A New Family of MMSE Multiuser Receivers for Interference Suppression in DS/CDMA Systems Employing BPSK Modulation

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***Abstract*—In this paper, we deal with interference suppression in asynchronous direct-sequence code-division multiple-access (CDMA) systems employing binary phase-shift keying modulation. Such an interference may arise from other users of the network, from external low-rate systems, as well as from a CDMA network coexisting with the primary network to form a dual-rate network. We derive, for all of these cases, a new family of minimum mean-square-error detectors, which differ from their conventional counterparts in that they minimize a modified cost function. Since the resulting structure is not implementable with acceptable complexity, we also propose some suboptimum systems. The statistical analysis reveals that both the optimum and the suboptimum receivers are near–far resistant, not only with respect to the other users, but also with respect to the external interference. We also present a blind and a recursive least squares-based, decision-directed implementation of the receivers wherein only the signature and the timing of the user to be decoded and the signaling time and the frequency offset of the external interferer are assumed known. Finally, computer simulations show that the proposed adaptive algorithm outperforms the classical decision-directed RLS algorithm.**

***Index Terms*—Code-division multiple access, interference suppression, minimum-mean-square-error, multiple-access interference, multirate systems, multiuser detection, narrow-band interference.**

I. Introduction

CODE-DIVISION multiple-access (CDMA) systems are now emerging as stronger and stronger candidates to ensure reliable communication through wireless networks, due to their inherent resistance to cochannel interference. Such an interference may be due, for example, to other users simultaneously accessing the channel—in which case it is referred to as multiple-access interference (MAI)—as well as to external systems. In the former case, the most customary countermeasures are either the usage of a tight power control or multiuser detection; both strategies ensure an efficient suppression of MAI and, eventually, enable random access to the network [1], [2].

The usage of spread-spectrum signals in overlay applications [3], wherein the CDMA network is to coexist with other external systems sharing the same frequency bandwidth, typically requires additional suppression blocks, in that the processing gain may turn out not to be sufficient to suppress strong external interferers. In [4], in particular, it is shown that perfect immunity to narrow-band interference (NBI) can be gained by simply generalizing the idea of the decorrelating detector, while in [5], the minimum mean-square-error (MMSE) strategy is applied to the same goal. In [6], instead, a new projection-type receiver, capable of simultaneously suppressing MAI and NBI, is introduced and assessed with reference to synchronous systems, while in [7] its generalization to the case that the system is asynchronous and that the channel introduces multipath distortion is presented. The main result of these studies is that suppressing NBI generally requires periodically time-varying processing.

More recently, a great deal of attention has been also devoted to the so-called multirate systems, wherein several CDMA networks, with different bit rates, are to share the same frequency bandwidth [8]. This application is of great importance in future third-generation cellular networks [9], as well as in multimedia communications, wherein each service is characterized by its own output bit rate and is required to achieve its own quality. In these situations, suitable processing schemes are to be adopted, in order to ensure that each CDMA system achieves the required quality of service. In [10], for example, it has been shown that also an asynchronous dual-rate CDMA system can take great advantage of the usage of periodically time-varying filtering.

From the previous arguments, it is understood that real systems are typically faced with the problem of suppressing both MAI and external interference, wherever the latter originates from. A further requirement that a system should comply with is that it lend itself to blind and/or adaptive implementations, namely that it can be suitably modified to account for possible prior uncertainty as to the interference.

In this paper, suppression of MAI and external interference is dealt with in a unified approach. In a conventional MMSE design strategy, the estimate of each user's bit is sought as a complex-valued function of the data, minimizing the square modulus of the estimation error; in our approach, the estimate is forced to be real, which entails the minimization of a modified cost function. We thus show that such a strategy leads to a new family of MMSE-based receivers, which generalizes the

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conventional ones, to which in fact the new structures reduce under particular conditions. Unfortunately, however, these new receivers require, in the general case, an intolerable computational complexity, and indeed we also propose a class of suboptimum systems which, while still outperforming the conventional MMSE, entail only a minor complexity increase. The issues of blind and adaptive detection are also addressed; in particular, we show that the newly proposed receivers may be made blind if an off-line estimate of the correlation properties of the data is available. Additionally, we present a new recursive least squares (RLS)-based, decision-directed implementation of both the optimum and the suboptimum receivers. Since these receivers are time-varying, direct application of conventional RLS tracking algorithm is *suboptimum*, and indeed we also motivate the need for the new adaptive procedure through a comparative performance assessment. It should be noted that the proposed technique applies to CDMA systems employing binary phase-shift keying (BPSK) modulation, while is not useful for quadrature phase-shift keying (QPSK) signaling constellations. However, even though the most part of the leading standard proposals for third-generation wireless networks recommends the use of QPSK signaling constellation, there also exist some standard proposals where adoption of BPSK modulation has been planned [11, p. 85], [12, p. 124], whence CDMA systems with BPSK modulation are still of interest, not only from a theoretical point of view, but also for commercial motivations.

The paper is organized as follows. Section II contains the problem formulation, while Section III is devoted to the synthesis of the receiver. Section IV presents a simplified suboptimum receiver, while in Section V we address the issue of blind and adaptive detection. Section VI is devoted to the statistical analysis and to the results presentation. Finally, concluding remarks and hints for future research form the object of Section VII.

II. Problem Formulation

Let us consider a CDMA network wherein a number, K say, of users asynchronously and simultaneously transmit the respective bit streams. Due to the presence of possible external interference, the baseband equivalent of the received signal is written as

$$r(t) = \sum_{k=0}^{K-1} A_k e^{j\phi_k} \sum_{m=-\infty}^{\infty} b_k(m) s_k(t - mT_b - \tau_k) + i(t) + w(t). \tag{1}$$

The meaning of the symbols in the above formula is as follows.

- $A_k e^{j\phi_k}$ is a complex gain, accounting for the channel effect and assumed known; $\{b_k(m)\}_{m=-\infty}^{+\infty}$ represents the bit stream of the kth user, modeled as a sequence of independent and identically distributed binary variates, each taking on values in the set $\{-1; 1\}$; $T_b = NT_c$ is the bit interval of the network; $[\tau_0, \ldots, \tau_{K-1}]^T = \boldsymbol{\tau}$ is a set of delays; from now on we assume, with no loss of generality, $0 = \tau_0 \leq \tau_1 \leq \cdots \leq \tau_{K-1} < T_b$; $s_k(t)$ is the *signature* assigned to the kth user, expressed as

$$s_k(t) = \sum_{n=0}^{N-1} c_{kn} u_{T_c}(t - nT_c)$$

  with N the processing gain, $\{c_{kn}\}_{n=0}^{N-1}$ the kth spreading code, T_c the chip interval, and $u_{T_c}(\cdot)$ a unit-height rectangular pulse supported on the interval $[0, T_c]$.
- $i(t)$ is the external interference. It may be of any type, but in the sequel we assume the general form

$$i(t) = \sum_{k=0}^{K'-1} A_{Ik} e^{j\phi_{Ik}} \sum_{m=-\infty}^{\infty} b_{Ik}(m) s_{Ik}(t - mT_I - \tau_{Ik}) e^{j2\pi f_I t} \tag{2}$$

  with $A_{Ik} e^{j\phi_{Ik}}$ a set of complex gains, $\{\{b_{Ik}(m)\}_{k=0}^{K'-1}\}_{m=-\infty}^{+\infty}$ K' streams of binary variates, $\{s_{Ik}(t)\}_{k=0}^{K'-1}$ the *interference signatures*, T_I its bit interval, $\{\tau_{Ik}\}_{k=0}^{K'-1} = \boldsymbol{\tau}_I$ a set of delays and f_I the interference frequency offset with respect to the carrier frequency of the CDMA system. Notice that several relevant cases of practical interest are subsumed in the model (2). Precisely, if $s_{Ik}(t) = u_{T_I}(t)\ \forall k$, $\tau_{Ik} = \tau_I\ \forall k$ and if the $\{b_{Ik}(m)\}_{m=-\infty}^{\infty}$ are sequences of independent variates, then $i(t)$ is a superposition of data-like interferers all sharing the same bit interval and carrier frequency. If, instead, $K' = 1$, $s_{I0}(t) = u_{T_I}(t)$, and $b_{I0}(m) = b_{I0}\ \forall m$, then $i(t)$ is a purely sinusoidal interferer. Moreover, if $\{\{b_{Ik}(m)\}_{k=0}^{K'-1}\}_{m=-\infty}^{+\infty}$ are K' streams of independent binary variates and

$$s_{Ik}(t) = \sum_{n=0}^{N'-1} c'_{kn} u_{T'_c}(t - nT'_c) \tag{3}$$

  then $i(t)$ is the interference induced by a coexisting asynchronous CDMA network with processing gain $N' = T_I/T'_c$, and models the situation encountered in a dual-rate CDMA system [10].
- $w(t)$ is the thermal noise term, modeled as a sample function from a complex white Gaussian process with power spectral density (PSD) $2\mathcal{N}_0$.

The received signal (1) may be conveniently recast to separate the useful term and the interferers; to fix the ideas, let us assume that we are willing to decode the user 0, and that a decision as to $b_0(\ell)$ is made by processing the interval $\mathcal{I}_\ell = [\ell T_b, (\ell+1)T_b]$. Even though, in principle, the receiver synthesis might be carried out by considering the continuous time model (1), it is customary to convert the waveform $r(t)$ received in $\mathcal{I}_\ell$ in a discrete-time sequence, so that the whole receiver processing may be digitally executed. This is a key point and needs special attention. We preliminarily notice that, if the quantities $\{\tau_k\}$, τ_{Ik}, and T_I are all integer multiples of T_c, and if $f_I = 0$, then it can be easily shown that the CDMA signals and the external interference, as observed in the interval $\mathcal{I}_\ell$, can be perfectly represented by projecting the waveform $r(t)$ along the

N-dimensional set $\{[1/\sqrt{T_c}]u_{T_c}(t-\ell T_b-iT_c)\}_{i=0}^{N-1}$. Otherwise stated, synchronous and chip-synchronous systems, affected by synchronous or chip-synchronous interference, admit a chip-by-chip representation with no loss of information. Unfortunately, however, for arbitrary values of the above parameters, adoption of such a representation set necessarily results into an information loss, and, supposedly, into increased bit-error rate (BER). A lossless representation of the signal in the interval $\mathcal{I}_\ell$ could be achieved by adopting either an infinite-dimensional basis for the functional space $L^2_{\mathcal{I}_\ell}$ of the square-integrable functions in $\mathcal{I}_\ell$, or a finite-dimensional basis depending on the particular realization of the cited parameters. Both these alternatives are unconvenient, in that the former would lead to unmanageable infinite-dimensional vectors, while the latter would ultimately lead to a parameter-dependent representation basis, which is clearly unsuited in all situations where these parameters are not *a priori* known. A possible solution is to adopt the "expanded" NM-dimensional orthonormal set

$$\left\{\frac{1}{\sqrt{T_{OS}}}\,u_{T_{OS}}(t-iT_{OS}-\ell T_b)\right\}_{i=0}^{NM-1} \tag{4}$$

wherein $T_{OS}=T_c/M$ and M is an integer positive number. Otherwise stated, in keeping with [13], we perform a sort of oversampling by a factor "M" with respect to the case of chip-matched filtering, so as to reduce, for increasing M, the representation error due to the system asynchrony and eventually, the system BER [14], [15]. As a side remark, notice that the above considerations hold for the case, herein considered, of rectangular (infinite-bandwidth) chip-waveform *and* overlay application. If, instead, a band-limited chip-waveform were adopted (like, for instance, the raised cosine pulses), and if no NBI were present at the receiver antenna, a lossless representation of the useful signals space could be ideally obtained by resorting to a low-pass (anti-aliasing) filter followed by a sampler at the Nyquist rate.

Projecting the waveform (1) onto the orthonormal set (4) yields the following NM-dimensional observable vector:

$$\begin{aligned}\boldsymbol{r}(\ell) &= A_0e^{j\phi_0}b_0(\ell)\boldsymbol{s}_0^0+\sum_{k=1}^{K-1}A_ke^{j\phi_k}\sum_{m\in\{-1,0\}}b_k(\ell+m)\boldsymbol{s}_k^m\\ &\quad+\sum_{k=0}^{K'-1}A_{Ik}e^{j\phi_{Ik}}\sum_{m\in\Omega'_k(\ell)}b_{Ik}(\ell+m)\boldsymbol{s}_{Ik}^m(\ell)e^{j2\pi\nu_I\ell NM}\\ &\quad+\boldsymbol{w}(\ell)=A_0e^{j\phi_0}b_0(\ell)\boldsymbol{s}_0^0+\boldsymbol{z}(\ell)+\boldsymbol{i}(\ell)+\boldsymbol{w}(\ell)\end{aligned} \tag{5}$$

where

$$\begin{aligned}\Omega'_k(\ell)=\Bigg\{&-\left(1+\left\lceil\frac{-T_b+\tau_{Ik}(\ell)}{T_I}\right\rceil\right),\ldots,-1,0,\\ &1,\ldots,\left\lceil\frac{T_b-\tau_{Ik}(\ell)}{T_I}\right\rceil\Bigg\}\end{aligned}$$

with $\tau_{Ik}(\ell)=T_I-((\ell T_b-\tau_{Ik})-\lfloor(\ell T_b-\tau_{Ik})/T_I\rfloor T_I)$ and $\nu_I=f_IT_{OS}$.

In (5), the first term on the right-hand side represents the contribution from the bit to be decoded, while the other terms represent the contributions from MAI ($\boldsymbol{z}(\ell)$), the external interference ($\boldsymbol{i}(\ell)$) and the thermal noise ($\boldsymbol{w}(\ell)$), respectively. In particular, following [2], the internal interference suffered by the 0th user in an asynchronous CDMA network can be regarded as equivalent to that generated in a synchronous system by a number of fictitious users employing the signatures $\{\{\boldsymbol{s}_k^i\}_{i\in\{-1,0\}}\}_{k=0}^{K-1}$. As regards external interference, the situation is similar to the MAI, with the relevant exception that the sets of the fictitious users [defined by $\Omega'_k(\ell)$] and their signatures $\{\{\boldsymbol{s}_{Ik}^m(\ell)\}_{i\in\Omega'_k(\ell)}\}_{k=0}^{K'-1}$ are now functions of ℓ, and, for $\nu_I\neq 0$, are in general complex-valued.

Equation (5) also highlights that MAI and external interference have quite different statistical properties. In fact, since the CDMA users share the same signaling interval, the projections $\boldsymbol{s}_k^m$ do not depend on the bit interval ℓ. From a geometrical point of view, this means that the subspace containing the MAI is one and the same for any bit interval, as can be easily checked by evaluating the covariance matrix of $\boldsymbol{z}(\ell)$. Conversely, both the sets $\Omega'_k(\ell)$ and the projections $\boldsymbol{s}_{Ik}^m(\ell)$ are in general time-varying measures, and so is the covariance matrix of $\boldsymbol{i}(\ell)$, implying that the subspace containing the external interference is itself time varying. Summing up, the decoding problem is now reduced to that of eliminating both the "stationary part" of the interference, i.e., $\boldsymbol{z}(\ell)$, and the "nonstationary" part, i.e., $\boldsymbol{i}(\ell)$. The presence of the latter contribution suggests that, in general, the required processing is time varying (see also [6]). Finally, notice that $\boldsymbol{w}(\ell)$ is a white complex Gaussian vector with covariance matrix $2\mathcal{N}_0\boldsymbol{I}_{NM}$, with $\boldsymbol{I}_{NM}$ the identity matrix of order NM. For future reference, we denote by $\mathcal{C}^{NM}$ the space of the complex NM-tuples on the complex field $\mathcal{C}$ with the usual internal and external operations, and by $\mathcal{S}(\ell)$ the *signal space*, namely the vector subspace of $\mathcal{C}^{NM}$ spanned by the desired signal, the MAI and the NBI.

Any linear receiver makes a decision as to the bit $b_0(\ell)$ according to the rule

$$\hat{b}_0(\ell)=\mathrm{sgn}(\Re\{\boldsymbol{d}_0^H(\ell)\boldsymbol{r}(\ell)\}) \tag{6}$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, $\Re\{\cdot\}$ denotes real part, and $(\cdot)^H$ conjugate transpose. The conventional MMSE detector[1] selects $\boldsymbol{d}_0(\ell)$ according to

$$\begin{aligned}\boldsymbol{d}_0(\ell)&=\boldsymbol{d}_0^C(\ell)\\ &=\arg\min_{\boldsymbol{x}\in\mathcal{S}(\ell)}E[|\boldsymbol{x}^H\boldsymbol{r}(\ell)-b_0(\ell)|^2]\\ &=\boldsymbol{M}^{-1}(\ell)A_0e^{j\phi_0}\boldsymbol{s}_0^0.\end{aligned} \tag{7}$$

In the previous equation $E[\cdot]$ denotes statistical expectation and $\boldsymbol{M}(\ell)=E[\boldsymbol{r}(\ell)\boldsymbol{r}^H(\ell)]$ the covariance matrix of the observables. On the other hand, since

$$\begin{aligned}&\min_{\boldsymbol{x}\in\mathcal{S}(\ell)}E[|\boldsymbol{x}^H\boldsymbol{r}(\ell)-b_0(\ell)|^2]\\ &\quad\geq\min_{\boldsymbol{x}\in\mathcal{S}(\ell)}E[(\Re\{\boldsymbol{x}^H\boldsymbol{r}(\ell)\}-b_0(\ell))^2]\end{aligned} \tag{8}$$

it is understood that defining

$$\boldsymbol{d}_0(\ell)=\arg\min_{\boldsymbol{x}\in\mathcal{S}(\ell)}E[(\Re\{\boldsymbol{x}^H\boldsymbol{r}(\ell)\}-b_0(\ell))^2] \tag{9}$$

[1]Here and in the following, we mean by "conventional MMSE detector" the receiver developed in [16], whose expression is given by (7) and minimizing the cost function in (7).

the decision rule (6) is necessarily not inferior to the conventional MMSE. Unfortunately, the solution to (9) is not easily found; the problem, of course, is that since $\Re\{\boldsymbol{x}^H \boldsymbol{r}(\ell)\}$ is *not* an admissible scalar product in $\mathcal{C}^{NM}$, the solution to (9) *cannot* be found by applying the usual orthogonality principle, i.e., (9) is not a conventional linear MMSE problem.

III. Detection Design

To solve (9), first notice that

$$\begin{aligned}\Re\{\boldsymbol{x}^H \boldsymbol{r}(\ell)\} &= \tfrac{1}{2}(\boldsymbol{x}^H \boldsymbol{r}(\ell) + \boldsymbol{x}^T \boldsymbol{r}^*(\ell)) \\ &= \tfrac{1}{2}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x}^* \end{pmatrix}^H \begin{pmatrix} \boldsymbol{r}(\ell) \\ \boldsymbol{r}^*(\ell) \end{pmatrix} \\ &= \boldsymbol{x}_a^H \boldsymbol{r}_a(\ell) \end{aligned} \tag{10}$$

with $(\cdot)^*$ denoting conjugate. As a consequence, solving the problem (9) in $\mathcal{S}(\ell)$ is *equivalent* to solving the problem

$$\boldsymbol{d}_a(\ell) = \arg \min_{\boldsymbol{x}_a \in \mathcal{S}_a(\ell)} E[(\boldsymbol{x}_a^H \boldsymbol{r}_a(\ell) - b_0(\ell))^2] \tag{11}$$

with

$$\boldsymbol{d}_a(\ell) = \tfrac{1}{2}\begin{pmatrix} \boldsymbol{d}_0(\ell) \\ \boldsymbol{d}_0^*(\ell) \end{pmatrix} \quad \boldsymbol{r}_a(\ell) = \begin{pmatrix} \boldsymbol{r}(\ell) \\ \boldsymbol{r}^*(\ell) \end{pmatrix}. \tag{12}$$

In (11), $\mathcal{S}_a(\ell)$ is a vector space on the field $\mathcal{C}$. Its elements are the augmented $2NM$-dimensional complex vectors whose first NM entries are the complex conjugates of the last NM, the internal operation is the usual component-wise vector sum in C^{2NM} and the external operation is

$$\times: \alpha \in \mathcal{C},\, \boldsymbol{x}_a \in \mathcal{S}_a \longrightarrow \alpha \boldsymbol{x}_a = \begin{pmatrix} \alpha \boldsymbol{x} \\ \alpha^* \boldsymbol{x}^* \end{pmatrix}. \tag{13}$$

Of course the transformation $f\colon \mathcal{S} \longrightarrow \mathcal{S}_a$ is bi-injective. The relevant advantage of the formulation (11) of the problem (9) is that, in $\mathcal{S}_a(\ell)$, (11) *is* a conventional MMSE problem, yielding

$$\boldsymbol{d}_a(\ell) = \boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(\ell)^{-1} \boldsymbol{v} \tag{14}$$

where

$$\begin{aligned}\boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(\ell) &= E[\boldsymbol{r}_a(\ell) \boldsymbol{r}_a^H(\ell)] \\ &= \begin{pmatrix} E[\boldsymbol{r}(\ell)\boldsymbol{r}(\ell)^H] & E[\boldsymbol{r}(\ell)\boldsymbol{r}(\ell)^T] \\ E[\boldsymbol{r}(\ell)^*\boldsymbol{r}(\ell)^H] & E[\boldsymbol{r}(\ell)^*\boldsymbol{r}(\ell)^T] \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{M}(\ell) & \boldsymbol{M}'(\ell) \\ \boldsymbol{M}'^*(\ell) & \boldsymbol{M}^*(\ell) \end{pmatrix} \end{aligned} \tag{15}$$

with $(\cdot)^T$ denoting transpose and $\boldsymbol{M}'(\ell) = E[\boldsymbol{r}(\ell)\boldsymbol{r}^T(\ell)]$. As to the vector $\boldsymbol{v}$, it is expressed as

$$\boldsymbol{v} = \boldsymbol{M}_{\boldsymbol{r}_a b_0} = E[b_0(\ell)\boldsymbol{r}_a(\ell)] = \begin{pmatrix} A_0 e^{j\phi_0} \boldsymbol{s}_0^0 \\ A_0 e^{-j\phi_0} \boldsymbol{s}_0^0 \end{pmatrix} \tag{16}$$

and is time invariant. The solution $\boldsymbol{d}_0(\ell)$ can be found, upon block-inversion of $\boldsymbol{M}_{\boldsymbol{r}_a \boldsymbol{r}_a}(\ell)$ in (14), as [17]

$$\boldsymbol{d}_0(\ell) = A_0 e^{j\phi_0} \boldsymbol{H}(\ell) \boldsymbol{s}_0^0 - A_0 e^{-j\phi_0} \boldsymbol{M}^{-1}(\ell) \boldsymbol{M}'(\ell) \boldsymbol{H}^*(\ell) \boldsymbol{s}_0^0 \tag{17}$$

with

$$\boldsymbol{H}(\ell) = 2(\boldsymbol{M}(\ell) - \boldsymbol{M}'(\ell)\boldsymbol{M}^{*-1}(\ell)\boldsymbol{M}'^*(\ell))^{-1}. \tag{18}$$

Notice that, unlike the conventional MMSE receiver (7), which is uniquely determined by the data covariance matrix, the solution (17) depends also on the matrix $\boldsymbol{M}'(\ell)$. This is a straigthforward, although not trivial, consequence of the fact that, when the radio-frequency signals are not wide-sense stationary (WSS) (and in fact neither the NBI nor the MAI are WSS), the correlation properties of the corresponding complex envelopes are specified by four real functions, or equivalently, by two complex functions. Likewise, the projections of these complex envelopes onto an orthonormal system are specified by two complex matrices, $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$, which are both present in the solution (17). As a consequence, it is (17) and *not* the conventional MMSE (7) that represents a baseband implementation of the MMSE receiver that would be obtained if radio-frequency (or intermediate-frequency) processing were performed [18]. Finally, notice that, if the radio-frequency signals are WSS,[2] i.e., $\boldsymbol{M}'(\ell) = \boldsymbol{0}$, (17) reduces to (7), as can be readily checked from (17) and (18).

Let us now give a deeper insight into the structure of the matrices $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$. Since

$$\begin{aligned}\boldsymbol{M}(\ell) &= A_0^2 \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0T} + E[\boldsymbol{z}(\ell)\boldsymbol{z}(\ell)^H] + E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^H] + 2\mathcal{N}_0 \boldsymbol{I}_{NM} \\ \boldsymbol{M}'(\ell) &= A_0^2 e^{j2\phi_0} \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0T} + E[\boldsymbol{z}(\ell)\boldsymbol{z}(\ell)^T] + E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^T] \end{aligned} \tag{19}$$

with $\boldsymbol{I}_{NM}$ the identity matrix of order NM, evaluating the expectations yields

$$\begin{aligned} E[\boldsymbol{z}(\ell)\boldsymbol{z}(\ell)^H] &= \sum_{k=0}^{K-1} A_k^2 \sum_{m \in \{-1,0\}} \boldsymbol{s}_k^m \boldsymbol{s}_k^{mT} \\ E[\boldsymbol{z}(\ell)\boldsymbol{z}(\ell)^T] &= \sum_{k=0}^{K-1} A_k^2 e^{j2\phi_k} \sum_{m \in \{-1,0\}} \boldsymbol{s}_k^m \boldsymbol{s}_k^{mT} \\ E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^H] &= \sum_{k=0}^{K'-1} A_{Ik}^2 \sum_{m \in \Omega'_k(\ell)} \boldsymbol{s}_{Ik}^m(\ell) \boldsymbol{s}_{Ik}^{mH}(\ell) \\ E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^T] &= e^{j4\pi\nu_I \ell NM} \sum_{k=0}^{K'-1} A_{Ik}^2 e^{j2\phi_{Ik}} \sum_{m \in \Omega'_k(\ell)} \boldsymbol{s}_{Ik}^m(\ell) \\ &\quad \cdot \boldsymbol{s}_{Ik}^{mT}(\ell). \end{aligned} \tag{20}$$

The above relationships allow envisaging a *general structure* for the receiver. In fact, the matrix $E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^H]$ is a periodically time-varying one, since the set of fictitious signatures $\Omega'_k(\ell)$ and the projections $\boldsymbol{s}_{Ik}^m(\ell)$ are themselves periodically time varying with period r, the smallest integer such that rT_b is an integer multiple of T_I (for a rigorous proof, see the arguments of [6]). Since the matrix $E[\boldsymbol{z}(\ell)\boldsymbol{z}^H(\ell)]$ is stationary, the matrix sequence $\boldsymbol{M}(\ell)$ is thus *periodical* with period r. Moreover, it is readily seen that the matrices $E[\boldsymbol{i}(\ell)\boldsymbol{i}(\ell)^T]$ define in general a polyperiodical sequence in the sense specified in

[2]Such a situation might occur, for instance, in the case that the signal is subject to fast fading, such that the phase offsets ϕ_k and ϕ_{Ik} would remain constant for very few signaling interval, or alternatively, if both the CDMA signals and the external signals employed a QPSK modulation format.

[19], containing as (discrete) fundamental frequencies $1/r$ and $2\nu_I NM$. As a consequence, the sequences $\boldsymbol{d}_a(\ell)$ and $\boldsymbol{d}_0(\ell)$ are themselves *polyperiodical,* with the same fundamental discrete frequencies. For this reason, from now on, the solution (17) will be referred to through the acronym PCMMSE (polyperiodical conjugate MMSE), where the meaning of the word "conjugate" will be explained in next section.

Notice also that, if the numerical frequency ν_I is a rational number, $\nu_I = q_1/q_2$, with q_1 and q_2 integer numbers, the sequence $e^{j4\pi\nu_I NM\ell}$ is itself periodical with period $r_* = q_3/(2\nu_I NM)$, with q_3 the smallest integer such that r_* is integer. As a consequence, the matrix sequence $\boldsymbol{M}'(\ell)$ can be also regarded as periodical with period $R = \text{l.c.m.}(r, r_*)$, with $\text{l.c.m.}(\cdot, \cdot)$ denoting least common multiple, and $\boldsymbol{d}_0(\ell)$ is periodical with the same period. Summing up, for rational ν_I, the optimum solution $\boldsymbol{d}_0(\ell)$ may be regarded either as periodical with period R or as a degenerate polyperiodical sequence with fundamental periods r and r^*. This fact, which is of marginal importance under a theoretical point of view, can instead be exploited at the receiver implementation level.

IV. Receiver Implementation: Suboptimum Structures

At first, we notice that since $\boldsymbol{d}_0(\ell)$ is polyperiodical with fundamental frequencies $1/r$ and $2\nu_I NM$, the solution (17) can be expanded in a generalized Fourier series [21], i.e.,

$$\boldsymbol{d}_0(\ell) = \sum_{\gamma \in I_\gamma} \boldsymbol{d}_0^{(\gamma)} e^{j2\pi\ell\gamma} \tag{21}$$

wherein $I_\gamma = I_\alpha \bigcup I_\beta$, with

$$I_\alpha = \left\{\frac{p}{r}\right\}_{p=0}^{r-1} \quad I_\beta = \left\{\left\{\frac{p}{r} + 2\nu_I NMm\right\}_{p=0}^{r-1}\right\}_{m=\pm 1, \pm 2, \ldots} \tag{22}$$

if ν_I is irrational, and with

$$I_\alpha = \left\{\frac{p}{r}\right\}_{p=0}^{r-1} \quad I_\beta = \left\{\left\{\frac{p}{r} + \frac{m}{r^*}\right\}_{p=0}^{r-1} \bmod 1\right\}_{m=1}^{r*-1} \tag{23}$$

if ν_I is rational.

Expressions (22) and (23) show that if ν_I is rational the set I_γ becomes finite—and in fact it contains all of the R distinct harmonical frequencies of the fundamental frequency $1/R$ (i.e., $I_\gamma = \{p/R\}_{p=0}^{R-1}$)—in keeping with the fact that in this situation the sequence $\boldsymbol{d}_0(\ell)$ is simply periodical with period R.

The coefficients of the series expansion (21) can be easily found as

$$\begin{aligned}\boldsymbol{d}_0^{(\gamma)} &= \langle \boldsymbol{d}_0(\ell) e^{-j2\pi\gamma\ell} \rangle \\ &= A_0 \left(e^{j\phi_0} \langle \boldsymbol{H}(\ell) e^{-j2\pi\gamma\ell} \rangle - e^{-j\phi_0} \langle \boldsymbol{M}^{-1}(\ell) \boldsymbol{M}'(\ell) \right. \\ &\quad \left. \cdot\, \boldsymbol{H}^*(\ell) e^{-j2\pi\gamma\ell} \rangle \right) \boldsymbol{s}_0^0\end{aligned} \tag{24}$$

where $\langle \cdot \rangle$ denotes (discrete) time-average.

The so called linear-conjugate/linear implementation [19], [20] of the receiver emerges by noticing that the solution $\boldsymbol{d}_0(\ell)$ has also an alternative interesting interpretation. In fact, if the data and their conjugate version are independently processed, the decision as to the bit $b_0(\ell)$ is made according to

$$\hat{b}_0(\ell) = \text{sgn}(\Re\{\boldsymbol{d}_{00}^H(\ell)\boldsymbol{r}(\ell) + \boldsymbol{d}_{01}^H(\ell)\boldsymbol{r}^*(\ell)\})$$

where $\boldsymbol{d}_{00}(\ell)$ and $\boldsymbol{d}_{01}(\ell)$ are chosen so that the estimate $\hat{b}_0(\ell) = \boldsymbol{d}_{00}^H(\ell)\boldsymbol{r}(\ell) + \boldsymbol{d}_{01}^H(\ell)\boldsymbol{r}^*(\ell)$ minimize the cost function $E[|\hat{b}_0(\ell) - b_0(\ell)|^2]$. It can be shown that the solution to this problem is $\boldsymbol{d}_{00}(\ell) = \boldsymbol{d}_{01}^*(\ell) = (1/2)\boldsymbol{d}_0(\ell)$, with $\boldsymbol{d}_0(\ell)$ given in (17); this explains why the solution (17) is named PCMMSE.

Notice that the form (21) of the solution allows implementing the time-varying decision rule defined by (14) through a FREquency-SHift (FRESH) structure, wherein the observable $\boldsymbol{r}(\ell)$ is first fed to a set of possibly infinitely many complex oscillators, keyed to the harmonical frequencies (22) and (23), and subsequently processed through as many linear time-invariant systems [19]. Additionally, notice that when ν_I is rational the FRESH implementation requires only R complex oscillators. However, if R is a large integer (or, even worse, if ν_I is not rational) the FRESH implementation is not directly applicable, since it would result in unacceptable complexity. On the other hand, even direct implementation of (17) would either require periodical switching among a large number of different coefficient sets (if ν_I is rational and R is large) or on-line computation of (17) and (18) (if ν_I is irrational).

As a consequence, in real cases a *suboptimum* approach is in order. To understand how to simplify the optimum structure, we notice that the solution (21) minimizes the point MMSE (P-MMSE) $\epsilon_P(\ell) = E[(\Re\{\boldsymbol{d}_0(n)^H\boldsymbol{r}(n)\} - b_0(n))^2]$ and, hence, also the time-averaged MMSE (TA-MMSE)

$$\begin{aligned}\epsilon_{\text{TA}} &= \lim_{P\to\infty} \frac{1}{2P+1} \sum_{n=-P}^{P} E[(\Re\{\boldsymbol{d}_0(n)^H\boldsymbol{r}(n)\} - b_0(n))^2] \\ &= \langle E[(\Re\{\boldsymbol{d}_0(n)^H r(n)\} - b_0(n))^2] \rangle.\end{aligned} \tag{25}$$

However, while the cost function defined by the problem (9) admits a unique minimum in the set of polyperiodical sequences, it can be shown that the functional (25) admits, in the same set, also some relative minima, besides the absolute minimum (21). Otherwise stated, the *minimum* set of harmonical frequencies ensuring equivalence between P-MMSE and TA-MMSE criteria is the one defined in (22) and (23), while any other set strictly contained in I_γ would represent at most a point of relative minimum for (25) and would *not* solve the P-MMSE minimization problem. The suboptimum approximation that we propose is to choose $\boldsymbol{d}_0(\ell)$ as a point corresponding to a relative minimum of (25), i.e.,

$$\boldsymbol{d}_0(\ell) = \sum_{\gamma \in I'_\gamma} \boldsymbol{d}_0^{(\gamma)} e^{j2\pi\ell\gamma} \tag{26}$$

where $I'_\gamma = I'_\alpha \bigcup I'_\beta \subset I_\gamma$ (with $I'_\alpha \subseteq I_\alpha$ and $I'_\beta \subset I_\beta$) is a finite subset of (22) and (23). From now on we denote by χ the cardinality of I'_γ. Of course, the suboptimality of this solution depends on the choice of I'_γ, which should thus result from a compromise between the conflicting requirements of achieving near-optimum performance and keeping the system complexity at the lowest possible level.

Interestingly, as I'_γ is finite, the coefficients $\boldsymbol{d}_0(\ell)$ can be given an explicit expression in terms of the coefficients of the generalized Fourier series expansions of the matrices $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$. Leaving the mathematical details to Appendix A, we have for the suboptimum solution

$$\boldsymbol{d}_0(\ell) = A_0 \boldsymbol{E}(\ell)\left(e^{j\phi_0}\tilde{\boldsymbol{H}} - e^{-j\phi_0}\tilde{\boldsymbol{M}}^{-1}\tilde{\boldsymbol{M}}'\tilde{\boldsymbol{H}}^*\right) \times (\boldsymbol{e}_1 \otimes \boldsymbol{I}_{NM})\boldsymbol{s}_0^0 \tag{27}$$

wherein

$$\tilde{\boldsymbol{H}} = 2(\tilde{\boldsymbol{M}} - \tilde{\boldsymbol{M}}'\tilde{\boldsymbol{M}}^{*-1}\tilde{\boldsymbol{M}}'^{*})^{-1} \tag{28}$$

$\boldsymbol{e}_1 = [1, 0, \ldots, 0]$ is a χ-dimensional vector, $\boldsymbol{E}(\ell) = [e^{-j2\pi\gamma_0\ell}\, e^{-j2\pi\gamma_1\ell} \, \cdots \, e^{-j2\pi\gamma_{\chi-1}\ell}] \otimes \boldsymbol{I}_{NM}$, and $\otimes$ denotes the kroneker product. In the above equation, $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{M}}'$ are square block-Toeplitz matrices of order $NM\chi$, whose (k_1, k_2) blocks are

$$\boldsymbol{M}_{(k_1, k_2)} = \left\langle \boldsymbol{M}(\ell) e^{-j2\pi(\gamma_{k_1} - \gamma_{k_2})\ell} \right\rangle$$
$$\boldsymbol{M}'_{(k_1, k_2)} = \left\langle \boldsymbol{M}'(\ell) e^{-j2\pi(\gamma_{k_1} - \gamma_{k_2})\ell} \right\rangle. \tag{29}$$

Notice that $\boldsymbol{M}_{(k_1, k_2)}$ and $\boldsymbol{M}'_{(k_1, k_2)}$ can also be interpreted as the generalized Fourier coefficients of the matrices $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$, respectively, corresponding to the harmonical frequency $(\gamma_{k_1} - \gamma_{k_2})$. Of course, the matrices defined in (29) are zero except for those frequencies such that $2\pi(\gamma_{k_1} - \gamma_{k_2}) \bmod 2\pi = 2\pi\gamma \bmod 2\pi$, with $\gamma \in I_\gamma$.[3] As a consequence, we have

$$\boldsymbol{M}_{(k_1, k_2)} = 0 \qquad \forall\, (\gamma_{k_1} - \gamma_{k_2}) \equiv \gamma \in I_\gamma - I_\alpha$$
$$\boldsymbol{M}'_{(k_1, k_2)} = 0 \qquad \forall\, (\gamma_{k_1} - \gamma_{k_2}) \equiv \gamma \in (I_\gamma - I_\beta - \{0\}). \tag{30}$$

Notice finally that $\boldsymbol{M}_{(k_1, k_2)} = (\boldsymbol{M}_{(k_2, k_1)})^H$ and $\boldsymbol{M}'_{(k_1, k_2)} = (\boldsymbol{M}'_{(k_2, k_1)})^*$. The evaluation of the matrices $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{M}}'$ thus requires the computation of only $(\chi_\alpha + 1)(\chi_\alpha + 2)/2$ different matrices $\boldsymbol{M}_{(k_1, k_2)}$ and $(\chi_\beta + 1)(\chi_\beta + 2)/2$ different matrices $\boldsymbol{M}'_{(k_1, k_2)}$, respectively, with χ_α the cardinality of I'_α and χ_β the cardinality of I'_β. Finally, we notice that if ν_I is a rational number and if $I'_\gamma = I_\gamma$, the matrices $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{M}}'$ are block-circulant. In this case the implementation of the proposed receiver requires computation of at most $(r + R)$ different matrices.

From now on, the receiver defined in (27) will be referred to as suboptimal PCMMSE (SPCMMSE) which reduces to the PCMMSE when $I'_\gamma = I_\gamma$. Two special cases of (27) are: a) $I'_\alpha = I_\alpha$ and $I'_\beta = \emptyset$, i.e., the SPCMMSE degenerates in a linear conjugate periodically time-varying MMSE system with period r dictated by T_b and T_I only (PTVCMMSE); b) $I'_\alpha = \{0\}$ and $I'_\beta = \emptyset$, i.e., the SPCMMSE degenerates in a linear time-invariant conjugate MMSE receiver (TICMMSE) (see [18]). Notice that both cases represent a generalization of two conventional MMSE detectors: the PTVCMMSE of the periodically time-varying MMSE (PTVMMSE) detector defined in (7) which can be obtained assuming $\tilde{\boldsymbol{M}}' = \boldsymbol{0}$ in (17) and does not process the conjugate of the observable vector;[4] the TICMMSE of the linear time-invariant MMSE (TIMMSE) detector proposed in [5] which can be obtained assuming $\tilde{\boldsymbol{M}}' = \boldsymbol{0}$ and substituting in (17) the covariance matrix of the observables by its time average, which is just the matrix to which the observable sample covariance matrix would converge in a conventional adaptive procedure.

[3]In what follows, this condition will be denoted as $(\gamma_{k_1} - \gamma_{k_2}) \equiv \gamma$

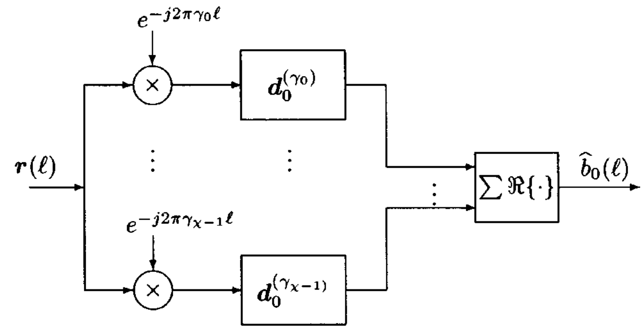

Fig. 1. FRESH implementation of the suboptimum MMSE receiver.

Equations (26) and (27) permit to simply derive the alternative FRESH implementation of the proposed suboptimum detector, whose operations are depicted in Fig. 1. The basic structure of a FRESH filter is retained, but now the number of complex oscillators, whose frequencies are dictated by the harmonical frequencies of I'_γ, is finite. The advantage of such an implementation is apparent, especially in the case that ν_I is not rational. In fact, the LTI transformations may be evaluated once and for all upon knowledge of the covariance matrices $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$, and the time-varying part of the processing confined to the complex oscillators.

A. Some Special Cases

Let us at first assume that no external interference is active, i.e., $\boldsymbol{i}(\ell) = 0$. In this case, the matrices $\boldsymbol{M}(\ell)$ and $\boldsymbol{M}'(\ell)$ end up time invariant, and, as already noticed in previous sections, the optimum detector (14) reduces to the TICMMSE receiver.

Consider now the far more interesting situation where the external interference is a digital BPSK signal with frequency offset f_I. If $f_I T_c = \nu_I = m$ with m an integer positive number ($m = 0$ implies that the data-like interference has the same carrier frequency as the CDMA system), the decision rule (6) ends up simply periodical with period r dictated by T_b and T_I only, and the optimum receiver is the PTVCMMSE. In more general cases, instead, the optimum MMSE receiver (9) is the PCMMSE.

Consider finally the case that we are dealing with a dual-rate CDMA system, namely that the external interference signatures have the form (3). In general, the processing scheme needs necessarily be polyperiodical, except than for particular values of the frequency offset f_I. For instance, if $f_I = 0$, decoding the user "0" of the "primary" CDMA network entails periodically time-varying processing with period r, while decoding the user "0" of the secondary network requires periodically time-varying

[4]Even though such a structure is not present in the open literature, it is based on concepts similar to those reported in [6] wherein a periodically time-varying decorrelating detector is presented, and in [7] and [15], wherein an MMSE-based receiver for fading channels is presented.

processing with period r', r' being the first integer such that $r'T_I$ is an integer multiple of T_b. This can be easily shown by redefining the desired signal, exchanging the roles of MAI and external interference, and solving the corresponding general MMSE problem. Finally, notice that, if the transmission system is a dual-rate CDMA with $T_c' = hT_c$ (with h an integer number), $N' = N$ *and* $f_I = 1/T_c - 1/T_c'$, namely if we consider a variable chip-rate frequency-shifted (VCRFS) system [10], with the primary network transmitting at a rate h times faster than the secondary CDMA network, then $\nu_I = (h-1)/Mh$, and the processing is again periodically time varying with period h or 1, whether the primary or the secondary network is considered; in both situations the optimum MMSE receiver (9) thus coincides with the PTVCMMSE.

V. Blind Adaptive Interference Cancellation

Both the PCMMSE and the SPCMMSE rely on the knowledge of the second-order statistics of the observables, and, hence, of the overall interference. However, in time-varying environments commonly encountered in wireless communications, it is desirable to resort to blind and/or adaptive receivers, which may be implemented based upon a very limited *a priori* knowledge of the interfering signals and achieve a performance very close to that of their nonadaptive counterparts. Leaving aside the PCMMSE, which is not actually realizable except when I_γ is finite, we focus here on the issue of blind and adaptive implementation of SPCMMSE. Obviously, the results found on SPCMMSE may be applied to a PCMMSE with finite I_γ.

We consider two algorithms: the former one is blind, i.e., it can be implemented with no need for training sequences, and executes an off-line estimate of the covariance matrix of the observables. The latter algorithm is instead based on a reformulation of the conventional RLS decision-directed algorithm.[5]

A. Blind Detection

Let us first assume that an estimate of the covariance matrix of the observables is available at the receiver, and that this estimate is updated after a given number of signaling intervals. Since the observable sequence $\boldsymbol{r}(\ell)$ is *cycloergodic* [19], i.e.

$$\begin{aligned}\hat{\boldsymbol{M}}_{(k_1,\,k_2)} &= \frac{1}{2Q+1}\sum_{n=-Q}^{Q}\boldsymbol{r}(n)\boldsymbol{r}(n)^H e^{j2\pi n(\gamma_{k_1}-\gamma_{k_2})}\\ &\longrightarrow \boldsymbol{M}_{(k_1,\,k_2)}\\ \hat{\boldsymbol{M}}'_{(k_1,\,k_2)} &= \frac{1}{2Q+1}\sum_{n=-Q}^{Q}\boldsymbol{r}(n)\boldsymbol{r}(n)^T e^{j2\pi n(\gamma_{k_1}-\gamma_{k_2})}\\ &\longrightarrow \boldsymbol{M}'_{(k_1,\,k_2)}\end{aligned} \tag{31}$$

wherein convergence is to be meant in the mean square sense, the matrices $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{M}}'$ in (27) and (28) may be estimated with sufficient accuracy. The direction $\boldsymbol{d}_0(\ell)$ can thus be estimated by replacing in (27) $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{M}}'$ with their estimated versions.

[5]Here and in the following we mean by classical or conventional RLS algorithm the one reported, for instance, in [23]. Such an algorithm assumes that the solution to be tracked is either time invariant or slowly time varying; notice that for the case at hand, instead, the solution to be tracked is polyperiodical.

B. Adaptive RLS-Based Interference Suppression

In this section, we present a modified RLS algorithm, taking into account the two major differences between the SPCMMSE and the conventional time-invariant MMSE, i.e., that they rely on minimizing different cost functions and that the former leads to a polyperiodical solution. Let us consider the set of observables $\{\boldsymbol{r}(i)\}_{i=0}^n$ and let us denote by $\boldsymbol{d}_{0b}(n,\,i)$ the estimate formed at epoch n of the true vector sequence $\boldsymbol{d}_0(i)$ (27). Obviously, $\boldsymbol{d}_{0b}(n,\,i)$ is a polyperiodical sequence (with respect to its second argument) with harmonical frequencies in I_γ', i.e.,

$$\boldsymbol{d}_{0b}(n,\,i) = \sum_{\gamma\in I_\gamma'}\boldsymbol{d}_{0b}^{(\gamma)}(n)e^{j2\pi i\gamma} \tag{32}$$

wherein $\{\boldsymbol{d}_{0b}^{(\gamma)}(n)\}_{\gamma\in I_\gamma'}$ are NM-dimensional vectors.

We seek $\boldsymbol{d}_{0b}(n,\,i)$ as the sequence minimizing the following exponentially-weighted square error:

$$\sum_{i=0}^{n}\lambda^{n-i}(\Re\{\boldsymbol{d}_{0b}(n,\,i)^H\boldsymbol{r}(i)\} - b_0(i))^2. \tag{33}$$

The scalar constant λ is usually referred to as "forgetting factor." It is slightly smaller than unity and its purpose is to ensure that the algorithm gradually forgets the past observations, so that it is able to track changes in the interference background. Substituting expression (32) into (33) and elaborating, we obtain

$$\sum_{i=0}^{n}\lambda^{n-i}(\Re\{\tilde{\boldsymbol{d}}_{0b}(n)^H\tilde{\boldsymbol{r}}(i)\} - b_0(i))^2 \tag{34}$$

with

$$\begin{aligned}\tilde{\boldsymbol{d}}_{0b}(n) &= \tfrac{1}{2}\left[\boldsymbol{d}_{0b}^{(\gamma_0)T}(n),\,\ldots,\,\boldsymbol{d}_{0b}^{(\gamma_{\chi-1})T}(n)\right]^T\\ \tilde{\boldsymbol{r}}(i) &= \left[\boldsymbol{r}(i)^T e^{-j2\pi\gamma_0 i},\,\ldots,\,\boldsymbol{r}(i)^T e^{-j2\pi\gamma_{\chi-1} i}\right]^T.\end{aligned} \tag{35}$$

Following the same steps which led to (14), the cost function (34) can be rewritten as

$$\sum_{i=0}^{n}\lambda^{n-i}(\tilde{\boldsymbol{d}}_{ab}(n)^H\tilde{\boldsymbol{r}}_a(i) - b_0(i))^2 \tag{36}$$

with $\tilde{\boldsymbol{d}}_{ab}(n)$ and $\tilde{\boldsymbol{r}}_a(i)$ the augmented $2NM\chi$-dimensional vectors

$$\tilde{\boldsymbol{d}}_{ab}(n) = \tfrac{1}{2}\begin{pmatrix}\tilde{\boldsymbol{d}}_{0b}(n)\\ \tilde{\boldsymbol{d}}_{0b}^*(n)\end{pmatrix} \quad \tilde{\boldsymbol{r}}_a(i) = \begin{pmatrix}\tilde{\boldsymbol{r}}(i)\\ \tilde{\boldsymbol{r}}^*(i)\end{pmatrix}. \tag{37}$$

Minimizing (36) with respect to $\tilde{\boldsymbol{d}}_{ab}(n)$ is a classical problem, which may be easily solved in the light of [23, ch. 13]. After some algebra, we obtain

$$\tilde{\boldsymbol{d}}_{ab}(n) = \left(\hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}(n)\right)^{-1}\hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0}(n) \tag{38}$$

wherein $\hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0}(n)$ and $\hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}(n)$ are defined as

$$\begin{aligned}\hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}(n) &= \sum_{i=0}^{n}\lambda^{n-i}\tilde{\boldsymbol{r}}_a(i)\tilde{\boldsymbol{r}}_a^H(i)\\ \hat{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0}(n) &= \sum_{i=0}^{n}\lambda^{n-i}\tilde{\boldsymbol{r}}_a(i)b_0(i).\end{aligned} \tag{39}$$

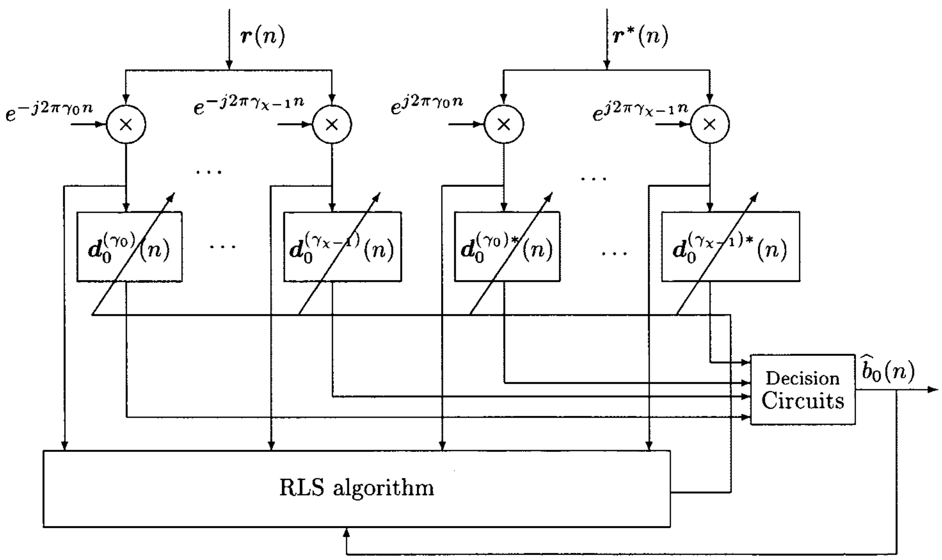

Fig. 2. RLS-based FRESH receiver implementation.

Now, following [23], it is easy to provide an RLS procedure with at most $O((2NM\chi)^2)$ computational complexity for the recursive evaluation of the solution $\tilde{\boldsymbol{d}}_{ab}(n)$ at the nth signaling interval. Of course, such a procedure is formally coincident with the conventional RLS algorithm, except for the fact that it operates on the augmented input sequence $\tilde{\boldsymbol{r}}_a(n)$. Summing up, the system operation is depicted in Fig. 2. The received signal is first fed to a bank of frequency oscillators, whose output is then forwarded to the block implementing the RLS algorithm and to the decision circuits, which, in turn, are driven by the output of the RLS algorithm. The new structure thus accounts for the polyperiodical nature of the observables, and is able to track polyperiodically time-varying receiver structures, at the price, however, of an increased computational complexity. As a final remark, notice that, like the conventional RLS, the new algorithm requires the usage of periodical training sequences. However, following the approach in [24] and [25], it can be readily reformulated in a blind fashion, so that the training period may be skipped.

VI. Analysis and Numerical Results

Commonly accepted performance measures for CDMA systems are the user BER and the *near–far resistance,* which characterizes the system behavior in the presence of interference [2], [5]. A closed-form, albeit implicit, expression for the BER can be directly found by observing (6), and indeed we have

$$P(e|\boldsymbol{i}(\ell), \boldsymbol{z}(\ell)) = \frac{1}{2}\,\text{erfc}\left(\frac{\mu(\boldsymbol{z}(\ell), \boldsymbol{i}(\ell))}{\sqrt{2\mathcal{N}_0}\,\|\boldsymbol{d}_0(\ell)\|}\right) \tag{40}$$

where

$$\mu(\boldsymbol{z}(\ell), \boldsymbol{i}(\ell)) = \Re\left\{A_0 e^{j\phi_0}\boldsymbol{d}_0(\ell)^H\boldsymbol{s}_0^0 + \boldsymbol{d}_0^H(\boldsymbol{z}(\ell)+\boldsymbol{i}(\ell))\right\} \tag{41}$$

and $\text{erfc}(\cdot)$ is the complementary error function. The unconditional BER is thus found by averaging (40) over $\boldsymbol{i}(\ell)$ and $\boldsymbol{z}(\ell)$. Lacking a closed form for the final result, we had to resort to computer simulations to evaluate the unconditional BER.

Conversely, the near–far resistance can be given a closed-form expression. Omitting the mathematical details, which are just an extension of those classically employed to show the convergence of the MMSE detectors to the zero-forcing receivers in the case of either vanishingly small $\mathcal{N}_0$ or arbitrarily large other-user amplitudes, we have the following.

Proposition: Let $\boldsymbol{M}_0(\ell) = \boldsymbol{M}(\ell) - A_0^2\boldsymbol{s}_0^0\boldsymbol{s}_0^{0T}$ and $\tilde{\boldsymbol{M}}_0$ a matrix defined as $\tilde{\boldsymbol{M}}$ in (29), with $\boldsymbol{M}(\ell)$ replaced by $\boldsymbol{M}_0(\ell)$. Then, if $\tilde{\boldsymbol{M}}_0$ is singular, the proposed nonadaptive receivers are near–far resistant.

Proof: For the proof and for an expression of the near–far resistance see Appendix B.

In the light of the arguments presented in Appendix B, it can be also easily shown that, the larger the set I'_γ, the larger the near–far resistance, with the obvious consequence that a PCMMSE receiver has better near–far resistance than any SPCMMSE. Even though we do not give an explicit proof, exploiting arguments similar to those reported in Appendix B it can be shown that an SPCMMSE with $I'_\beta = \emptyset$ and $I'_\alpha = I_\alpha$ (i.e., a PTVCMMSE) achieves a larger near–far resistance than a PTVMMSE. Likewise, a TICMMSE is always superior to a TIMMSE, consistent with [18]. It can be also readily shown that, if no external interference is present (and thus is not accounted for at the design stage), *and* the system is synchronous, so that no oversampling is necessary, then the system near–far resistance coincides with that of the optimum detector presented in [2].

In what follows, we provide some numerical results illustrating the performance of the proposed detection structures under several instances of interest, i.e.:

a) as the CDMA network is allocated in a frequency bandwidth partially occupied by a digital BPSK, which henceforth is to be considered as an NBI;
b) as the CDMA network is to coexist with another CDMA network, in a dual-rate architecture.

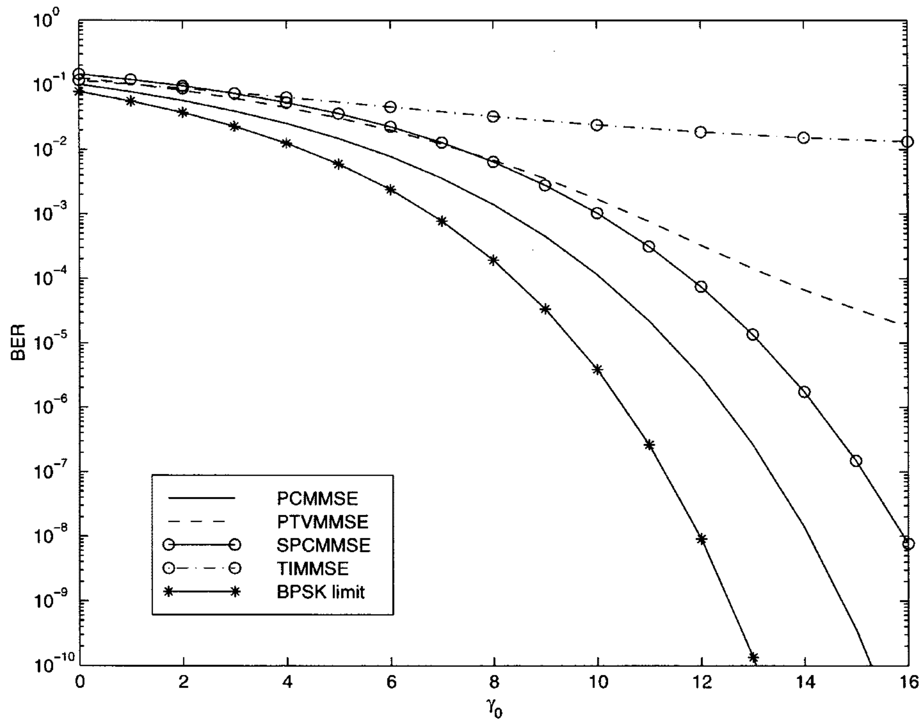

Fig. 3. BER for $K = 5$, $N = 7$, and a digital NBI with $T_I = 5T_c$.

We assume that the CDMA network adopts Gold codes as spreading sequences, with processing gain $N = 31$ or $N = 7$. The average with respect to the random parameters of the internal and the external interference (i.e., the bits falling in the processing window) has been carried out numerically. As to the delays of the NBI and the MAI, they have been modeled as random variates uniformly distributed in $[0, T_I]$ and $[0, T_b]$, respectively. In the following, the performance corresponding to asynchronous systems has been thus evaluated by averaging the BERs corresponding to 100 realizations of these last parameters.

As regards situation $\mathbf{a}$, in Fig. 3 we have represented the BER of the PCMMSE, SPCMMSE, PTVMMSE, and TIMMSE receivers versus the received energy contrast $\gamma_0 = A_0^2 T_b / 2\mathcal{N}_0$ for $K = 5$ users. The SPCMMSE receiver has been implemented by letting $I'_\alpha = \{p/r\}_{p=0}^{r-1}$ and $I'_\beta = \{1/r^*, (r^* - 1)/r^*\}$. The considered CDMA network is a synchronous one with processing gain $N = 7$, while the NBI has $T_I = 5T_c$ (thus $r = 5$) and $f_I T_c = 1/3$. The figure thus refers to the case that ν_I is a rational number, whereby the SPCMMSE degenerates into a linear periodically time-varying receiver with period $R = 15$. The interference-to-signal ratio (ISR), defined as $\mathrm{ISR} = A_{I0}^2 / A_0^2$, has been set to 20 dB, while the other users amplitudes $\{A_k\}_{k=1}^{K-1}$ are 5 dB larger than A_0. For comparison purposes, we also report the performance of an uncoded BPSK transmission over a single-user channel (BPSK limit). The curves clearly show that a suitably designed SPCMMSE outperforms the PTVMMSE achieving near-optimum performance. This result is even more interesting in the light of the fact that the computational complexity per bit is proportional to R for the PCMMSE, to $(r + 2)$ for the SPCMMSE and to r for the PTVMMSE, thus showing that the SPCMMSE represents an effective compromise between optimality and complexity.

Again, with reference to situation $\mathbf{a}$, in Fig. 4 we consider the performance of the same systems for an asynchronous CDMA system with processing gain $N = 31$, assuming an interfering BPSK with $T_I = (31/3)T_c$ and with a uniformly distributed frequency offset. Notice that, since T_b/T_I is an integer, the polyperiodical structure under consideration degenerates into a linear periodically time-varying filter whose periodicity is dictated by the frequency offset only. As a consequence, the conventional MMSE detector (7) degenerates in the classical TIMMSE. Also for this case the plots show that the newly proposed detector outperforms the TIMMSE.

Let us now consider situation $\mathbf{b}$, where the CDMA system is to coexist with a lower-rate CDMA network, arranged in a VCRFS architecture [10]. The two CDMA systems share the same processing gain, but the chip interval of the secondary system is twice that of the primary system, so that the bit rate of the primary network is twice faster than that of the secondary network. We consider the BERs of the PCMMSE and the PTVMMSE. In Fig. 5 the BER of the zeroth user of the high-rate network is represented; notice that, due to the particular frequency shift of the VCRFS architecture, the PCMMSE defined in (27) for the high-rate users is the PTVCMMSE, and it largely outperforms the PTVMMSE introduced in [10]. The figure refers to the case that $K = K' = 12$. The case that the user 0 of the low-rate network is to be decoded is instead represented in Fig. 6. The PCMMSE (27) now coincides with a TICMMSE. In both cases, the amplitudes of all of the interfering signals, from both CDMA systems, have been set 5 dB larger than that of the user of interest. Summing up the results clearly show that the newly proposed architecture is to be largely preferred both for the high-rate and for the low-rate system.

As a final remark, we notice that the results confirm that there always exists an SPCMMSE structure which, at the price of a

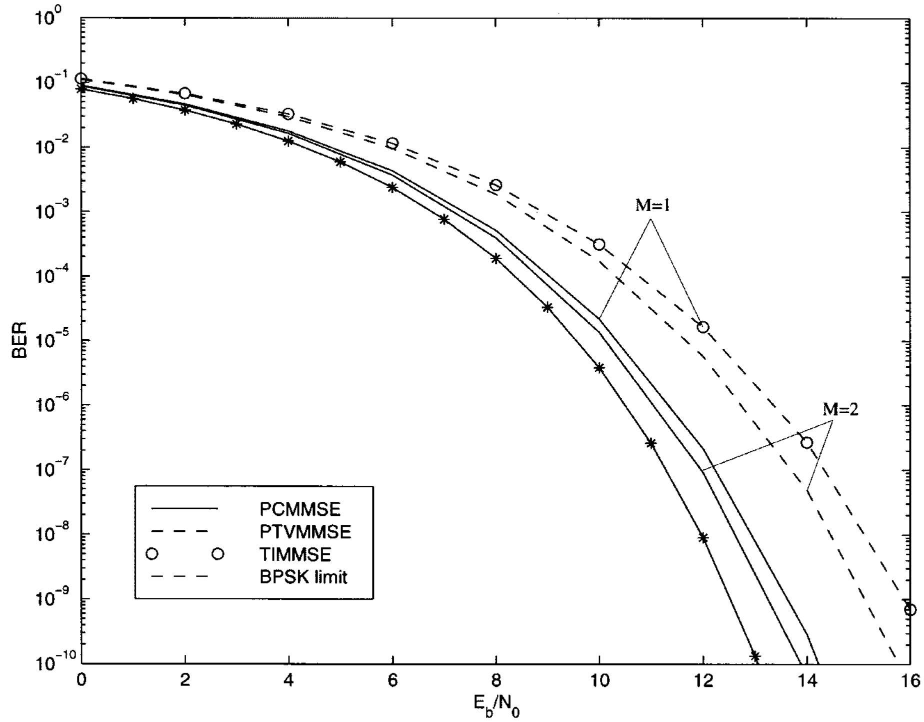

Fig. 4. BER for $K = 5$ and a digital NBI with $T_b/T_I = 3$.

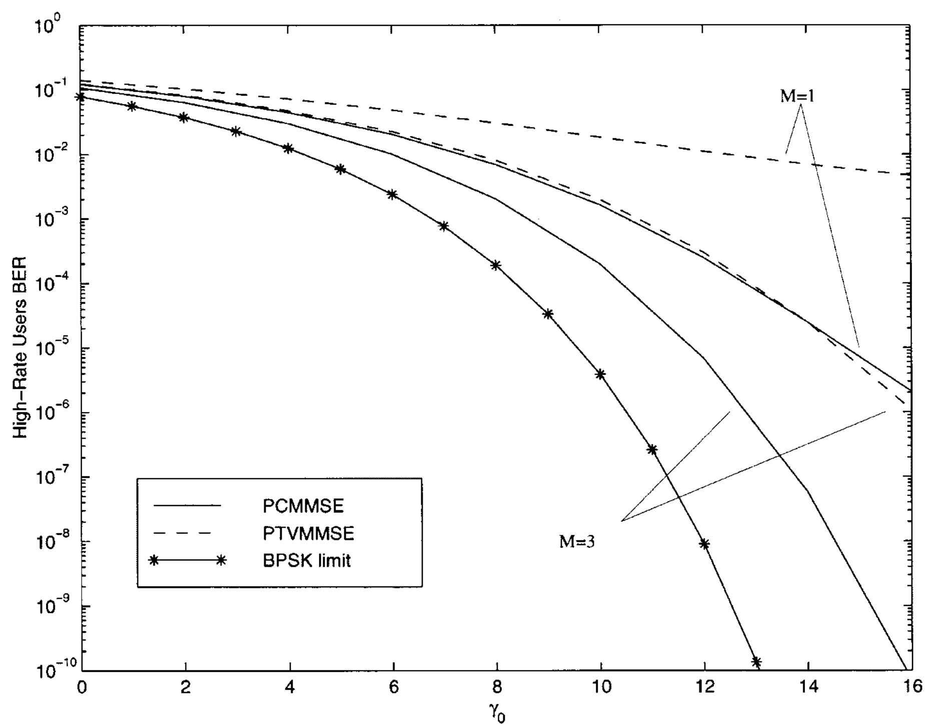

Fig. 5. BER of the high rate zeroth user. $K = K' = 12$.

limited complexity increase, as compared to the PTVMMSE, achieves a performance very close to that of the PCMMSE, which instead presents an often prohibitive computational complexity.

Finally, let us consider the assessment of the convergence properties of the RLS-based adaptive algorithm. Due to lack of space, we do not dwell on a rigorous analysis, and give a plot of the convergence dynamics. Here, we have considered adaptive detection of high-rate users in a VCRFS dual-rate system with $h = 3$. The spreading sequences are again Gold codes with $N = 31$, and, in order to avoid the average over the random delays, we have focused on the case of a synchronous network. This hypothesis, as illustrated in [5] and [25], does not entail any loss of generality. In Fig. 7 we have represented the correlation

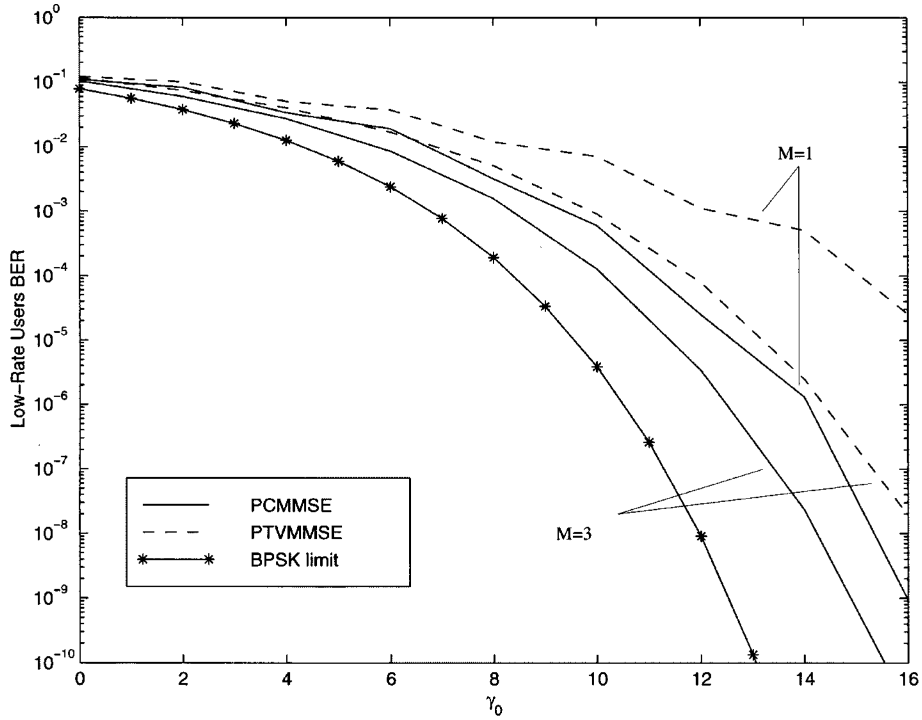

Fig. 6. BER of the low rate zeroth user. $K = K' = 12$.

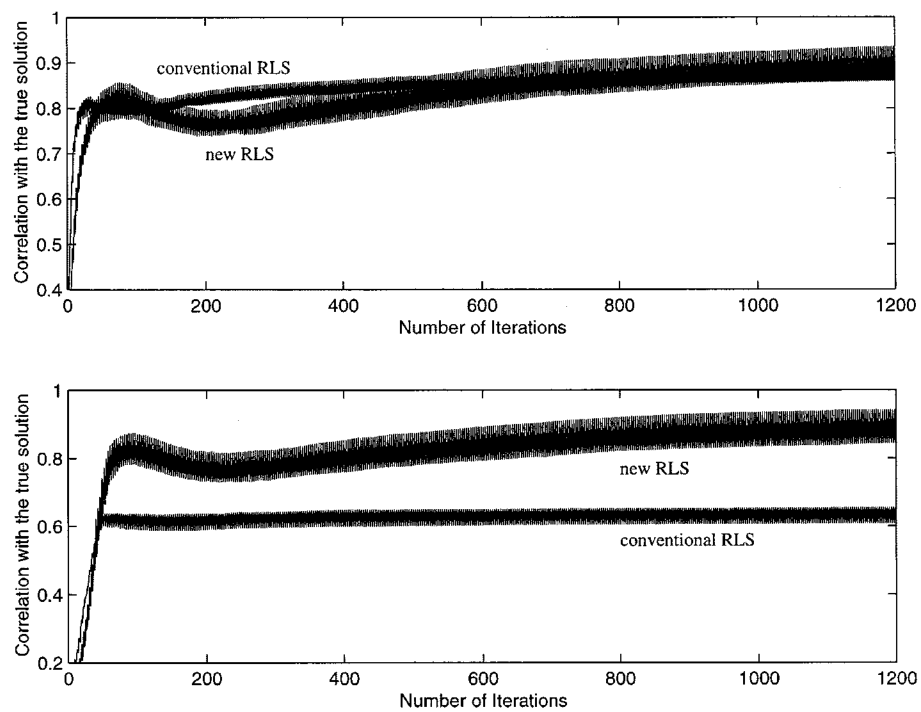

Fig. 7. Convergence dynamics of the RLS algorithm.

coefficient between the estimated filter, available at the output of the block implementing the RLS algorithm, and the true solution fulfilling (17), versus the number of iterations, for both the new adaptive algorithm and a classical RLS algorithm [23, ch. 13] operating on the augmented array $\boldsymbol{r}_a(\ell)$ and not taking into account the periodicity of the solution to be tracked; this latter algorithm will be referred to as conventional RLS. The plots are the result of an average over 100 random independent runs; furthermore, a training preamble of 100 bits is adopted, while for $n > 100$ both algorithms operate in decision-directed operating mode. The upper curve refers to a scenario with $K = 5$ and $K' = 12$; all of the signals have the same amplitude (i.e., perfect power-control is assumed) and the average received energy contrast is 13 dB. The lower curve, instead, is similar to the former one, except for the fact that the low-rate users amplitudes are assumed to be 20 dB stronger than the high-rate amplitudes.

It is seen that in the upper curve the gap between the new algorithm and the conventional RLS is very limited (even though, as n increases, results not shown here have confirmed that the steady-state performance of the new algorithm is always superior to that of the classical RLS) *but* the convergence time of the new algorithm is larger than that of the conventional one. In the lower curve, instead, the new RLS algorithm largely outperforms the conventional one. Such a behavior is easily justified. Indeed, the need for a periodically time-varying detection rule is due to the presence of the low-rate users. In a perfectly power-controlled situation, the time variations of the covariance matrix of the observables are not very strong, whence the conventional RLS algorithm (which is suited for the tracking of a time-invariant, or slowly time-varying, interference covariance matrix) achieves satisfactory performance. In this situation the new algorithm achieves good performance as well, *but* converges more slowly due to the fact that its input vector has a dimensionality h times larger than the dimensionality of the vector $\boldsymbol{r}_a(\cdot)$ which is processed by the conventional algorithm. In the lower curve, instead, the severe near–far scenario emphasizes the periodicity of the matrix $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}(\cdot)$, and so the conventional RLS algorithm is not able to track its variations. The new algorithm, on the contrary, achieves very satisfactory performance and enables adaptive high-rate users demodulation. Other simulations, whose results are not shown here for sake of brevity, have revealed that the conventional algorithm performs poorer and poorer as the users number increases, even if perfect power control is assured; the new algorithm, instead, is quite insensitive to the network load.

VII. Conclusions

In this paper, we have introduced and assessed a new family of MMSE detectors for joint suppression of multiaccess and external interference in CDMA networks. The main contribution of this study is, at a theoretical level, to show that full exploitation of the available information on the second-order statistics of the observables entails polyperiodical processing; thus, previously known MMSE detectors, whether time invariant or periodically time varying, represent either special cases or suboptimum systems. We also present a statistical analysis of the newly proposed systems, giving a closed-form formula for the near–far resistance. As case studies, we considered both the situation where the external interference source is a narrow-band system (i.e., an overlay architecture) and the situation that the external interference is another CDMA network with different bit rate (i.e., a dual-rate architecture). The results show that the newly proposed systems largely outperform all of the other systems currently known in the literature.

We also deal with the issue of adaptive implementation of the proposed receivers, deriving a new RLS, decision-directed algorithm, taking into account the polyperiodical nature of the solution to be tracked. A comparative performance analysis, carried on through computer simulations, shows the superiority of this algorithm with respect to the conventional RLS algorithms, e.g., designed to track time-invariant solutions.

Even though the proposed technique has been shown to be effective in suppressing MAI and external interference and to outperform previously derived detection structures, one of its major drawbacks lies in the fact that it can be applied only to systems employing BPSK modulation. Indeed, in an M-PSK modulation (with $M > 2$) format, the symbol to be decoded is complex, thus implying the inadequacy of the cost function (9); it is thus evident that, in this context, the optimization criterion is to be properly reformulated so as to cope with the complex nature of the information symbols. This issue forms the object of current investigation. Preliminary results have revealed that the criterion (9) may be generalized to the case that the quantity to be estimated is complex, but the corresponding estimator ends up coincident with the conventional linear MMSE estimator unless the external interference is BPSK-modulated. These results will be possibly reported elsewhere.

Appendix A

Here, we demonstrate that the cost function (25) admits, in the sequence set defined by (26), the minimum (27). To begin with, substituting expression (26) into (25), we have

$$\begin{aligned}\epsilon_{TA} &= \left\langle E\left[\left(\Re\left\{\sum_{\gamma\in I'_\gamma} \boldsymbol{d}_0^{(\gamma)H} e^{j2\pi n\gamma}\boldsymbol{r}(n)\right\} - b_0(n)\right)^2\right]\right\rangle \\ &= \left\langle E\left[(\tilde{\boldsymbol{d}}_a^H \tilde{\boldsymbol{r}}_a(n) - b_0(n))^2\right]\right\rangle. \end{aligned} \tag{42}$$

In the above expression, $\tilde{\boldsymbol{d}}_a$ is the augmented version of the $NM\chi$-dimensional vector containing the χ NM-dimensional vectors of the Fourier series expansion coefficients of $\boldsymbol{d}_0(\ell)$, i.e.,

$$\tilde{\boldsymbol{d}}_a = \tfrac{1}{2}\left[\boldsymbol{d}_0^{(\gamma_0)T}, \ldots, \boldsymbol{d}_0^{(\gamma_{\chi-1})T}, \boldsymbol{d}_0^{(\gamma_0)H}, \ldots, \boldsymbol{d}_0^{(\gamma_{\chi-1})H}\right]^T. \tag{43}$$

Likewise, $\tilde{\boldsymbol{r}}_a(n)$ is the augmented version of the vector $\tilde{\boldsymbol{r}}(n)$ of (35), and is defined in (37). Taking the gradient of (42) with respect to $\tilde{\boldsymbol{d}}_a$ and nullifying it yields

$$\tilde{\boldsymbol{d}}_a = \left(\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}\right)^{-1}\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0} \tag{44}$$

wherein $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}$ and $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b0}$ are a $2NM\chi$-dimensional square matrix and a $2NM\chi$-dimensional vector, respectively, defined as:

$$\begin{aligned}\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a} &= \langle E[\tilde{\boldsymbol{r}}_a(n)\tilde{\boldsymbol{r}}_a^H(n)]\rangle \\ &= \begin{pmatrix}\tilde{\boldsymbol{M}} & \tilde{\boldsymbol{M}}' \\ \tilde{\boldsymbol{M}}'^* & \tilde{\boldsymbol{M}}^*\end{pmatrix} \\ \tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0} &= \langle E[\tilde{\boldsymbol{r}}_a(n)b_0(n)]\rangle \\ &= \left[A_0 e^{j\phi_0}\boldsymbol{s}_0^{0T}, \underbrace{0, \ldots, 0}_{NM(\chi-1)}, A_0 e^{-j\phi_0}\boldsymbol{s}_0^{0T}, \underbrace{0, \ldots, 0}_{NM(\chi-1)},\right]^T. \end{aligned} \tag{45}$$

Evaluating $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a\tilde{\boldsymbol{r}}_a}^{-1}$ through block-inversion, plugging it in (44), and taking the first $NM\chi$ entries of the augmented vector $\tilde{\boldsymbol{d}}_a$, we obtain the χ coefficients of the generalized Fourier series expansion of the solution. Recombining these coefficients according to (26), i.e., left-multiplying the said $NM\chi$-dimensional vector by the matrix $\boldsymbol{E}(\ell)$ defined in Section IV, we finally obtain (27).

APPENDIX B

At first, we show that the system (27) is near–far resistant under certain circumstances. In this case too, it is convenient to resort to the augmented vectors $\tilde{\boldsymbol{d}}_a$ and $\tilde{\boldsymbol{r}}_a(\ell)$. Adopting the notations already defined in Appendix A, the decision criterion (6) can be rewritten as

$$\hat{b}_0(\ell) = \operatorname{sgn}(\tilde{\boldsymbol{d}}_a^H \tilde{\boldsymbol{r}}_a(\ell)) \tag{46}$$

with $\tilde{\boldsymbol{d}}_a$ given in (44). For vanishingly small $\mathcal{N}_0$ we have

$$\lim_{\mathcal{N}_0 \to 0} \tilde{\boldsymbol{d}}_a = \begin{pmatrix} \tilde{\boldsymbol{M}}_0 & \tilde{\boldsymbol{M}}' \\ \tilde{\boldsymbol{M}}'^* & \tilde{\boldsymbol{M}}_0^* \end{pmatrix}^\dagger \tilde{\boldsymbol{M}}_{\boldsymbol{r}_a b_0} \tag{47}$$

with $\tilde{\boldsymbol{M}}_0 = \lim_{\mathcal{N}_0 \to 0} \tilde{\boldsymbol{M}}$ and $(\cdot)^\dagger$ denoting pseudoinverse. On the other hand, it can be shown that (47) also coincides—except that for an irrelevant positive factor—with a solution to the constrained minimization problem

$$\begin{cases} \langle E[|\tilde{\boldsymbol{d}}_a^H (\tilde{\boldsymbol{z}}_a(\ell) + \tilde{\boldsymbol{i}}_a(\ell))|^2]\rangle = \min \\ \text{subject to } \tilde{\boldsymbol{d}}_a^H \tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0} = 1. \end{cases} \tag{48}$$

where $\tilde{\boldsymbol{z}}_a(\ell)$ and $\tilde{\boldsymbol{i}}_a(\ell)$ are $2NM\chi$-dimensional vectors defined as the vector $\tilde{\boldsymbol{r}}_a(\ell)$ in (37). We explicitly notice here that $\tilde{\boldsymbol{r}}_a(\ell)$, $\tilde{\boldsymbol{z}}_a(\ell)$ and $\tilde{\boldsymbol{i}}_a(\ell)$ depend on which harmonical frequencies are present in I'_γ, and so do the respective covariance matrices; we do not explicitly indicate such a dependence whenever possible, resuming it just in the last part of the present appendix. Now, if the matrix

$$\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{z}}_a + \tilde{\boldsymbol{i}}_a} = \tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a \tilde{\boldsymbol{r}}_a} - \tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0} \tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{r}}_a b_0}^H \tag{49}$$

is *singular*, then the minimum attained by the solution to (48) is zero, i.e., $\tilde{\boldsymbol{d}}_a$ becomes orthogonal to the range span of $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{z}}_a + \tilde{\boldsymbol{i}}_a}$. Since the functional in (48) is a time-average of a nonnegative functional, it is necessarily true that, for $\mathcal{N}_0 \to 0$

$$\begin{aligned} \tilde{\boldsymbol{d}}_a^H (\tilde{\boldsymbol{z}}_a(\ell) + \tilde{\boldsymbol{i}}_a(\ell)) &= (\boldsymbol{d}_a^H(\ell)(\boldsymbol{z}_a(\ell) + \boldsymbol{i}_a(\ell))) \\ &= \Re\{\boldsymbol{d}_0^H(\ell)(\boldsymbol{z}(\ell) + \boldsymbol{i}(\ell))\} = 0 \qquad \forall\, \ell \end{aligned} \tag{50}$$

with $\boldsymbol{d}_a(\ell) = \begin{pmatrix} \boldsymbol{E}(\ell) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{E}^*(\ell) \end{pmatrix} \tilde{\boldsymbol{d}}_a$. The near–far resistance is thus given by

$$\eta(\ell) = \frac{\boldsymbol{d}_a^H(\ell)\boldsymbol{v}}{2\|\boldsymbol{s}_0^0\|^2} = \frac{\|\boldsymbol{d}_a(\ell)\|^2}{2\|\boldsymbol{s}_0^0\|^2} \tag{51}$$

where $\boldsymbol{v}$ is defined in (16).

Let us finally consider the χ-dimensional set I'_γ and the $(\chi - 1)$-dimensional set $I''_\gamma = I'_\gamma - \{\gamma_\chi\}$. Now, denoting by $\tilde{\boldsymbol{x}}_a(I''_\gamma)$ a $2NM(\chi - 1)$-dimensional vector of the null space of $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{z}}_a + \tilde{\boldsymbol{i}}_a}(I''_\gamma)$, then any $2NM\chi$-dimensional vector obtained by adding $2NM$ zeros to $\tilde{\boldsymbol{x}}_a(I''_\gamma)$—i.e., NM zeros at the end of the first half of $\tilde{\boldsymbol{x}}_a(I''_\gamma)$ and as many at the end of the second half—belongs to the null space of $\tilde{\boldsymbol{M}}_{\tilde{\boldsymbol{z}}_a + \tilde{\boldsymbol{i}}_a}(I'_\gamma)$. Otherwise stated, seeking a solution with harmonical frequencies in I''_γ is equivalent to seeking a solution with harmonical frequencies in I'_γ, with the additional constraint that the said $2NM$ entries of the solution be zero. As a consequence, allowing a larger number of harmonical frequencies in the sets I'_γ results in less and less constrained solutions, which explains why, if $I'_\gamma \supset I''_\gamma$, the near–far resistance of a system built on I''_γ is not larger than that of a system built on I'_γ.

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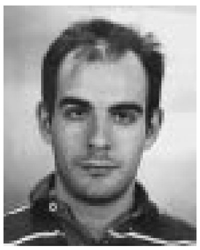

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