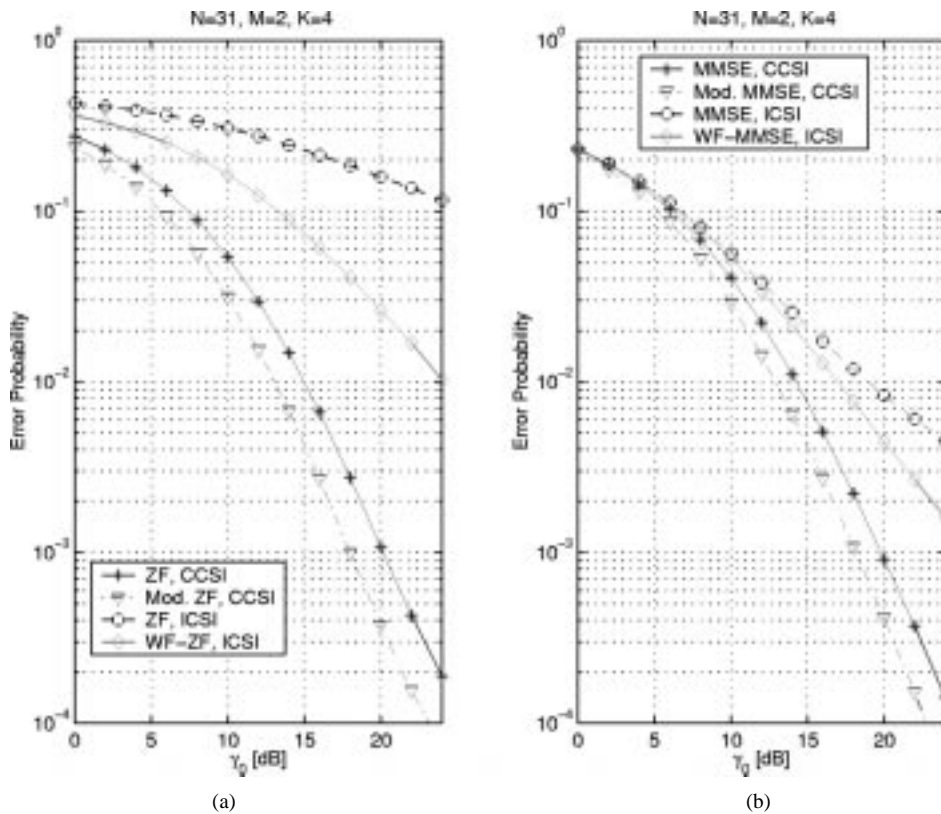\end{aligned}
\right. \quad (102)
$$

Fig. 10. Error probabilities for the multiuser receivers in fading channels. (a) ZF receivers. (b) MMSE receivers. There are $K = 4$ users and the oversampling ratio is $M = 2$. Digital NBI has a power advantage of 15 dB over the user of interest and has $T_b/T_I = 3$. Propagation channels have been assumed to have $L = 3$ resolvable paths.

is of primary concern to ensure the existence of at least one cochannel-interference-free direction upon which to project the observables. Since it has already been explained how adopting TI solutions in PTV situations amounts to overestimating the dimensionality of the NBI space, it is understood that, under ICSI, it is important to retain the optimal PTV solution.

Different arguments can be put forward in the case of CCSI. Indeed, we have already commented on the equivalence, under certain points of view, of a fading channel with CCSI with a nonfading channel operating with modified signatures. This allows us to understand what happens, in terms of near–far resistance, at each single fading realizations, but is of no help in determining a closed-form formula for the ensemble of such realizations. Conditioned upon a given fading realization, in the limit of vanishingly small noise floor, the MMSE and the MMOE receivers for CCSI still approach a ZF receiver and the useful signal is projected along a suitable cochannel-interference-free direction. The crucial difference with the previous situation is that the receiver does know the fading realizations, whereby, for vanishingly small noise floor, the projection direction becomes orthogonal to the MAI and the ISI resulting from the particular realization of the fading. In case of ICSI, instead, it has already been pointed out that the unavailability of the fading realizations forces the receiver to be orthogonal to the range spanned by MAI, ISI, and interpath interference in the ensemble of fading realizations. Thus, exploitation of

CCSI allows a dimension saving of a factor of up to $1/L$, with L the number of different paths, as explained in [18]. This fact also has some nice consequences for the limiting performance of adaptive systems under block-constant fading channels, as will be discussed in the upcoming section on adaptive interference suppression.

E. On the Availability of CSI in Overlay Channels

So far nothing has been said about how to obtain CSI at the receiver end. In spite of the significant efforts that are being made to develop completely blind schemes, it is still customary to transmit pilot signals for channel estimation purposes, by multiplexing them with the information-bearing signals. Even so, the issue of pilot signal demultiplexing, channel estimation, and symbol detection in the presence of overlay channels is far from being a mature topic. However, some studies are already available in the open literature [26] and we briefly outline the main results for the sake of completeness. We consider only the case in which the receiver is interested in acquiring the channel tap weights of the user of interest, so that the situation is that of ICSI.

Two main formats for multiplexing a pilot signal with a CDMA multiplex in overlay channels that have been considered so far are the parallel pilot channel (PPC) format and the time-multiplexed pilot channel (TMPC) format.

In the PPC format the pilot signal is assigned a dedicated spreading code and is, thus, essentially an additional user. Its use in conjunction with MMSE detection in the context of
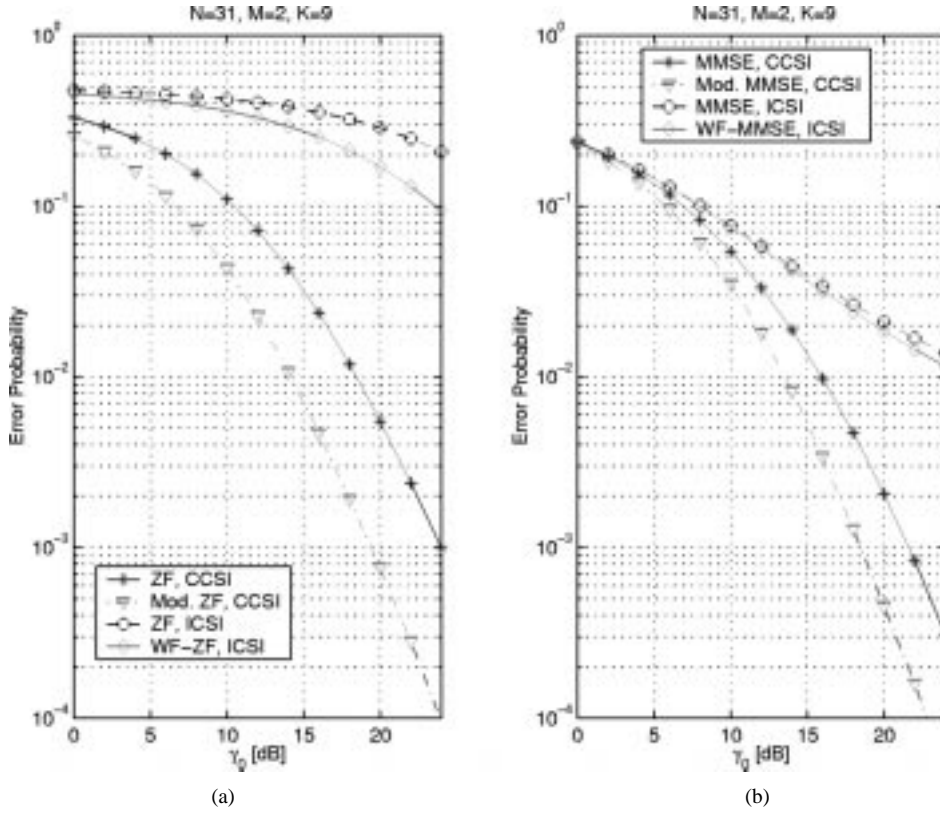
Fig. 11. Error probabilities for the multiuser receivers in fading channels. (a) ZF receivers. (b) MMSE receivers. There are $K = 9$ users and the oversampling ratio is $M = 2$. Digital NBI has a power advantage of 15 dB over the user of interest and has $T_b/T_I = 3$. Propagation channels have been assumed to have $L = 3$ resolvable paths.

MAI removal has been studied in [32]. Of course, resorting to such a multiplexing format in the downlink of a cellular network involves the assignment of one spreading code to the unique pilot to be transmitted and its impact on the network capacity is relatively modest. Conversely, its adoption in the uplink appears problematic in that it would require assigning two spreading codes to each user, i.e., one code for information and a second code for pilot transmission, which would result in halving the network capacity. Thus, we can assume its adoption in the downlink only.

The TMPC format inserts a block of V_T training symbols between consecutive blocks of V_I information symbols. If V_I is chosen such that $V_I T_b$ is small with respect to the channel coherence time, but $V_I \gg V_T$, the net information rate decrease due to multiplexing is negligible and reliable channel estimates can be achieved. The pilot is, thus, extracted by processing V_T observables and then the corresponding estimate is used for the following V_I signalling intervals. In principle, the alternation between training and detecting phases generates a further periodicity, which must be accounted for at the design level, unless the estimator is linear *and* the training and information-transmission phase are statistically undistinguishable in their second-order moments (i.e., have the same covariance). If this is not the case, then channel-estimation-based interference suppression would result in a total period given by the least common multiple of Q and $(V_I + V_T)$, where Q is the period of the NBI covariance matrix. On the other hand, if the training and the information

phases are performed using the same signatures and powers *and* if the propagation delay is the same, which is definitely the case in the downlink and can be assumed true also in the uplink, then this periodicity does not have any dramatic effect on the receiver complexity [26].

The presence of pilot signals has some influence on the receiver structure. Keeping the discussion at a qualitative level, it can easily be seen that, in a PPC format, the pilot signal does not alter the structure of the covariance matrix of the observables. A schematic of a receiver performing pilot demultiplexing, channel estimation and interference suppression is shown in Fig. 12. The channel tap weights are linearly estimated from the received signal as

$$\widehat{\boldsymbol{\alpha}}_0(p) = \boldsymbol{H}_c^H(p)\boldsymbol{r}(p) \qquad (105)$$

and are then forwarded to the BER optimization blocks to be used in MRC. The matrix $\boldsymbol{H}_c(p)$ can be chosen as the linear MMSE estimator of $\boldsymbol{\alpha}_0(p)$, but obviously any other optimization criterion can be adopted as, for instance the least-squares estimator adopted in [32]. Since the observables undergo, on a parallel branch, an interference-blocking transformation prior to MRC, the resulting receiver is a quadratic one, possibly PTV depending on the NBI model.

Slightly different arguments are instead needed for a TMPC format. In this format, subject to the validity of the previous assumptions, the simultaneous presence of PTV NBI with period Q and of the time-multiplexed pilot symbols yields an observable covariance matrix that is actually
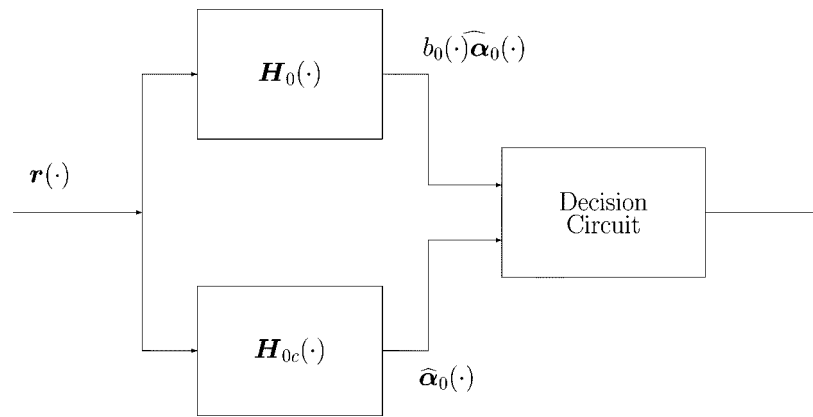
Fig. 12. Receiver structure for the PPC format. Upper filter suppresses the interference end extracts an estimate for the quantity $b_0(p)\boldsymbol{\alpha}_0(p)$, while the lower filter performs channel tap weight estimation. The two estimates are then processed in order to obtain an estimate of the transmitted symbol $b_0(p)$.

periodic with period l.c.m.$(Q, V_I + V_T)$. Luckily, this matrix can be regarded as the product of a PTV matrix sequence with period Q times a scalar (windowing) function with period $V_I + V_T$, whose task is just to window the observables during the training and the transmission phases. We will not dwell further on this issue, referring the interested reader to [26] in which a preliminary study of joint channel estimation and data demodulation in overlay CDMA channels is considered. As emphasized in the following, the problem of channel estimation in overlay applications has not yet received much attention and is still very much open.

Another very important issue that has not yet been sufficiently investigated is the effect of imperfect timing and channel estimation on the performance of overlaid CDMA networks. Intuition suggests that, the worst the channel and delay estimates, the larger the gap between the attained performance and that achievable in the ideal situation of no estimation errors. Some studies on this issue are currently available in the open literature with reference to the case of no overlay applications (see, e.g., [37], [67], [82], [139], [193], and [226] and references therein), while nothing is currently available with reference to the case that the cochannel interference consists of a superposition of MAI and NBI.

V. (BLIND) ADAPTIVE NBI REJECTION

The receivers discussed so far depend on knowledge of the second-order statistics of the observables, i.e., on the covariance and pseudocovariance matrices of the CDMA multiplex, the NBI, and the additive ambient noise. As a consequence, implementing these receivers requires knowledge of the parameters of the desired signal, of the parameters of the MAI (i.e., the number of interfering users, their spreading codes, propagation delays, and, possibly, their channel tap weights) and of the NBI parameters (i.e., of the type of external interference and of its covariance matrices). Leaving aside the effect of the external interference for the moment, it is clear that relying on so much prior knowledge is plausible in the reverse link of a cellular system, where the base station has the tasks of synchronizing and detecting the information symbols transmitted by all active users. Conversely, when considering the forward link, both complexity and privacy reasons lead to the conclusion that each mobile transceiver can rely upon knowledge of the relevant parameters (i.e., spreading code, propagation delay, and complex channel gains) of the signal of interest only. The problem, thus, arises of adaptive interference suppression, i.e., of devising detection structures that can be implemented with no prior information on the overall interference and that exhibit performance levels close to those of the nonadaptive multiuser receivers discussed so far.

Adaptive multiuser detectors may also be useful in the base station. In particular, new users may enter the channel and some others may leave it at random instants, due to the birth and death of calls and to handoffs and these variations in the interference structure need to be incorporated into the nonadaptive multiuser receivers. This process may, in some instances, be too cumbersome and, also, requires heavy signaling protocols. Additionally, considering the presence of external NBI and, for a cellular system, the *extracell* interference, i.e., the signals originated by the active users outside the cell under study, it is clear that adaptivity may be needed in base station processing as well. Moreover, in densely populated urban areas, wireless communications may rely exclusively upon multipath, which is in turn time varying due to the relative motion between transmitter and receiver. This dynamism naturally leads to the issue of joint adaptive equalization and interference suppression.

Fortunately, most techniques for adaptive equalization [69] for band-limited single-user channels can be imported to this new scenario, since these two problems have much in common. Among adaptive multiuser detectors we find both trained and untrained (or blind) techniques. The former procedures rely on periodic interleaving of a known symbol sequence with the data stream, typically in the midamble of a packet, which allows channel state estimation and receiver updating. The main drawback of trained procedures is that the insertion of training sequences may need to be done frequently in dynamic situations and can thus affect the transmission efficiency.

Blind systems, instead, are able to extract the CSI and accomplish system adaptation without training sequences. As outlined in [14], the inherent "self-recovery" ability of blind systems may be of great importance in point-to-multipoint communications. The problem of adaptive interference suppression in CDMA systems has been investigated extensively [21], [23]–[27], [31], [32], [59]–[63], [70]–[72], [90], [104], [108], [113], [121], [129], [131], [134], [152], [153], [160], [183], [184], [196], [204]–[206]. In this section, we will describe these results as they apply to the NBI suppression problem. In what follows, we assume that the receiver has knowledge of the channel tap weights, spreading code, and propagation delay for the user of interest. Even though these are the most customary assumptions, it is worth pointing out that, for fading dispersive channels, a more recent trend is to relax the assumption of knowing the propagation channel impulse response and to obtain (possibly blind) adaptive algorithms that do not need any prior CSI [31], [60], [183], [184], [204]. Also, alternative algorithms that exploit the finite-alphabet size of the CDMA multiplex signaling constellation and/or its higher order statistics and do not require knowledge of the desired user' spreading code have been proposed as well (see, e.g., [103], [113], [171], [204], and [222]). These techniques, however, typically require substantial computational effort and are of interest mainly for specialized applications at this time.

Paralleling the organization of the previous sections, we first consider the case of nonfading channels and then move on to the case of fading dispersive channels, wherein adaptive equalization is required as well. In the following, we focus on the problem of (possibly blind) adaptive implementation of the MMSE receivers. These receivers have been shown to outperform the ZF receivers and can be used in recursive adaptive form through the aid of the well-known LMS and recursive-least-squares (RLS) algorithms. As to the adaptive implementation of the ZF, we note that this problem has been considered and solved recently in [205], [206], wherein it is shown that the ZF receiver for MAI suppression can be implemented in a blind recursive fashion by resorting to subspace tracking algorithms [39], [44], [174], [176], [219]. The results presented in [205], [206] can be applied, with no modification, for blind implementation of a ZF receiver in the case that the overall interference amounts to a stationary NBI and to the MAI. The blind adaptation of the ZF detector in the case of nonstationary NBI, instead, has not yet been addressed in the open literature and will not be considered here.

A. Adaptive TI Systems in Non-Fading Channels

To begin with, let us consider the case of a nonfading overlay channel with stationary NBI, as outlined in [152], [153]. The variations of the observables' covariance matrix are thus due to possible (long-term) variations in either the user signal parameters, in their number, or in the NBI signal. In this context, "long-term" just means that these covariance properties may be considered to be constant for several hundreds (if not thousands) of symbol intervals of the CDMA network which, for typical symbol rate values, implies as-

suming a channel coherence time at most on the order of fractions of seconds.

To understand the blind algorithms, we first consider more traditional trained adaptive algorithms. The problem of interest can thus be stated as follows. Given the observables $\boldsymbol{r}(0), \ldots, \boldsymbol{r}(n)$ of (14), the signature $\boldsymbol{s}_0$ of user 0, and the information symbols $b_0(0), \ldots, b_0(n)$, find an estimate (possibly up to a positive scaling factor) at epoch n, $\widehat{\boldsymbol{d}}_{\mathrm{MMSE}}$, say, of the MMSE multiuser receiver $\boldsymbol{d}_{\mathrm{MMSE}}$ in (44). Luckily enough, in this situation, which is indeed similar to what happens if no NBI is active, adaptive implementations of the MMSE multiuser receiver can be readily obtained through a direct application of the large body of available knowledge on adaptive equalization for band-limited communication channels [69].

For example, the LMS (or stochastic gradient) adaptive strategy implements the following $\mathcal{O}(NM)$ updating rule[6]:

$$\widehat{\boldsymbol{d}}_{\mathrm{LMS}}(n) = \widehat{\boldsymbol{d}}_{\mathrm{LMS}}(n-1)$$
$$-\mu \left(\boldsymbol{r}(n)\boldsymbol{r}^H(n)\widehat{\boldsymbol{d}}_{\mathrm{LMS}}(n-1) - b_0(n)\boldsymbol{r}(n) \right). \quad (106)$$

In the above equation, the step size μ must be chosen carefully to avoid, on one hand, system instability and, on the other hand, too slow convergence toward the steady state. Recently, modified version of the LMS algorithm (based on the use of iterate averaging and adaptive step sizes) have been proposed and applied to CDMA systems in order to speed up convergence (see [90], [92], and references therein) and, even more recently, to multirate CDMA systems [16]. These algorithms, which can be directly applied to suppress NBI, too, will not be reviewed in the present survey. Of course the system (106) needs a training phase, since the symbols of the user of interest appear explicitly; once the training phase is over (i.e., beyond the nth signaling interval), the system can switch to decision-directed mode.

Besides the LMS algorithm, another useful adaptive algorithm is RLS, which achieves faster convergence at the cost of increased computational complexity. The RLS-based implementation of the MMSE multiuser receiver may be obtained through the minimization of the following exponentially windowed time average:

$$\sum_{i=0}^{n} \lambda^{n-i} \left| b_0(i) - \widehat{\boldsymbol{d}}_{\mathrm{RLS}}^H(n)\boldsymbol{r}(i) \right|^2 \quad (107)$$

with λ a close-to-unity forgetting factor aimed at ensuring the tracking capabilities of the algorithm. On setting the gradient of the above function to zero and solving, we have the optimal solution [152]

$$\widehat{\boldsymbol{d}}_{\mathrm{RLS}}(n) = \boldsymbol{R}^{-1}(n) \sum_{i=0}^{n} \lambda^{n-i} b_0(i)\boldsymbol{r}(i) \quad (108)$$

where the matrix

$$\boldsymbol{R}(n) = \sum_{i=0}^{n} \lambda^{n-i} \boldsymbol{r}(i)\boldsymbol{r}^H(i) \quad (109)$$

---

[6]Herein, we use the usual Landau notation $O(\ell)$; hence, an algorithm is $O(\ell)$ if its implementation requires a number of flops proportional to ℓ [66].

is the exponentially weighted sample covariance matrix, i.e., it is an estimate of the covariance matrix of the observables. Applying standard signal processing techniques, the solution (108) can be computed recursively with the following $\mathcal{O}((NM)^2)$ procedure:

$$\boldsymbol{k}(n) = \frac{\boldsymbol{R}^{-1}(n-1)\boldsymbol{r}(n)}{\lambda + \boldsymbol{r}^H(n)\boldsymbol{R}^{-1}(n-1)\boldsymbol{r}(n)}$$

$$\boldsymbol{R}^{-1}(n) = \frac{1}{\lambda}\left[\boldsymbol{R}^{-1}(n-1) - \boldsymbol{k}(n)\boldsymbol{r}^H(n)\boldsymbol{R}^{-1}(n-1)\right]$$

$$\widehat{\boldsymbol{d}}_{\text{RLS}}(n) = \widehat{\boldsymbol{d}}_{\text{RLS}}(n-1) + \epsilon(n)\boldsymbol{k}(n) \quad (110)$$

where $\epsilon(n) = b_0(n) - \widehat{\boldsymbol{d}}_{\text{RLS}}^H(n-1)\boldsymbol{r}(n)$ is the prediction error at time n and $\boldsymbol{k}(n)$ is the Kalman gain. As with LMS, once the training phase is over, the receiver can switch to decision-directed mode.

The issue of the convergence of these adaptation rules to the true solution is rather lengthy and has been dealt with in the context of joint NBI and MAI suppression in [152] and [153] for the RLS and the LMS procedures, respectively. It suffices here to recall that the steady-state SIR of the RLS algorithm (110) can be expressed, under mild conditions (see [152] for the details) as a function of the SIR of the nonadaptive MMSE SIR_{MMSE} of (64) as

$$\text{SIR}_{\text{RLS}} = \frac{\text{SIR}_{\text{MMSE}}}{1 + \beta + \frac{\beta}{\text{SIR}_{\text{MMSE}}}} \quad (111)$$

where $\beta = NM(1-\lambda)/2\lambda$ determines the steady-state excess mean-square error.

Even though simulation results show that the steady-state SIR of RLS may, in many instances of practical interest, be very close to optimum, both RLS and LMS may be inadequate for coping with adverse scenarios. In particular, if the system is operating in decision-directed mode and a sudden variation occurs in either the NBI or the MAI, the effect on the performance may be catastrophic, due to the inherent lack of any self-recovering capability. Thus, decision-directed systems, like decision-directed equalizers, when used on radio channels, should either rely on frequent training phases or be complemented by blind self-recovering systems that do not require any training.

The theoretical background to derive blind systems for this purpose has been laid in Section III. In particular, recall the equivalence between the MMSE and the MMOE criteria. While the MSE cost function depends on the transmitted symbols, the MOE cost function depends only on quantities that are available at the receiver (i.e., the observables themselves), which means that it can be minimized with no need for training symbols. We now use this idea to develop blind adaptive algorithms for the interference suppression problem. For simplicity, in what follows, we assume that α_0 is real, i.e., that the phase offset of the signal of interest has been compensated for. This is not an unrealistic assumption since the phase of α_0 must be known to the receiver in order to be able to carry out coherent detection.

An LMS-based blind implementation of the MMSE multiuser receiver has been proposed in [71] and in [153] in the context of MAI suppression and NBI suppression in SS sys-

tems, respectively, while modified versions of this algorithm have been reported, e.g., in [43], [162], and [170]. This algorithm is based on the following canonical decomposition of any linear multiuser receiver:

$$\boldsymbol{d} = \boldsymbol{s}_0 + \boldsymbol{x}, \quad \text{with } \boldsymbol{s}_0^H\boldsymbol{x} = 0. \quad (112)$$

Otherwise stated, the vector $\boldsymbol{d}$ is written as the superposition of the useful signature and of a vector orthogonal to this signature. Note that, for any choice of $\boldsymbol{x}$ orthogonal to $\boldsymbol{s}_0$, the vector $\boldsymbol{d}$ in (112) fulfills the constraint in (42) and the adaptive algorithm updates only the vector $\boldsymbol{x}$. Following [153], the appropriate LMS algorithm for blind MMOE detection can be derived as

$$\boldsymbol{x}_{\text{BLMS}}(n) = \boldsymbol{x}_{\text{BLMS}}(n-1) - \mu$$
$$\left[(\boldsymbol{s}_0 + \boldsymbol{x}_{\text{BLMS}}(n-1))^H \boldsymbol{r}(n)\right]^* \left[\boldsymbol{r}(n) - \left(\boldsymbol{s}_0^H\boldsymbol{r}(n)\right)\boldsymbol{s}_0\right]$$
$$(113)$$

where, as before, μ is a properly chosen step size and the fact that $\|\boldsymbol{s}_o\| = 1$ has been exploited (the acronym BLMS stands for blind LMS).

A blind RLS (BRLS) algorithm can be obtained by considering the following exponentially windowed constrained optimization problem:

$$\begin{cases} \widehat{\boldsymbol{d}}_{\text{BRLS}}(n) = \arg\min_{\boldsymbol{d}} \sum_{i=0}^n \lambda^{n-i} \left|\boldsymbol{d}^H\boldsymbol{r}(i)\right|^2 \\ \text{subject to} \quad \boldsymbol{d}^H\boldsymbol{s}_0 = 1 \end{cases} \quad (114)$$

Applying standard Lagrangian minimization techniques to solve the constrained minimization (114) we find [152]:

$$\widehat{\boldsymbol{d}}_{\text{BRLS}}(n) = \frac{1}{\boldsymbol{s}_0^H\boldsymbol{R}^{-1}(n)\boldsymbol{s}_0}\boldsymbol{R}^{-1}(n)\boldsymbol{s}_0 = \gamma(n)\boldsymbol{R}^{-1}(n)\boldsymbol{s}_0. \quad (115)$$

Notice that the positive factor $\gamma(n)$ can be excluded from the recursion, whereby an equivalent solution is $\widehat{\boldsymbol{d}}_{\text{BRLS}}'(n) = \boldsymbol{R}^{-1}(n)\boldsymbol{s}_0$, which is amenable to an $\mathcal{O}((NM)^2)$ updating through coupling first two equations in (110) with

$$\widehat{\boldsymbol{d}}_{\text{BRLS}}'(n) = \frac{1}{\lambda}\left[\widehat{\boldsymbol{d}}_{\text{BRLS}}'(n-1) - \boldsymbol{k}(n)\boldsymbol{r}^H(n)\widehat{\boldsymbol{d}}_{\text{BRLS}}'(n-1)\right]. \quad (116)$$

Note that the lack of a reference signal in the cost function (114) results, as expected, in a blind procedure. A detailed convergence study of the BRLS algorithm is reported in [152]. In particular, it is shown therein that, under mild conditions, the vector $\widehat{\boldsymbol{d}}_{\text{BRLS}}(n)$ converges in the mean value to the nonadaptive MMOE solution and that this convergence is independent of the eigenvalue distribution of the covariance matrix of the observables. Additionally, with regard to the steady-state SIR, the following relation can be proven:

$$\text{SIR}_{\text{BRLS}} \simeq \frac{\text{SIR}_{\text{MMSE}}}{1 + \beta + \beta\text{SIR}_{\text{MMSE}}}. \quad (117)$$

Usually, the RLS algorithm operates in the range $\beta \ll 1$; in particular, for large values of the optimum SIR, it is seen that the steady-state SIR of the BRLS algorithm is upper bounded by $1/\beta$. Conversely, from (111), it is seen that the trained RLS algorithm does not exhibit this performance impairment. The price to be paid in order to avoid the
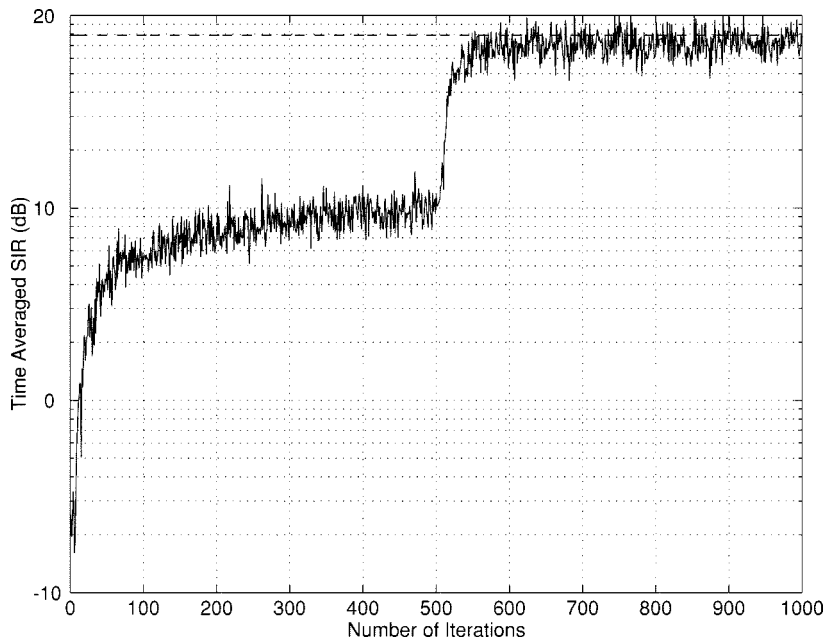
Fig. 13. Convergence dynamics of the blind and decision-directed RLS algorithms. At time epoch $n = 500$, the receiver switches from blind to decision-directed RLS. NBI is a second-order AR process with both poles at 0.99. Other parameters: $\gamma_0 = 20$ dB, $K = 4$, $N = 31$. Dashed line corresponds to the optimum SIR, achieved by the nonadaptive MMSE receiver.

transmission of known training symbols is thus a degraded steady-state performance with respect to nonblind RLS. As a consequence, a good strategy to keep the advantages of both systems is to start with the blind system and switch to decision-directed mode as soon as the blind system converges. Of course, decision-directed adaptation is still subject to heavy performance degradation and error propagation in the event of sudden changes in the environment. Whenever such situations occur, the receiver should, thus, immediately switch back to the blind adaptation mode and stay in that mode until performance is restored. Alternatively, one could use the system recently proposed in [31], wherein a decision-directed system is driven by the decisions of the BRLS algorithm. Finally, another possibility for improving on the performance of the BRLS algorithm is to adopt the subspace approach, which was introduced in [205], [206]. Basically, in these papers it is shown that both the linear MMSE and ZF receivers can be expressed in terms of signal space parameters, i.e., in terms of the dominant eigenvalues (and corresponding eigenvectors) of the observables' covariance matrix. It, thus, follows that subspace tracking algorithms, which recursively update the dominant eigenvalues and eigenvectors of the covariance matrix of the received signal, can be employed to obtain blind implementations of the linear multiuser receivers. In [205], it is shown, with reference to a system with no NBI, that the subspace-based blind MMSE receiver achieves a steady-state SIR larger than that of the BRLS algorithm, is robust to the signature waveform mismatch problem (which might arise due, for example, to imperfect timing estimation), and can be implemented with a computational complexity lower than the BRLS. In this survey, we will not dwell further on the application of subspace tracking algorithms to blind adaptive MUD, since

their application to joint suppression of NBI and MAI has not yet appeared in the literature. Nonetheless, we observe that a preliminary study on the application of this approach in multirate CDMA systems has recently appeared in [17].

The superiority of the trained RLS algorithm relative to the BRLS procedure is confirmed by the simulation results in Fig. 13, which is taken from [152]. This example assumes a synchronous CDMA system with $\gamma_0 = 20$ dB, $K = 4$ users, processing gain $N = 31$, and NBI consisting of a second-order AR process with both poles at 0.99. The MAI signals have a 10-dB power advantage with respect to the user of interest, while the NBI power advantage is 20 dB. Fig. 13 shows the TA output SIR averaged over 100 independent simulation runs versus time for the BRLS algorithm, which is employed in the first 500 signaling intervals and for the trained RLS algorithm, operating in decision-directed mode, which is employed starting from epoch 500. The forgetting factor λ has been set to 0.995. Fig. 13, thus, confirms the noticeable performance gap that may exist between the blind and the trained RLS algorithm, which achieves an SIR very close to the optimum one (represented by the dashed line in the figure).

In the above discussions, we have considered only the issue of blind adaptive implementation of the classical MMSE receiver. As already mentioned, this receiver represents a suboptimum structure if the observables are a complex improper random process. In this case, full exploitation of the correlation properties of the interference requires use of the modified receivers of Section III-C. In what follows, we discuss briefly the problem of adaptive implementation of these receivers and, in particular, we focus on the RLS and BRLS algorithms.

Consistent with the above derivations, the key issue is to devise proper TA cost functions to be used in lieu of their *ensemble* averaged counterparts. Thus, for a trained procedure, the weighted time-average equivalent of the minimization problem (53) is

$$\widehat{\boldsymbol{d}}_{\text{RLS}}^{H}(n) = \arg\min_{\boldsymbol{d}} \sum_{i=0}^{n} \lambda^{n-i} \left[\Re\left\{ \boldsymbol{d}^{H}\boldsymbol{r}(i) \right\} - b_{0}(i) \right]^{2} \quad (118)$$

while, in order to derive the BRLS algorithm, we have to consider the following constrained optimization problem:

$$\begin{cases} \widehat{\boldsymbol{d}}_{\text{BRLS}}^{H}(n) = \arg\min_{\boldsymbol{d}} \sum_{i=0}^{n} \lambda^{n-i} \left[\Re\left\{ \boldsymbol{d}^{H}\boldsymbol{r}(i) \right\} \right]^{2} \\ \text{subject to} \quad \Re\left\{ \boldsymbol{d}^{H}\boldsymbol{s}_{0} \right\} = 1 \end{cases}.$$
$$(119)$$

Skipping the mathematical details, we simply recall here that the above problems can be reduced to conventional RLS problems once they are formulated in terms of the augmented vectors introduced in Section III-C. Thus, the solutions are easily found in terms of such augmented vectors. In particular, solving problem (118) and denoting by $\widehat{\boldsymbol{d}}_{a,\text{RLS}}(n)$ its augmented solution, it can be shown that the following $\mathcal{O}((2NM)^2)$ updating procedure can be used:

$$\boldsymbol{k}_{a}(n) = \frac{\boldsymbol{R}_{a}^{-1}(n-1)\boldsymbol{r}_{a}(n)}{\lambda + \boldsymbol{r}_{a}^{H}(n)\boldsymbol{R}_{a}^{-1}(n-1)\boldsymbol{r}_{a}(n)}$$
$$\boldsymbol{R}_{a}^{-1}(n) = \frac{1}{\lambda}\left[\boldsymbol{R}_{a}^{-1}(n-1) - \boldsymbol{k}_{a}(n)\boldsymbol{r}_{a}^{H}(n)\boldsymbol{R}_{a}^{-1}(n-1) \right]$$
$$\widehat{\boldsymbol{d}}_{a,\text{RLS}}(n) = \widehat{\boldsymbol{d}}_{a,\text{RLS}}(n-1) + \epsilon'(n)\boldsymbol{k}_{a}(n) \quad (120)$$

where $\epsilon'(n) = b_0(n) - \boldsymbol{r}_a^H(n)\widehat{\boldsymbol{d}}_{a,\text{RLS}}(n-1)$, while $\boldsymbol{k}_a(n)$ is the augmented Kalman gain.

Likewise, the *enlarged* solution to the problem (119), which we denote by $\widehat{\boldsymbol{d}}_{a,\text{BRLS}}(n)$ can be updated by coupling the first two equations in (120) with the following recursion:

$$\widehat{\boldsymbol{d}}_{a,\text{BRLS}}(n) = \frac{1}{\lambda}$$
$$\cdot \left[\widehat{\boldsymbol{d}}_{a,\text{BRLS}}(n-1) - \boldsymbol{k}_{a}(n)\boldsymbol{r}_{a}^{H}(n)\widehat{\boldsymbol{d}}_{a,\text{BRLS}}(n-1) \right]. \quad (121)$$

The computational complexity is $\mathcal{O}((2NM)^2)$ in this case as well.

B. Adaptive PTV Systems in Nonfading Channels

If the hypothesis of stationary NBI is relaxed, e.g., if the NBI either is a data-like signal whose symbol interval is not an integer submultiple of the CDMA symbol interval or is a secondary CDMA system transmitting at a lower rate, then the covariance matrices of the observables exhibit, besides the long-term variations induced by the channel, a symbol-rate variation, i.e., they are PTV. Thus, even though the channel coherence time is large with respect to the CDMA symbol interval, tracking the receiver via the previously discussed TI systems would lead to an unsatisfactory steady state SIR, as discussed in Section III, especially for overloaded networks. Thus, these adaptive algorithms must be properly modified, so as to explicitly take into account the

fact that the solution to be tracked is PTV. In what follows, we present a cyclic version of the RLS algorithm, which has been introduced and analyzed in [20], [23]–[26]. Cyclic versions of the LMS algorithm may be derived as well, along the same lines presented here. Preliminary results on this issue have recently appeared in [16].

To fix the ideas, let us assume that we are interested in the estimation of a period Q PTV linear MMSE receiver. The estimation problem requires estimating, based upon the observables $\{\boldsymbol{r}(i)\}_{i=0}^{n}$, a sequence $\widehat{\boldsymbol{d}}(n,i)$, periodic in its second argument with period Q, which is the best estimate, according to the MMSE criterion, available at time n. The adaptive PTV receiver implements the decision rule

$$\widehat{b}_{0}(p) = \Re\left\{ \widehat{\boldsymbol{d}}^{H}(p, p \bmod Q)\boldsymbol{r}(p) \right\}. \quad (122)$$

In principle, one could solve this problem by running Q parallel RLS algorithms to update the Q different vectors of the desired PTV sequence. Each of the Q RLS procedures would, thus, accept decimated versions of the observables, i.e., the sample $\boldsymbol{r}(i)$ would be fed to the $i \bmod Q$th RLS processor.

Alternatively, the RLS problem can be reformulated in terms of the desired PTV estimate $\widehat{\boldsymbol{d}}(n,i)$, wherein the former index denotes the number of observations, while the latter index denotes which of the Q different receivers is being estimated. To illustrate further, let us focus on the derivation of the blind cyclic RLS algorithm. We, thus, consider the following constrained optimization problem:

$$\begin{cases} \sum_{i=0}^{n} \lambda^{n-i} \left| \widehat{\boldsymbol{d}}_{\text{BRLS}}^{H}(n,i)\boldsymbol{r}(i) \right|^{2} = \min \\ \widehat{\boldsymbol{d}}_{\text{BRLS}}^{H}(n,i)\boldsymbol{s}_{0} = 1, \quad \forall i = 1, \ldots, Q \end{cases}. \quad (123)$$

The apparent difficulty in solving the above problem is the time-varying structure of the desired solution. However, (123) can be readily reduced to a TI form as follows. The idea is that, instead of tracking a PTV solution, it is possible to track the supervector of its Fourier-series expansion, which is QNM-dimensional. In particular, the unknown vector sequence $\widehat{\boldsymbol{d}}_{\text{BRLS}}(n,i)$ can be replaced by its Fourier series expansion, i.e.,

$$\widehat{\boldsymbol{d}}_{\text{BRLS}}(n,i) = \sum_{i=0}^{Q-1} \widehat{\boldsymbol{d}}_{\text{BRLS}}^{(i)}(n)e^{j(2\pi in/Q)}. \quad (124)$$

After some lengthy algebraic manipulations, the problem (123) can, thus, be reformulated as

$$\begin{cases} \sum_{i=0}^{n} \lambda^{n-i} \left| \widetilde{\boldsymbol{d}}_{\text{BRLS}}^{H}(n)\widetilde{\boldsymbol{r}}(i) \right|^{2} = \min \\ \widetilde{\boldsymbol{s}}_{0}^{H}\widetilde{\boldsymbol{d}}_{\text{BRLS}}(n) = \boldsymbol{e}_{1} \end{cases} \quad (125)$$

with $\widetilde{\boldsymbol{d}}_{\text{BRLS}}(n)$ the QNM-dimensional vector of the Fourier coefficients of the filter $\widehat{\boldsymbol{d}}_{\text{BRLS}}(n,i)$, i.e.,

$$\widetilde{\boldsymbol{d}}_{\text{BRLS}}(n) = \left[\widehat{\boldsymbol{d}}_{\text{BRLS}}^{(0)T}(n), \widehat{\boldsymbol{d}}_{\text{BRLS}}^{(1)T}(n), \ldots, \widehat{\boldsymbol{d}}_{\text{BRLS}}^{(Q-1)T}(n) \right]^{T}.$$
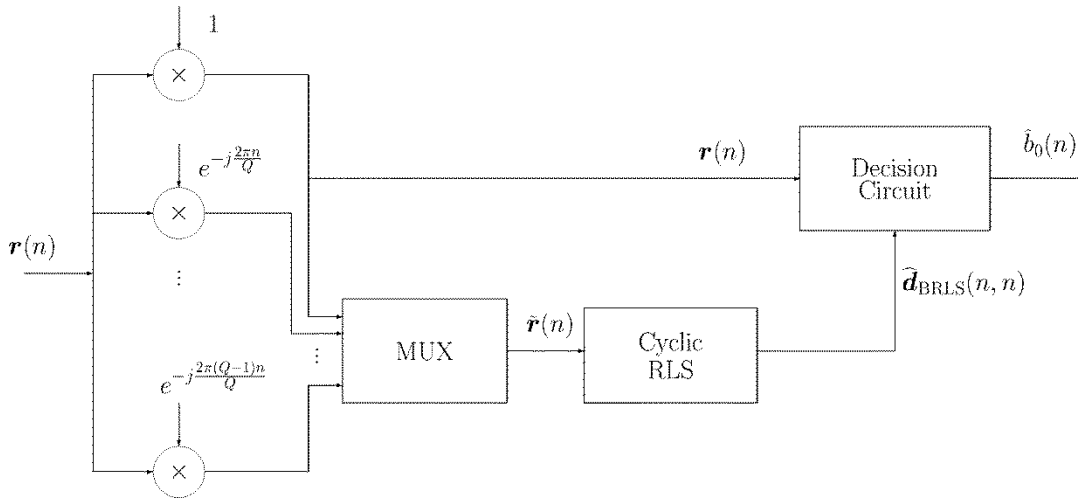
Fig. 14. FRESH implementation of the cyclic RLS algorithm.

The vectors $\widetilde{r}(i)$ are the frequency-shifted versions of the observables, i.e.,

$$\widetilde{r}(i) = \left[r^H(i), r^H(i)e^{j(2\pi i/Q)}, \ldots, \right.$$
$$\left. r^H(i)e^{j(2\pi(Q-1)i/Q)}\right]^H \tag{126}$$

and

$$\widetilde{s}_0 = I_Q \otimes s_0, \quad e_1 = \left[1, \underbrace{0, \ldots, 0}_{Q-1}\right]^T \tag{127}$$

where $\otimes$ denotes the Kronecker product. The minimization of (125) is a canonical problem in that the desired quantity is time invariant. As a consequence, applying standard Lagrangian techniques, we obtain the following recursions [25]:

$$\widetilde{k}(n) = \frac{\widetilde{R}^{-1}(n-1)\widetilde{r}(n)}{\lambda + \widetilde{r}^H(n)\widetilde{R}^{-1}(n-1)\widetilde{r}(n)}$$

$$\widetilde{R}^{-1}(n) = \frac{1}{\lambda}\left(\widetilde{R}^{-1}(n-1) - \widetilde{k}(n)\widetilde{r}^H(n)\widetilde{R}^{-1}(n-1)\right)$$

$$H(n,n) = \Psi(n)\widetilde{R}^{-1}(n)$$

$$= \frac{1}{\lambda}\left(\Psi(1)H(n-1,n-1)\right.$$

$$\left. - \Psi(1)H(n-1,n-1)\widetilde{r}(n)\widetilde{k}^H(n)\right)$$

$$g(n,n) = e_2^H \otimes I_{NM} H(n,n)\widetilde{s}_0 e_1$$

$$\widehat{d}_{BRLS}(n,n) = \frac{g(n,n)}{(s_0^H g(n,n))} \tag{128}$$

where

$$\Psi(n) = \text{Diag}$$
$$\cdot \left(\left[1, e^{j(2\pi n/Q)}, e^{j(2\pi 2n/Q)}, \ldots, e^{j(2\pi(Q-1)n/Q)}\right]^T\right)$$
$$\otimes I_{NM},$$
$$e_2 = \underbrace{[1, 1, \ldots, 1]}_{Q}^T \tag{129}$$

$$\widetilde{R}(n) = \sum_{i=1}^n \lambda^{n-i}\widetilde{r}(i)\widetilde{r}^H(i). \tag{130}$$

A diagram of this receiver is shown in Fig. 14. Also in this case, a bank of complex oscillators keyed to the harmonic frequencies of the observables' covariance matrix is used. The output of this bank is then forwarded to a block that implements the cyclic RLS algorithm (128) and forms the estimate $\widehat{d}_{BRLS}(n,n)$ at each symbol interval n. It is worth pointing out that adaptive suboptimal systems, i.e., those that retain only a subset of the harmonic frequencies of their optimum counterparts, can be easily obtained with the above procedure by simply forcing some coefficients in (124) to zero. This ease in devising suboptimal simplified structures is a significant advantage of cyclic RLS over running Q parallel TI RLS procedures. Both of these two approaches lead to the same computational complexity; since the matrix (130) is block-circulant, it can be shown that the above blind cyclic recursive algorithm entails a computational complexity $\mathcal{O}(Q(NM)^2)$. However, no definite conclusion can be drawn as to which approach is superior in terms of tracking capability and convergence speed. There is no way to simplify the parallel TI system, since not less than Q parallel branches must be built and operated; in a cyclic scheme, on the other hand, a number of parallel branches can be dropped, based, e.g., on the energy content of the harmonic components of the signal correlation.

In order to highlight the merits of the cyclic BRLS algorithm, in Fig. 15 (taken from [25]), we show the normalized correlation coefficient between the true solution and the estimated one for the TI and the cyclic BRLS algorithms. We consider a synchronous DS/CDMA system with processing gain $N = 31$. The external interference is a secondary DS/CDMA system transmitting at a rate three times slower than that of the primary network (this implies that $Q = 3$). The plots are the result of an average over 100 random independent runs. The upper curve refers to a scenario with $K = 5$ users and $K_I = 12$; all of the signals have the same amplitude (i.e., perfect power-control is assumed) and $\gamma_0 = 13$ dB. The lower curve, instead,
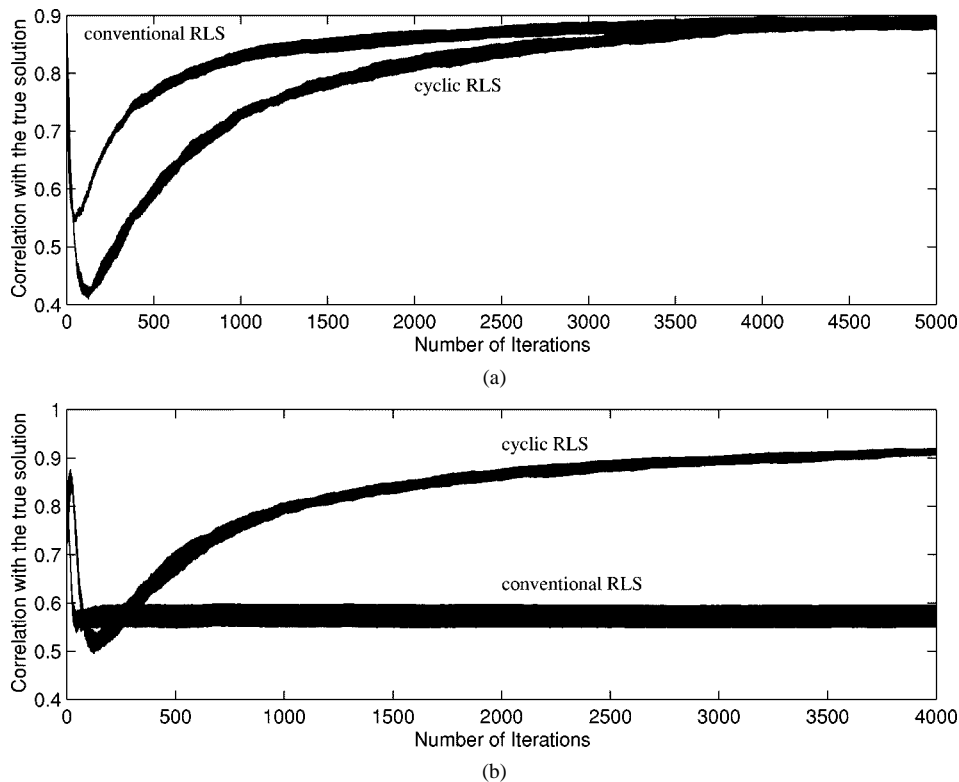
Fig. 15. Convergence dynamics of the cyclic and of the conventional BRLS algorithms. The NBI is a lower rate CDMA system. System parameters: $N = 31$, $K = 5$, $K_I = 12$, and $Q = 3$. (a) Power-controlled scenario. (b) Low-rate users' amplitudes are 20 dB stronger than the high-rate users' amplitudes.

is similar to the former one, except for the fact that the low-rate users' amplitudes are assumed to be 20 dB stronger than the high-rate amplitudes. It is seen that in the upper curve the cyclic algorithm and the conventional BRLS achieve the same asymptotic value, but the convergence time of the cyclic algorithm is noticeably larger than that of the conventional one. In the lower curve, instead, the cyclic RLS algorithm largely outperforms the conventional one. This behavior may be explained by noticing that, in a perfectly power-controlled situation, the time variations of the interference covariance matrix are not very strong, so that the conventional RLS algorithm achieves satisfactory performance. Likewise, the cyclic algorithm achieves good performance as well, but converges more slowly due to the fact that it operates on vectors with a dimensionality Q times larger. In the lower curve, instead, the severe near–far scenario emphasizes the periodicity of the NBI covariance matrix and so the conventional RLS algorithm is not able to track its variations. Other simulations, whose results are not shown here for the sake of brevity, have revealed that the conventional algorithm performs worse and worse as the number of users increases, even if perfect power control is assumed; the cyclic algorithm, instead, is quite insensitive to the network load (see also [23] and [25] for further details).

So far, we have considered the cyclic version of the BRLS algorithm only. Decision-directed procedures can easily be derived following the same steps, as can the corresponding trained and blind procedures for the modified receivers exploiting pseudocovariances. In this latter case, the RLS

problem formulation and its solution parallel exactly the one just presented, once the augmented vectors are introduced. A caveat here is that the size of the vectors to be tracked in adaptive modified receivers may end up noticeably larger, since, on one hand, the period Q becomes now the common period of the covariance and the pseudocovariance, while, on the other hand the augmentation operation produces a further doubling of the vector sizes. This fact obviously has some impact on the convergence speed of the adaptive solution. A trained cyclic RLS implementation of the modified MMSE receiver has been presented in [24].

C. Adaptive Systems in Fading Dispersive Channels

Adaptive techniques for suppressing cochannel interference in fading dispersive channels parallel those just described for the case of distortionless channels. In particular, LMS algorithms can be obtained through the application of stochastic-gradient adaptation rules, while, to derive RLS-based algorithms, the first step is to consider exponentially weighted TA cost functions in place of statistical expectations.

Thus, MMSE receivers give rise to decision-directed procedures, while MMOE problems result in blind receivers. Once again, unlike entropic NBI, quasi-deterministic NBI may result in either time invariant or PTV adaptive systems, depending on whether they are isochronous or anisochronous with respect to the bit-interval of the CDMA network. Likewise, modified receivers, i.e., those exploiting, under CCSI,

both the covariance and the pseudocovariance of the observables, can easily be derived by resorting to a set of augmented vectors and rewriting the relevant cost functions of MMSE and MMOE in an exponentially weighted form.

To give the general idea of these approaches, we consider the issue of blind adaptive TI systems under ICSI, which also sheds light on what happens under CCSI. The interested reader is referred to [18], [23], and [26] for a treatment of more general situations. Before proceeding in the discussion, though, it is worthwhile to make some preliminary remarks on the new issues arising when dealing with fading channels. To be more definite, let us consider the MMSE solution $\boldsymbol{H}_{\text{MMSE}}(p)$ given in (81) for the interference blocking matrix under ICSI and the MMSE criterion. Leaving aside complexity issues, it is obvious that a viable means to make the system adaptive is to form a batch estimate $\widehat{\boldsymbol{M}_{\boldsymbol{rr}}}$ of the observable covariance matrix and to substitute it into (81) in place of the true covariance, which yields

$$\widehat{\boldsymbol{H}}_{\text{MMSE}} = \sqrt{\mathcal{E}_0}\widehat{\boldsymbol{M}}_{\boldsymbol{rr}}^{-1}\boldsymbol{S}_{00}\Sigma_0. \quad (131)$$

In the above equation, the dependence on the epoch p has been omitted, since we are dealing with a time-invariant situation. The estimate $\widehat{\boldsymbol{M}_{\boldsymbol{rr}}}$ could instead be the sample covariance matrix of a set of N_E samples taken at some previous epochs. Two limiting situations are of interest when commenting on (131), with the understanding that practical cases lie somewhere between the following.

1) The channel coherence time is small with respect to $N_E T_b$, so that a given realization of the channel tap weights, when observed over time intervals on the order of $N_E T_b$, spans the same set of values spanned by the whole random process in the *ensemble* of the realizations. Otherwise stated, the channel tap weights appear as ergodic processes over time intervals on the order of $N_E T_b$.

2) The channel coherence time is large with respect to $N_E T_b$, so that the channel tap weights are constant random vectors for the whole duration of the estimation phase. Otherwise stated, the channel, as observed over time intervals on the order of $N_E T_b$, is *not* ergodic.

Applying standard tools of statistical analysis, it is readily seen that, in situation 1, under mild regularity conditions, we have, for sufficiently large N_E

$$\widehat{\boldsymbol{M}_{\boldsymbol{rr}}} \simeq \boldsymbol{M}_{\boldsymbol{rr}} \quad (132)$$

while, in situation 2 and under the same constraint on N_E, we have

$$\widehat{\boldsymbol{M}_{\boldsymbol{rr}}} \simeq \boldsymbol{M}_{\boldsymbol{rr}}|\boldsymbol{A}. \quad (133)$$

The above distinction, which may appear to be rather obvious and of limited theoretical importance, has a remarkable impact on the performance of adaptive systems in fading channels. In situation 1, it can easily be shown that the matrix $\widehat{\boldsymbol{H}}_{\text{MMSE}}$ in (131) is approximately equal to its nonadaptive counterpart (81), thus, implying that, for a sufficiently large

value of N_E, the performance of the adaptive system is practically indistinguishable from that of the nonadaptive MMSE receiver. Conversely, in situation 2, the matrix $\widehat{\boldsymbol{H}}_{\text{MMSE}}$ does not approach, for increasing N_E, the solution (81), as it depends on the particular realization of the channel tap weights. As already noticed when ICSI systems have been contrasted to CCSI systems (see the results in Figs. 10 and 11), this represents a net dimension saving (by a factor up to L, the diversity order). The conclusion is that slow fading may result for ICSI systems in a performance that is much closer to that of CCSI nonadaptive systems[7] than to that of ICSI nonadaptive systems.

After this lengthy discussion, we can move on to the issue of adaptive reduced-complexity implementation of the systems of Section IV-B. It is preliminarily important to notice that, under ICSI and in keeping with the discussion of Section IV-B, an adaptive receiver must track an entire matrix, i.e., it must form an estimate $\widehat{\boldsymbol{H}}(p)$, say, of the $NM \times L$ blocking matrix $\boldsymbol{H}(p)$, rather than a single direction. Once such a blocking stage is made adaptive, the output is either forwarded to a WF (entailing an $\mathcal{O}(L^3)$ complexity to eigendecompose the matrix $\widehat{\boldsymbol{H}}^H(p)\widehat{\boldsymbol{H}}(p)$) and a combiner or to a combiner only. On the other hand, as already mentioned, a BRLS estimate of the matrix $\boldsymbol{H}_{\text{MMSE}}(p)$ can be achieved upon reformulating the MMOE problem (84) in a TA form. Letting $\{\boldsymbol{r}(i)\}_{i=0}^n$ be the observations up to time n, the RLS-based blind adaptive implementation of the MMSE filter at epoch n can be obtained by solving the problem

$$\begin{cases} \widehat{\boldsymbol{H}}_{\text{BRLS}}^H(n) = \arg\min_{\boldsymbol{H}} \sum_{i=0}^n \lambda^{n-i}\left\|\boldsymbol{H}^H\boldsymbol{r}(i)\right\|^2 \\ \text{subject to trace}\left[\boldsymbol{H}^H\boldsymbol{S}_{0,0}\Sigma_0\right] = 1 \end{cases} \quad (134)$$

which yields the solution

$$\widehat{\boldsymbol{H}}_{\text{BRLS}}(n) = \frac{\boldsymbol{R}^{-1}(n)\boldsymbol{S}_{0,0}\Sigma_0}{\text{tr}\left(\Sigma_0^2\boldsymbol{S}_{0,0}^H\boldsymbol{R}^{-1}(n)\boldsymbol{S}_{0,0}\right)}. \quad (135)$$

Applying standard linear algebra, a recursive updating procedure is obtained by coupling the first two equations in (110) with

$$\boldsymbol{Q}(n) = \frac{1}{\lambda}\left[\boldsymbol{Q}(n-1) - \boldsymbol{k}(n)\boldsymbol{r}^H(n)\boldsymbol{Q}(n-1)\right]$$

$$\widehat{\boldsymbol{H}}_{\text{BRLS}}(n) = \frac{\boldsymbol{Q}(n)}{\Sigma_0\boldsymbol{S}_{0,0}^H\boldsymbol{Q}(n)}. \quad (136)$$

A rigorous convergence analysis of the above procedure, whose computational complexity is $\mathcal{O}(L(NM)^2)$, is outside the scope of the present paper. However, it is important here to adjust the considerations presented with reference to batch-estimation to the case of RLS adaptation. Situations 1 and 2 above, in the context of RLS-type bit-by-bit adaptation, give rise to the following two scenarios.

1) The channel coherence time is large with respect to the CDMA bit interval, but shorter than the convergence time of the adaptive procedure (136). This scenario is fairly realistic in that RLS procedures' convergence

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[7]Perfect coincidence is not achieved, in that ICSI systems also contain the fading covariance matrix.

times are typically on the order of hundreds of symbol intervals, which, for low bit rates, may yield a convergence time larger than the average decorrelation time of the channels' tap weights in rapidly time-varying channels.

2) The channel coherence time is large with respect to the convergence time of the RLS procedure as well, as will be the case if the CDMA network bit rate is on the order of hundreds of kilobits per second.

In situation 1, the exponentially weighted observables' covariance matrix can be shown, through standard convergence analysis techniques, to approach a biased version of the unconditional covariance matrix of the observables in the mean square sense, whereby the steady-state performance of the adaptive procedure is expected to be quite close, at least for the trained RLS algorithm, to that of its nonadaptive counterpart. In situation 2, instead, the estimated matrix will approach a biased version of the conditional covariance matrix of the observables, given $\boldsymbol{A}$, whereby the expected steady-state performance is *superior* to that of its nonadaptive counterpart, due to the already illustrated relevant dimensions-saving. For the sake of brevity, we do not present here plots of the convergence dynamics of the above algorithms and refer the interested reader to [23], [26], where extensive simulation results are provided that corroborate the effectiveness of the outlined procedures.

Finally, we now briefly consider the adaptive implementation of the modified receivers in fading channels. Once again, the derivation in this case parallels those already discussed. The only different and relevant issue is that, in situation 1, the modified receivers cannot be given an adaptive form. Indeed, in this case, the small channel coherence time implies that the sample pseudocovariance matrix, i.e., the time average over N_E symbol intervals of the quantity $\boldsymbol{r}(n)\boldsymbol{r}^H(n)$, converges to the unconditional pseudocovariance matrix, which, due to the properness of the fading process, is zero. In this situation, modified receivers thus reduce to the classical ones and do not bring any performance gain.

VI. Perspectives on New Research Areas and Open Problems

Despite the significant body of work that has been reviewed in the previous sections, there still remain a number of challenging and interesting open problems and unexplored research tracks on the topic of DS/CDMA overlay systems. This section is devoted to the discussion of what are some of the most interesting and promising open issues in this area.

As a starting point, we recall that the major characteristic of most of the proposed receivers is their PTV nature, induced by cyclostationary NBI whose period is not an integer submultiple of the CDMA bit interval, albeit being in a rational ratio with T_b. As far as the resulting period is small enough, there is no actual objection to using the optimized PTV systems in the original structure resulting from the optimization criterion. Also, it has been shown that the receiver complexity may be reduced by resorting to a FRESH structure and retaining just a limited number of harmonics,

e.g., those corresponding to larger signal energies. Section V, however, has also demonstrated that, when adaptive detection comes into play, the solution to be tracked is of crucial importance and ultimately determines the rate of convergence to the steady-state performance. Also, it has been shown that resorting to time invariant, reduced complexity systems in a PTV scenario does not produce a dramatic performance impairment when the users' powers are well balanced and the system is not overloaded. Putting together all of these considerations, it is natural to focus on hybrid solutions, i.e., solutions in which the interference is tracked through either a PTV or a TI adaptive algorithm and the receiver can switch between these two systems based on some quality of service parameter, such as the SIR. Also, considering that the bit rates of different-rate signals often differ by integer powers of two, it may be possible to establish relationships between the harmonic coefficients of PTV solutions with different periods through standard signal processing techniques. Summing up, the idea here is to insert both the overall complexity and the convergence speed as design parameters. As anticipated, this requires first devising a suitable test aimed at assessing the real need for PTV processing, then estimating the sufficient period for that processing and finally selecting the most convenient system, wherein convenience should be assessed as the best compromise between reliability, convergence speed, and complexity. To date, much work on this topic remains to be done, both for the analytical formulation of the problem and for the design of the entire algorithm.

Another interesting parallel issue on PTV processing is robustness against mismatch between nominal and the actual bit rates. Indeed, the inevitable timing jitter that affects transmitters causes the actual bit interval to float around its average (nominal) value, according to the well-known model

$$T(k) = T_0 + \tau(k)$$

where $T(k)$ is the actual bit interval at bit interval k, T_0 is the nominal one, and $\tau(k)$ is a random process with given statistics. The above model refers to both the CDMA users and to the NBI interfering signal. Obviously, the accuracy of the model for $\tau(k)$ is of critical importance in making use of this model, since overly severe jitter may endanger, if not completely destroy, cyclostationarity (in the ensemble of realizations or, relevant for the case of adaptive systems, for long time series). Preliminary simulation results have revealed that the PTV algorithms are robust to very moderate jitter, but their performance degrades considerably as the jitter variance increases. It is thus important to investigate the effect of jitter on the performance of PTV systems and, ultimately, incorporate the system's robustness to jitter into performance indices.

Furthermore, with regard to the issue of adaptive PTV processing, in this paper we have illustrated the cyclic version of the RLS algorithm, which is the only cyclic algorithm currently available in the literature. However, blind adaptive implementations of linear multiuser detectors can be obtained also through the use of the LMS algorithm and of subspace tracking algorithms [205], [206]. While, on

the one hand, the cyclic RLS algorithm has been shown to achieve satisfactory performance, on the other hand it has some well-known drawbacks, such as bad numerical behavior in ill-conditioned situations, sensitivity to signature waveform mismatch, large computational complexity, and nonoptimal steady-state performance. Based on these considerations, it is of interest to develop cyclic versions of the LMS and of subspace tracking procedures, which are known to overcome some of these drawbacks. Preliminary steps along these lines have been already taken in [16], [17], but work still remains to be done, especially with regard to the convergence analysis of these cyclic algorithms.

Another problem of significant interest is that of designing effective detectors for situations in which a strong NBI is overlaid on a DS/CDMA network employing long (aperiodic) spreading codes. Indeed, neither of the two most natural solutions, i.e., simple matched filtering or chip-by-chip processing coupled with prediction-subtraction of the NBI, appear to yield satisfactory performance in such situations. On the other hand, linear *code-aided* techniques are hardly practical here, in that the covariance matrix of the MAI is time varying and must be inverted from scratch at each bit interval. An even thornier issue arises in adaptive interference suppression for long-code systems, since the aperiodicity of the spreading codes destroys the cyclostationarity properties of the received signal, thus, implying that standard adaptive algorithms, like RLS and LMS, cannot be employed in this context. Thus, the issue of how to deal effectively with long codes in overlay situations is still an open and challenging problem. Some recent results have appeared in the literature [28]–[30], [51], [182], [212], [218], especially with reference to systems with no NBI, but there is still much to be done in this area.

A further situation that merits some attention is the case of fading dispersive channels. Indeed, all of the systems reviewed in this paper rely on the assumption that the channel (i.e., path gains and delays) of the user of interest are either known or have been perfectly estimated. While a shared pilot channel might be used for channel estimation in multiple downlink channels (thus, sacrificing only one code for training), the corresponding situation in the uplink requires each user to transmit its own training signals. Notice also that, if modified receivers are adopted, the accuracy of the channel estimate is a critical issue, because the gain they achieve over their conventional counterparts relies heavily on exploitation of the relative phase information of the channel tap weights. If this information is either imprecise or wrong, adopting modified receivers may result in an increase in system complexity with no actual advantage in terms of network reliability.

From the above considerations, it is clear that the issue of fully blind systems for joint suppression of NBI and MAI will be an important research topic in the future.[8] The starting

point for the problem statement should be the powerful representation, already adopted in [30], [184], and [204], wherein all of the unknown channel quantities are lumped into an unknown vector. Thus, the received signal, observed over two consecutive bit intervals of the user of interest, is written as

$$\boldsymbol{r}(p) = \boldsymbol{C}_0 \boldsymbol{g}_0 b_0(p) + \boldsymbol{z}(p) + \boldsymbol{i}(p) + \boldsymbol{n}(p) \qquad (137)$$

where now the vectors are $2NM$-dimensional. The matrix $\boldsymbol{C}_0$ is entirely determined by the spreading code of the user of interest, the vector $\boldsymbol{g}_0$ contains all of the unknown channel quantities, $\boldsymbol{z}(p)$ is the interference induced by MAI and ISI, while $\boldsymbol{i}(p)$ is the NBI. Finally, $\boldsymbol{n}(p)$ represents white noise. A possible detection strategy at this point could be to adopt one of several known schemes (see, e.g., [31], [184], [204], and references therein) conceived for CDMA systems with no NBI to the new scenario. Obviously, this requires taking into account the possible periodicity induced by the NBI in the received signal second-order statistics.

As a final remark, we note that key issues, such as timing acquisition and tracking, channel estimation, power-control, iterative (turbo) MUD and decoding, space–time coding, recursive approximations of the maximum-likelihood optimum multiuser receiver, etc., are still largely unexplored in the context of overlaid DS/CDMA systems. It is reasonable to expect that existing solutions, which have been designed for CDMA systems with no overlaid NBI, may exhibit significant performance impairment in the presence of strong NBI. Thus, it is of considerable interest to assess the impact of NBI on the performance of these algorithms and, then, to design new algorithms as needed, explicitly taking into account the presence of the external NBI.

VII. CONCLUSION

This paper has reviewed techniques for active interference suppression in DS/CDMA systems overlaid on narrower bandwidth networks. This general area has been a very active one for nearly a quarter century, due primarily to the fact that the use of such techniques can bring substantial performance improvement at the cost of manageable complexity increases. This paper has mainly considered progress in this area in the last fifteen years, since the appearance of Milstein's excellent survey [122] in 1988 of developments to that time. In the intervening years, significant advances have taken place, both due to the increasing commercial interest in this problem and through the recognition that techniques developed in related areas, such as MUD, adaptive filtering, and beamforming, may be adapted for use in this area.

The focus of this survey has been on the so-called *code-aided* algorithms, a term coined in [151] to refer to those techniques explicitly taking into account the knowledge of the spreading code of the user to be detected. Special emphasis has been devoted to PTV detection rules, which have been shown to be needed in systems in which the external NBI is a digital communication signal whose signaling interval is not an integer submultiple of the CDMA signals' symbol interval. Indeed, in this situation the NBI introduces new periods into the second-order statistics of the received waveform, thus, implying that linear multiuser

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[8]In using the term "fully blind," we refer here to the situation in which the receiver has no prior information about the interferers or about the useful signal, except for its spreading code. Note that, in some papers, the term fully blind detection refers to the case that the receiver does not even know the spreading code of the user of interest. In this case, some detection strategies, e.g., exploiting the finite alphabet size of the employed modulation format, can be conceived. Due to their heavy computational complexity, these receivers are of interest mainly for specialized applications and have not been considered in this paper.

detectors are themselves PTV. Significant attention has been further devoted to the issue of simultaneous suppression of NBI and MAI in frequency-selective fading channels. In particular, the cases that the receiver has either CCSI or ICSI have been considered and their implications on the receiver performance and on its computational complexity have been discussed in detail. When considering the issue of blind adaptive interference suppression, we have discussed a recently proposed cyclic version of the RLS algorithm, which is able to bridge the gap between PTV processing of the observables and the need for adaptivity.

Despite the substantial body of work that has been summarized in this paper, we note that there are still many interesting open issues in this area that deserve further investigation, some of which are discussed in Section VI. Indeed, due to the proliferation of wireless communications applications and to the increasing deployment of overlaid multirate networks, it is reasonable to expect that the demand for sophisticated signal processing techniques able to guarantee a satisfactory quality of service to users of such networks will increase. As a consequence, it is anticipated that many further advances in this area will take place in the coming years.

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Stefano Buzzi (Member, IEEE) was born in Piano di Sorrento, Italy, on December 10, 1970. He received the Dr. Eng. (honors) and the Ph.D. degrees in electronic engineering and computer science from the University of Naples Federico II, Italy, in 1994 and 1999, respectively.

In 1996, he spent six months at the Centro Studi e Laboratori Telecomunicazioni, Turin, Italy. From November 1999 through December 2001, he was with the Department of Electrical Engineering, Princeton University, Princeton, NJ, as a Visiting Research Fellow. He is currently an Assistant Professor with the University of Cassino, Italy. His current research interests are in the area of statistical signal processing, with emphasis on signal detection in non- Gaussian noise and multiple access communications.

Dr. Buzzi was awarded by the Associazione Elettrotecnica ed Elettronica Italiana the G. Oglietti Scholarship in 1996, the NATO/CNR Advanced Fellowship in 1999, and a CNR Short-Term Mobility Grant in 2000 and 2001.

Marco Lops (Senior Member, IEEE) was born in Naples, Italy, on March 16, 1961. He received the Dr. Eng. and Ph.D. degrees in electronic engineering from the University of Naples, Italy, in 1986 and 1992, respectively.

From 1986 to 1987, he was an Engineer with the Air Traffic Control Systems Group, Selenia, Roma, Italy. From 1991 to 2000, was an Associate Professor of Radar Theory and Digital Transmission Theory with the University of Naples. Since March 2000, he has been a Full Professor with the University of Cassino, Italy, engaged in research in the field of statistical signal processing, with emphasis on radar processing and spread spectrum multiuser communications. He also held teaching positions at the University of Lecce and, during 1991 and 1998, he was on sabbatical leaves at University of Connecticut and Rice University, respectively.

H. Vincent Poor (Fellow, IEEE) received the Ph.D. degree in electrical engineering and computer science in 1977 from Princeton University, Princeton, NJ.

He is currently a Professor of Electrical Engineering with Princeton University. He is also affiliated with Princeton's Department of Operations Research and Financial Engineering and with its Program in Applied and Computational Mathematics. From 1977 until he joined the Princeton faculty in 1990, he was a Faculty Member with the University of Illinois at Urbana-Champaign. He has also held visiting and summer appointments at several universities and research organizations in the United States, Britain, and Australia. Among his publications, he has authored *Wireless Communications: Signal Processing Perspectives* (Englewood Cliffs, NJ: Prentice-Hall, 1998). His current research interests are in the area of statistical signal processing and its applications, primarily in wireless multiple-access communication networks.

Dr. Poor is a Member of the National Academy of Engineering and is a Fellow of the Acoustical Society of America, the American Association for the Advancement of Science, the Institute of Mathematical Statistics, and the Optical Society of America. He received the Frederick E. Terman Award of the American Society for Engineering Education in 1992, the Distinguished Member Award from the IEEE Control Systems Society in 1994, the IEEE Third Millennium Medal in 2000, the IEEE Graduate Teaching Award in 2001, and the Joint Paper Award of the IEEE Communications and Information Theory Societies in 2002. He has been involved in a number of IEEE activities, including serving as President of the IEEE Information Theory Society in 1990 and as a Member of the IEEE Board of Directors in 1991 and 1992.