A Generalized Minimum-Mean-Output-Energy Strategy for CDMA Systems With Improper MAI

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***Abstract*—It has been recently shown that in a direct-sequence/code-division multiple-access (DS/CDMA) system employing binary phase-shift keying (BPSK) modulation the baseband equivalent of the CDMA multiplex is, under very mild assumptions, an improper complex random process, i.e., it has a nonzero pseudoautocorrelation function. In this correspondence, the problem of linear multiuser detection for asynchronous DS/CDMA systems with improper multiaccess interference (MAI) is considered. A new mean-output-energy (MOE) cost function is introduced, whose constrained minimization leads to two new linear multiuser detectors, exploiting the information contained in the pseudoautocorrelation of the observables, and which generalize the classical decorrelating and minimum mean-square error (MMSE) receivers. The problem of blind adaptive receiver implementation based on subspace tracking is also briefly tackled. Finally, the superiority of the new detectors with respect to the classical linear detection structures present in the literature is demonstrated through both theoretical considerations and computer simulations.**

***Index Terms*—Code division multiple access (CDMA), decorrelating detector, improper complex noise, minimum mean-square error (MMSE) detector, multiuser detection, subspace tracking.**

I. Introduction

Multiuser detection [1] for direct-sequence/code-division multiple-access (DS/CDMA) systems is an intriguing research topic that has attracted the interest of many researchers over the past two decades. Indeed, since the seminal work by Verdú [2], who showed that multiaccess interference (MAI), if explicitly accounted for at the design stage, is not a performance-limiting factor, and derived the optimum multiuser detector, a great deal of attention has been devoted to the design of suboptimal multiuser detection receivers, so as to surmount the cumbersome complexity (exponential in the number of users) of the optimum multiuser detector and, at the same time, attain close-to-optimality performance. The most popular suboptimal multiuser detectors are the decorrelating detector [3] and the minimum mean-square error (MMSE) receiver [4]: interestingly, both receivers have a complexity which is linear in the number of users and achieve optimum performance in terms of near–far resistance.

Most of the existing studies on linear multiuser detection, however, propose detection structures which exploit only the information contained in the autocorrelation function of the observables. While this is the optimum strategy when dealing with proper complex random processes, it turns out to be suboptimal in situations where the disturbance is an improper complex random process. A complex random process $n(t)$ is said to be proper if its pseudoautocorrelation function $R_n(t, u) = \mathrm{E}[n(t)n(u)]$ is zero $\forall\, t,\, u$, and it is said to be improper in the opposite case that $R_n(t, u)$ is nonzero [5]. From a physical point of view, proper processes are circularly symmetric, while in improper processes such a phase symmetry is not present. Recently, it has been shown in [6], [7] that in a DS/CDMA system employing a BPSK modulation format the MAI can be modeled as an improper complex noise. As a consequence, it is expected that designing receiving structures capable of exploiting the information contained in the pseudoautocorrelation function of the observables would permit achieving better performance with respect to classical linear multiuser receivers neglecting this information.

In this work, we thus deal with the problem of linear multiuser detection under such a relevant scenario. Specifically, when only one of the dimensions of the complex decision statistic at the output of a linear receiver is of interest, it is possible to define a more efficient cost function for optimizing the receiver. It was shown in [7] that substantial performance gains may be obtained by an appropriate modification of the linear MMSE cost criterion. The present correspondence considers an analogous modification of the minimum mean-output energy (MOE) criterion. More generally, our derivations in this paper make it evident that our *modified* cost functions for deriving correlators operating on a complex received vector of dimension d are actually *standard* cost functions for an equivalent correlator operating on a real received vector of dimension $2d$. The corresponding new linear multiuser detectors exploit information contained in the pseudoautocorrelation of the received signal. Furthermore, we also address the problem of blind adaptive multiuser detection, showing that the tools of subspace tracking theory may be successfully applied for blind adaptive implementation of the proposed receivers. With regard to the performance analysis, the superiority of the new detectors with respect to the classical MMSE and decorrelating schemes is shown through both theoretical considerations and numerical results. Moreover, note that it has been recently shown that, for a synchronous CDMA system with random spreading codes, the new detectors provide an asymptotic performance advantage of 3 dB when both the processing gain and the users number grow large with their ratio fixed [8]. Plots of the convergence dynamics of a subspace-tracking algorithm for blind adaptive implementation of the new receivers are also given.

It should be noted that the proposed technique applies to CDMA systems employing BPSK modulation (with a possibly complex spreading), while is not useful for quaternary phase-shift keying (QPSK) signaling constellations. Indeed, QPSK signaling makes the CDMA multiplex a proper random process, thus implying that the newly proposed detection structures do not achieve any performance improvement with respect to the classical linear multiuser detectors. However, even though the most part of the leading standard proposals for third-generation wireless networks recommends the use of QPSK signaling constellation, there also exist some standard proposals where adoption of BPSK modulation has been planned [9, p. 85], [10, p. 124], whence CDMA systems with BPSK modulation are still of interest, not only from a theoretical point of view but also for commercial reasons. Additionally, it is worth pointing out that the proposed approach may also be extended to the case of CDMA systems employing QPSK modulation and with improper cochannel interference.

The correspondence is organized as follows. In the next section, we dwell on the system model, while in Section III, the receiver's synthesis is outlined. Section IV contains a performance assessment and a discussion of the numerical results, while the problem of blind adaptive receivers implementation is tackled in Section V. Finally, concluding remarks are given in Section VI.

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II. System Model

Let us consider an asynchronous, BPSK modulated, DS/CDMA network with K active users. The baseband equivalent of the received signal is written as

$$r(t) = \sum_{k=0}^{K-1} A_k e^{j\phi_k} \sum_{m=-P}^{P} b_k(m) s_k(t - mT_b - \tau_k) + w(t). \quad (1)$$

In the above expression, A_k is the amplitude of the kth user signal, accounting for the transmitted energy and the channel propagation loss; $\{b_k(m)\}_{m=-\infty}^{+\infty}$ represents the bit stream of the kth user, modeled as a sequence of independent and identically distributed binary variates, each taking on values in the set $\{-1; 1\}$; $T_b = NT_c$ is the bit interval of the network, with T_c the chip interval and N the processing gain; $[\tau_0, \ldots, \tau_{K-1}]^T = \boldsymbol{\tau}$ is a set of delays; from now on, we assume, without loss of generality, that $0 = \tau_0 \leq \tau_1 \leq \cdots \leq \tau_{K-1} < T_b$; $2P+1$ is the transmitted frame length; $s_k(t)$ is the *signature* assigned to the kth user, expressed as

$$s_k(t) = \frac{1}{\sqrt{NT_c}} \sum_{n=0}^{N-1} c_{kn} u_{T_c}(t - nT_c)$$

with $\{c_{kn}\}_{n=0}^{N-1}$ the kth (possibly complex) spreading code[1] and $u_{T_c}(\cdot)$ a unit-height rectangular pulse supported on the interval $[0, T_c]$; $w(t)$ is the thermal noise term, modeled as a sample function from a proper complex white Gaussian process with power spectral density (PSD) $2\mathcal{N}_0$. Finally, the term $e^{j\phi_k}$ is a phase term, accounting for the possible channel propagation effects, the relative delay, and the initial phase offset of the kth-user frequency oscillator. Even though these parameters are obviously random and time-variant, it is customary [3], [4], [6], [11], [12] to assume that these phase terms remain constant at least for the whole frame duration $(2P+1)T_b$, unless we are dealing with a fast-fading channel, a case that we do not consider here.

The received signal (1) may be conveniently recast to separate the useful term and the interferers; to fix the ideas, let us assume that we are willing to decode the user 0 bit stream, and that a decision as to $b_0(p)$ is made by processing the interval $[pT_b, (p+1)T_b]$. In order to convert the received signal to discrete time, we project it along the unit vectors of the NM-dimensional orthonormal system

$$\left\{ \frac{1}{\sqrt{T_{OS}}} u_{T_{OS}}(t - pT_b - iT_{OS}) \right\}_{i=0}^{MN-1} \quad (2)$$

with $T_{OS} = T_c/M$ and M, the number of samples per chip, a positive integer.

Assuming, without loss of generality, that $\phi_0 = 0$, we thus obtain the following NM-dimensional observable vector:

$$\boldsymbol{r}(p) = A_0 b_0(p) \boldsymbol{s}_0^0 + \sum_{k=1}^{K-1} A_k e^{j\phi_k} \sum_{m \in \{-1,0\}} b_k(p+m) \boldsymbol{s}_k^m + \boldsymbol{w}(p) = A_0 b_0(p) \boldsymbol{s}_0^0 + \boldsymbol{z}(p) + \boldsymbol{w}(p). \quad (3)$$

In (3), the first term on the right-hand side represents the contribution from the bit to be decoded (in particular, $\boldsymbol{s}_0^0$ is the discretized version of the signature waveform from the user of interest), while the other terms represent the contributions from MAI $(\boldsymbol{z}(p))$ and the thermal noise $(\boldsymbol{w}(p))$, respectively. The vectors $\boldsymbol{s}_k^m$ are the discrete-time versions of the interfering users' signatures. Notice that $\boldsymbol{w}(p)$ is a proper white complex Gaussian vector with covariance matrix $2\mathcal{N}_0 \boldsymbol{I}_{NM}$, with $\boldsymbol{I}_{NM}$ the identity matrix of order NM. The covariance matrix of the observables, $\boldsymbol{M_{rr}}$, say, can be shown to admit the following expression:

$$\boldsymbol{M_{rr}} = \mathrm{E}\left[\boldsymbol{r}(p)\boldsymbol{r}^H(p)\right] = A_0^2 \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0H} + \sum_{k=1}^{K-1} A_k^2 \sum_{m \in \{-1,0\}} \boldsymbol{s}_k^m \boldsymbol{s}_k^{mH} + 2\mathcal{N}_0 \boldsymbol{I}_{NM}$$

with $(\cdot)^H$ denoting conjugate transpose. The pseudocovariance matrix $\boldsymbol{M}'_{\boldsymbol{rr}}$ is instead written as [5]

$$\boldsymbol{M}'_{\boldsymbol{rr}} = \mathrm{E}\left[\boldsymbol{r}(p)\boldsymbol{r}^T(p)\right] = A_0^2 \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0T} + \sum_{k=1}^{K-1} A_k^2 e^{j2\phi_k} \sum_{m \in \{-1,0\}} \boldsymbol{s}_k^m \boldsymbol{s}_k^{mT} \quad (4)$$

with $(\cdot)^T$ denoting transpose. It is thus seen that, since $\boldsymbol{M}'_{\boldsymbol{rr}}$ is nonzero, the baseband equivalent of the CDMA signals is an improper random process.

For future reference, we denote by $\mathcal{C}^{NM}$ the space of the complex NM-tuples on the complex field $\mathcal{C}$ with the usual internal and external operations, and by $\mathcal{S}$ the *signal space*, namely, the vector subspace of $\mathcal{C}^{NM}$ spanned by the desired signal and the MAI.

III. Detector Design

A. The Minimum-Output Interference Energy Approach

Given the NM-dimensional observable vector $\boldsymbol{r}(p)$, any linear receiver takes a decision as to the bit $b_0(p)$ according to the rule

$$\hat{b}_0(p) = \mathrm{sgn}\left(\Re\left\{\boldsymbol{d}_0^H \boldsymbol{r}(p)\right\}\right) \quad (5)$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, $\Re\{\cdot\}$ denotes real part, and the vector $\boldsymbol{d}_0 \in \mathcal{C}^{NM}$ is to be designed according to some optimization criterion. Notice that since the final decision on the bit $b_0(p)$ is taken on the basis of the sign of the test statistic, the decision rule (5) is invariant to any positive scaling of the vector $\boldsymbol{d}_0$. As already stated, the most popular suboptimum detectors are the decorrelating and the MMSE receivers. Following [13], it can be shown that the decorrelating detector, first introduced in [3], can be obtained as the unique solution to the following constrained minimization of the output interference energy:

$$\begin{cases} \mathrm{E}\left[\left|\boldsymbol{d}_0^H \boldsymbol{z}(p)\right|^2\right] = \min \\ \boldsymbol{d}_0^H \boldsymbol{s}_0^0 = 1. \end{cases} \quad (6)$$

Likewise, the classical MMSE detector, according to [4], [11], [13], may be obtained as the solution to the problem

$$\mathrm{E}\left[\left|b_0(p) - \boldsymbol{d}_0^H \boldsymbol{r}(p)\right|^2\right] = \min \quad (7)$$

or, alternatively, to the following constrained minimization problem:

$$\begin{cases} \mathrm{E}\left[\left|\boldsymbol{d}_0^H (\boldsymbol{z}(p) + \boldsymbol{w}(p))\right|^2\right] = \min \\ \boldsymbol{d}_0^H \boldsymbol{s}_0^0 = 1. \end{cases} \quad (8)$$

Applying standard Lagrangian techniques, it is easy to show that the solutions to (6) and (8) can be written as

$$\boldsymbol{d}_{0,\mathrm{DEC}} = \frac{\left(A_0^2 \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0H} + \boldsymbol{M_{zz}}\right)^+ \boldsymbol{s}_0^0}{\boldsymbol{s}_0^{0H} \left(A_0^2 \boldsymbol{s}_0^0 \boldsymbol{s}_0^{0H} + \boldsymbol{M_{zz}}\right)^+ \boldsymbol{s}_0^0}$$

and

[1]Notice that the users' spreading codes are independent of the temporal index, thus implying that the case of periodic codes, which repeat at each symbol interval, is being considered here.

$$\boldsymbol{d}_{0,\mathrm{MMSE}} = \frac{\boldsymbol{M}_{\boldsymbol{rr}}^{-1}\boldsymbol{s}_0^0}{\boldsymbol{s}_0^{0H}\boldsymbol{M}_{\boldsymbol{rr}}^{-1}\boldsymbol{s}_0^0} \tag{9}$$

respectively.[2] In the above expression, the symbol $(\cdot)^+$ denotes the Moore–Penrose generalized inverse, while the matrices $\boldsymbol{M}_{\boldsymbol{zz}}$ and $\boldsymbol{M}_{\boldsymbol{rr}}$ denote the covariance matrices of the MAI $z(p)$ and of the observables $\boldsymbol{r}(p)$.

As already highlighted, receivers (9) do not exploit information contained in the pseudoautocorrelation function of the observables. Our ultimate goal is thus to devise new minimum MOE optimization criteria in order to come up with new linear receiving structures exploiting this information. To this end, notice that, inspecting the decision rule (5), it can be inferred that what actually matters is not the square modulus of the mean output disturbance contribution, *but* its mean square real part: more precisely, the ultimate disturbance energy impairing the receiver performance is $\mathrm{E}[(\Re\{\boldsymbol{d}_0^H(z(p)+\boldsymbol{w}(p))\})^2]$, while the companion interfering term $\Im\{\boldsymbol{d}_0^H(z(p)+\boldsymbol{w}(p))\}$ does *not* affect the system error probability ($\Im(\cdot)$ denotes the coefficient of the imaginary part of a complex number). Likewise, the constraint $\boldsymbol{d}_0^H\boldsymbol{s}_0^0 = 1$ appears suboptimal, since only the real part of the inner product $\boldsymbol{d}_0^H\boldsymbol{s}_0^0$ is effective in the decision rule (5). Based on these arguments, we adopt here, in place of (6) and (8), a *generalized* minimum MOE optimization criterion, i.e., we consider the following constrained minimization problems:

$$\begin{cases} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{d}_0^H\boldsymbol{z}(p)\right\}\right)^2\right] = \min \\ \Re\left\{\boldsymbol{d}_0^H\boldsymbol{s}_0^0\right\} = 1 \end{cases} \tag{10}$$

and

$$\begin{cases} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{d}_0^H\left(\boldsymbol{z}(p)+\boldsymbol{w}(p)\right)\right\}\right)^2\right] = \min \\ \Re\left\{\boldsymbol{d}_0^H\boldsymbol{s}_0^0\right\} = 1. \end{cases} \tag{11}$$

These new cost functions can be shown to achieve better performance than their counterparts (6) and (8). To see this, let us focus on problems (6) and (10). First of all, note that, due to the constraints, the desired term $\Re\{\boldsymbol{d}_0^H\boldsymbol{s}_0^0\}$ in the decision statistic $\Re\{\boldsymbol{d}_0^H\boldsymbol{r}(p)\}$ is the same for both the original and modified MOE. Further, the constraint $\boldsymbol{d}_0^H\boldsymbol{s}_0^0 = 1$ is obviously equivalent to the following pair of real constraints:

$$\begin{cases} \boldsymbol{d}_{0R}^T\boldsymbol{s}_{0R}^0 + \boldsymbol{d}_{0I}^T\boldsymbol{s}_{0I}^0 = 1 \\ \boldsymbol{d}_{0R}^T\boldsymbol{s}_{0I}^0 - \boldsymbol{d}_{0I}^T\boldsymbol{s}_{0R}^0 = 0 \end{cases} \tag{12}$$

where the subscripts $(\cdot)_R$ and $(\cdot)_I$ denote real part and coefficient of the imaginary part, respectively. The constraint $\Re\{\boldsymbol{d}_0^H\boldsymbol{s}_0^0\} = 1$, instead, leads just to one real constraint, i.e., the first equation in (12). As a consequence, denoting by $\mathcal{Q}_1 \subset \mathcal{S}$ and $\mathcal{Q}_2 \subset \mathcal{S}$ the sets containing all of the vectors $\boldsymbol{d}_0 \in \mathcal{S}$ fulfilling the first and the second equation in (12), respectively, problems (6) and (10) can be rewritten as follows:

$$\boldsymbol{d}_{0,\mathrm{DEC}} = \arg\min_{\boldsymbol{x}\in\mathcal{Q}_1\cap\mathcal{Q}_2} \mathrm{E}\left[\left|\boldsymbol{x}^H\boldsymbol{z}(p)\right|^2\right]$$

and

$$\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}} = \arg\min_{\boldsymbol{x}\in\mathcal{Q}_1} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{x}^H\boldsymbol{z}(p)\right\}\right)^2\right] \tag{13}$$

[2]Strictly speaking, the vectors $\boldsymbol{d}_{0,\mathrm{DEC}}$ and $\boldsymbol{d}_{0,\mathrm{MMSE}}$ are not coincident, but only proportional through a positive constant, to the decorrelating and MMSE detectors. Nonetheless, due to the said invariance of the decision rule to any positive scaling of the test statistic, here and in the following we refer to the vectors in (9) as to the decorrelating and MMSE receivers.

with $\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}}$ denoting the solution to (10). Now, given (13), the following inequalities can be easily proven:

$$\begin{aligned} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{d}_{0,\mathrm{DEC}}^H\boldsymbol{z}(p)\right\}\right)^2\right] &\geq \min_{\boldsymbol{x}\in\mathcal{Q}_1\cap\mathcal{Q}_2} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{x}^H\boldsymbol{z}(p)\right\}\right)^2\right] \\ &\geq \min_{\boldsymbol{x}\in\mathcal{Q}_1} \mathrm{E}\left[\left(\Re\left\{\boldsymbol{x}^H\boldsymbol{z}(p)\right\}\right)^2\right] \\ &= \mathrm{E}\left[\left(\Re\left\{\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}}^H\boldsymbol{z}(p)\right\}\right)^2\right]. \end{aligned} \tag{14}$$

In (14), the first inequality follows from the fact that minimizing only the expected value of the squared real part of $\boldsymbol{x}^H\boldsymbol{z}(p)$ obviously yields a smaller minimum than $\mathrm{E}[(\Re\{\boldsymbol{d}_{0,\mathrm{DEC}}^H\boldsymbol{z}(p)\})^2]$, with $\boldsymbol{d}_{0,\mathrm{DEC}}$ given by the first line in (13). The second inequality, moreover, follows from the fact that, since $\mathcal{Q}_1 \cap \mathcal{Q}_2 \subseteq \mathcal{Q}_1$, the right-hand-most minimization is performed in a larger set and can achieve an even smaller minimum, while the last equality follows trivially from the second line in (13). Overall, inequalities (14) reveal that, while contribution from the desired term to the test statistic is the same in both the classical and the new approach, the vector $\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}}$ is capable of suppressing interference more efficiently than the vector $\boldsymbol{d}_{0,\mathrm{DEC}}$. It is thus shown that the new approach achieves better performance than the classical approach. Likewise, the same steps can be applied to demonstrate the superiority of the solution to the problem (11), which will be denoted by $\tilde{\boldsymbol{d}}_{0,\mathrm{MMSE}}$, with respect to the solution (8).

B. Receiver Synthesis

Let us now give an explicit formula to the vectors $\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}}$ and $\tilde{\boldsymbol{d}}_{0,\mathrm{MMSE}}$, which denote the solutions to problems (10) and (11). To begin with, notice that the operator $\Re\{\boldsymbol{x}^H\boldsymbol{y}\}$ (with $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{C}^{NM}$) is not an admissible inner product in $\mathcal{C}^{NM}$, whence solving the said problems requires special attention. First of all, notice that

$$\Re\left\{\boldsymbol{x}^H\boldsymbol{y}\right\} = \left(\boldsymbol{x}_R^T\boldsymbol{y}_R + \boldsymbol{x}_I^T\boldsymbol{y}_I\right) = \begin{pmatrix}\boldsymbol{x}_I \\ \boldsymbol{x}_R\end{pmatrix}^H \begin{pmatrix}\boldsymbol{y}_I \\ \boldsymbol{y}_R\end{pmatrix} = \boldsymbol{x}_a^H\boldsymbol{y}_a \tag{15}$$

with

$$\boldsymbol{x}_a = \begin{bmatrix}\boldsymbol{x}_R \\ \boldsymbol{x}_I\end{bmatrix} \quad \text{and} \quad \boldsymbol{y}_a = \begin{bmatrix}\boldsymbol{y}_R \\ \boldsymbol{y}_I\end{bmatrix}.$$

As a consequence, problems (10) and (11) can be written as

$$\begin{aligned} &\begin{cases} \mathrm{E}\left[\left(\boldsymbol{d}_{0a}^T\boldsymbol{z}_a(p)\right)^2\right] = \min \\ \boldsymbol{d}_{0a}^T\boldsymbol{s}_{0a}^0 = 1 \end{cases} \\ &\begin{cases} \mathrm{E}\left[\left(\boldsymbol{d}_{0a}^T\left(\boldsymbol{z}_a(p)+\boldsymbol{w}_a(p)\right)\right)^2\right] = \min \\ \boldsymbol{d}_{0a}^T\boldsymbol{s}_{0a}^0 = 1 \end{cases} \end{aligned} \tag{16}$$

with

$$\begin{aligned} \boldsymbol{d}_{0a} &= \begin{bmatrix}\boldsymbol{d}_{0R} \\ \boldsymbol{d}_{0I}\end{bmatrix} & \boldsymbol{z}_a(p) &= \begin{bmatrix}\boldsymbol{z}_R(p) \\ \boldsymbol{z}_I(p)\end{bmatrix} \\ \boldsymbol{w}_a(p) &= \begin{bmatrix}\boldsymbol{w}_R(p) \\ \boldsymbol{w}_I(p)\end{bmatrix} & \boldsymbol{s}_{0a}^0 &= \begin{bmatrix}\boldsymbol{s}_{0R}^0 \\ \boldsymbol{s}_{0I}^0\end{bmatrix}. \end{aligned} \tag{17}$$

In the above expressions, the vectors $\boldsymbol{z}_a(p)$, $\boldsymbol{w}_a(p)$, and $\boldsymbol{s}_{0a}^0$ are augmented $2NM$-dimensional *real* vectors. The relevant advantage of the formulation (16) of the problems (10) and (11) is that now standard differentiation techniques can be applied for their solution, yielding

$$\tilde{\boldsymbol{d}}_{0a,\mathrm{DEC}} = \frac{\left(A_0^2\boldsymbol{s}_{0a}^0\boldsymbol{s}_{0a}^{0T} + \boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a}\right)^+\boldsymbol{s}_{0a}^0}{\boldsymbol{s}_{0a}^{0T}\left(A_0^2\boldsymbol{s}_{0a}^0\boldsymbol{s}_{0a}^{0T} + \boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a}\right)^+\boldsymbol{s}_{0a}^0}$$

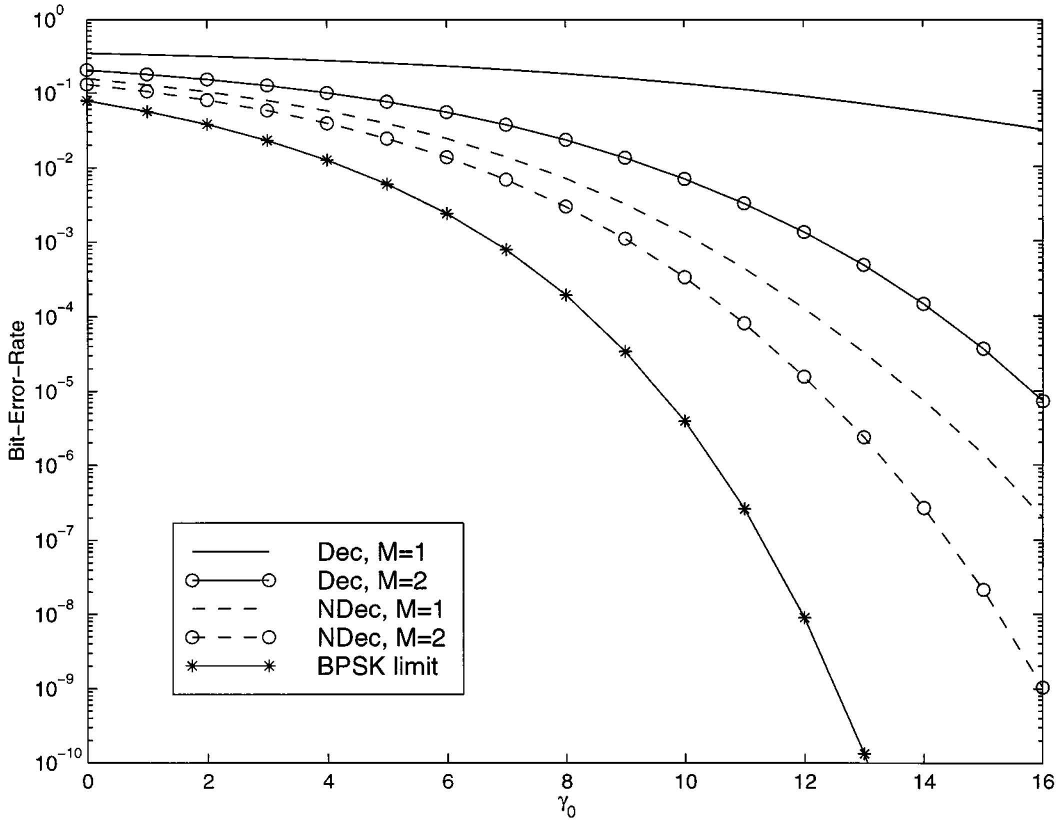

Fig. 1. Bit error rate (BER) of the classical and of the newly proposed decorrelating detector for two values of the oversampling factor M and for $K = 15$.

and

$$\tilde{d}_{0a,\,\mathrm{MMSE}} = \frac{\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}^{-1}\boldsymbol{s}_{0a}^{0}}{\boldsymbol{s}_{0a}^{0T}\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}^{-1}\boldsymbol{s}_{0a}^{0}}, \tag{18}$$

where

$$\tilde{\boldsymbol{d}}_{0a,\,\mathrm{DEC}} = \begin{bmatrix}\tilde{\boldsymbol{d}}_{0R,\,\mathrm{DEC}}\\ \tilde{\boldsymbol{d}}_{0I,\,\mathrm{DEC}}\end{bmatrix} \qquad \tilde{\boldsymbol{d}}_{0a,\,\mathrm{MMSE}} = \begin{bmatrix}\tilde{\boldsymbol{d}}_{0R,\,\mathrm{MMSE}}\\ \tilde{\boldsymbol{d}}_{0I,\,\mathrm{MMSE}}\end{bmatrix} \tag{19}$$

$$\begin{aligned}\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a} &= \mathrm{E}\left[\boldsymbol{r}_a(p)\boldsymbol{r}_a^H(p)\right]\\ &= \begin{pmatrix}\mathrm{E}\left[\boldsymbol{r}_R(p)\boldsymbol{r}_R^T(p)\right] & \mathrm{E}\left[\boldsymbol{r}_R(p)\boldsymbol{r}_I(p)^T\right]\\ \mathrm{E}\left[\boldsymbol{r}_I(p)\boldsymbol{r}_R(p)^T\right] & \mathrm{E}\left[\boldsymbol{r}_I(p)\boldsymbol{r}_I(p)^T\right]\end{pmatrix}\end{aligned} \tag{20}$$

and $\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a}$ is defined similarly to $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}$. It is now understood that the new linear receivers, solving problems (10) and (11), can be obtained by evaluating the $2NM$-dimensional real vectors $\tilde{\boldsymbol{d}}_{0a,\,\mathrm{DEC}}$ and $\tilde{\boldsymbol{d}}_{0a,\,\mathrm{MMSE}}$ through (18), and by properly combining the upper NM components with the lower NM components so as to obtain complex-valued vectors.

Remark 1: The new solutions (18) can be shown to be equivalent to the classical ones (9) when the pseudoautocovariance of the vector $\boldsymbol{r}(p)$ is zero. This property descends from the fact that, when the CDMA signals are proper, the following relations are fulfilled:

$$\mathrm{E}\left[\boldsymbol{r}_R(p)\boldsymbol{r}_R^T(p)\right] = \mathrm{E}\left[\boldsymbol{r}_I(p)\boldsymbol{r}_I^T(p)\right]$$

and

$$\mathrm{E}\left[\boldsymbol{r}_R(p)\boldsymbol{r}_I(p)^T\right] = -\mathrm{E}\left[\boldsymbol{r}_I(p)\boldsymbol{r}_R(p)^T\right].$$

For the sake of brevity, however, the complete proof of this is omitted.

Remark 2: With regard to the computational complexity, notice that the computation of the classical solutions (9) entails inversion of an MN-dimensional matrix with complex entries, which requires $4(NM)^3$ real multiplications. Computation of the new solutions (18), instead, entails inversion of a $2NM$-dimensional matrix with real entries, which requires $(2NM)^3 = 8NM$ real multiplications. Adopting the new solutions thus leads to a slight computational complexity increase.

IV. Computer Simulation Examples

In order to corroborate the effectiveness of the proposed receivers, and to carry out comparisons with the classical linear multiuser receivers, in the following we illustrate the results of some computer simulations. In particular, we focus on a CDMA system employing Gold codes with spreading length $N = 31$. In Fig. 1, we have represented the error probability versus the average received energy contrast $\gamma_0 = (A_0^2)/(2\mathcal{N}_0)$, for the classical decorrelating detector (9), labeled in the figure as "Dec," and for the newly proposed decorrelating detector, labeled as "NDec." The number of users is $K = 15$, and two different values of the oversampling ratio M have been considered; the interfering users amplitudes are assumed to be 5 dB larger than that of the desired signal. Since the system is asynchronous, the curves are the average over 100 random realizations of the delays τ_k and of the phases ϕ_k of the interfering users. The results clearly show the superiority of the new strategy, which largely outperforms the classical decorrelating structure. For comparison purposes, we also report the error probability corresponding to an uncoded BPSK transmission over a MAI-free additive white Gaussian noise (AWGN) channel. As to M, it is seen that increasing M can provide huge performance gains. In Fig. 2, instead, we report the bit-eror rate (BER) for the newly proposed MMSE receiver, labeled as "NMMSE" and for the classical MMSE receiver [4], labeled as "MMSE." The simulation scenario is the same as in the previous plot. Once again, it is seen that the new structure outperforms the

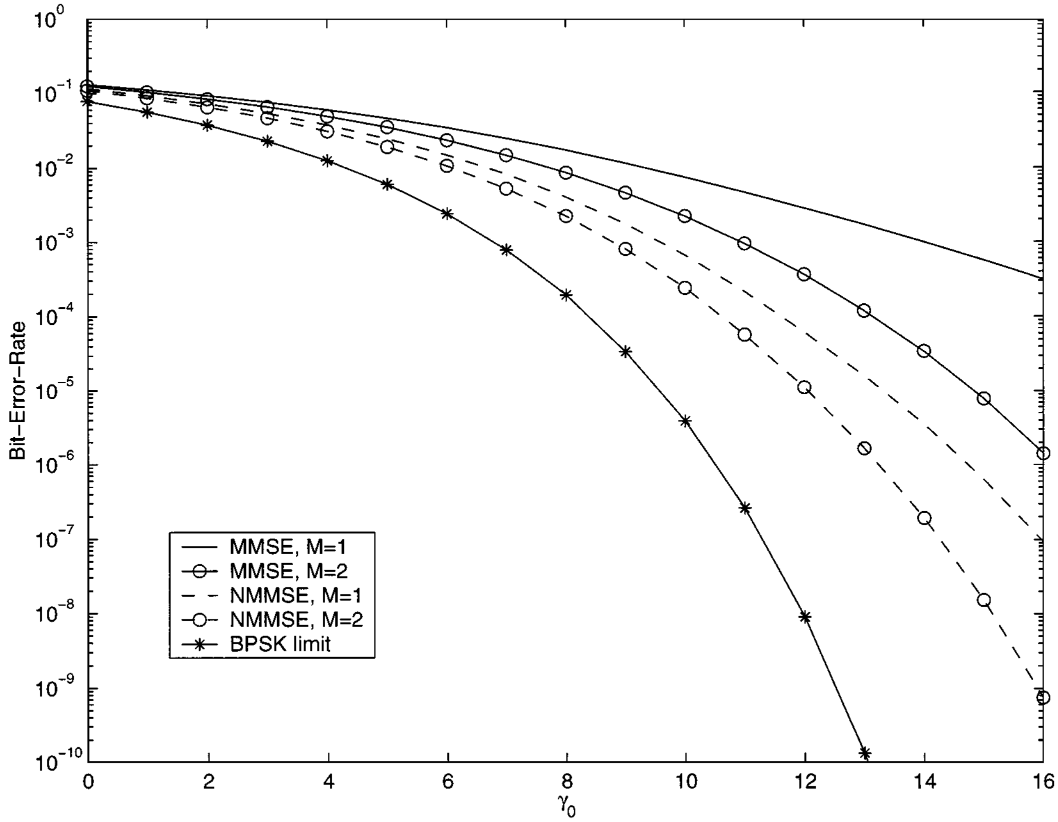

Fig. 2. BER of the classical and of the newly proposed MMSE detector for two values of the oversampling factor M and for $K = 15$.

classical one, and that oversampling is beneficial in improving performance. Notice also that, in keeping with well-known results [11], the MMSE receiver is shown to achieve better performance than the decorrelating detector.

Finally, in Fig. 3 the classical linear detection structures (9) are contrasted to the newly proposed receivers in terms of near–far resistance, which has been represented versus the number of network users. Again, it is seen that the new approach permits attaining a noticeable performance improvement. Likewise, it is again shown that M has a beneficial effect on the system capacity.

V. Blind Adaptive Receiver Implementation

In this section, we briefly address the problem of blind adaptive implementation of the proposed receivers, showing that well-known adaptive algorithms may be successfully applied in this context.

To begin with, notice that two very popular approaches to blind adaptive multiuser detection are those based on the recursive-least-squares (RLS) algorithm and on the subspace tracking theory. As regards the former algorithm, following [12], an RLS-based implementation of the newly proposed MMSE receiver may be obtained by solving the following exponentially windowed constrained minimization problem:

$$\begin{cases} \sum_{i=1}^{n} \lambda^{n-i} \left(\hat{\boldsymbol{d}}_{0a,\,\mathrm{MMSE}}^{H}(n) \boldsymbol{r}_a(i) \right)^2 = \min \\ \boldsymbol{d}_{0a,\,\mathrm{MMSE}}^{H} \boldsymbol{s}_{0a}^{0} = 1 \end{cases} \tag{21}$$

with λ the forgetting factor ensuring the algorithm tracking capability. A procedure for recursive computation of the solution to the above problem with computational complexity $\mathcal{O}((NM)^2)$ can be obtained through direct application of the results in [12]. Based on (15), the decision rule, at the nth signaling interval, is written as

$$\hat{b}_0(n) = \mathrm{sgn}\left\{ \hat{\boldsymbol{d}}_{0a,\,\mathrm{MMSE}}^{H}(n) \boldsymbol{r}_a(n) \right\} \tag{22}$$

with $\hat{\boldsymbol{d}}_{0a,\,\mathrm{MMSE}}^{H}(n)$ the RLS algorithm output at the nth epoch.

Even though the RLS algorithm is capable of tracking the desired solution with very satisfactory performance, as noted in [12], it suffers from a steady-state excess mean-square error (MSE), and exhibits a poor numerical stability in the case that the input data are ill-conditioned. Additionally, it cannot be adopted for the blind implementation of the decorrelating receiver. In order to circumvent these problems, and to provide blind adaptive implementations of both the MMSE and decorrelating receivers, it is customary to resort to a subspace approach [13]. To be more definite, notice that the covariance matrix $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}$ admits the following eigendecomposition:

$$\begin{aligned} \boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a} &= [\boldsymbol{U}_{s,a}\boldsymbol{U}_{n,a}] \begin{bmatrix} \boldsymbol{\Lambda}_{s,a} & \\ & \boldsymbol{\Lambda}_{n,a} \end{bmatrix} [\boldsymbol{U}_{s,a}\,\boldsymbol{U}_{n,a}]^H \\ &= \boldsymbol{U}_{s,a}\boldsymbol{\Lambda}_{s,a}\boldsymbol{U}_{s,a}^H + 2\mathcal{N}_0 \boldsymbol{U}_{n,a}\boldsymbol{U}_{n,a}^H. \end{aligned} \tag{23}$$

In the above equation, $\boldsymbol{U}_{s,a}$ is a $2NM \times (2K-1)$-dimensional matrix containing on its columns the eigenvectors corresponding to the $2K-1$ largest eigenvalues of the matrix $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}$, $\boldsymbol{\Lambda}_{s,a}$ is a diagonal matrix of order $2K-1$ containing on its diagonal the said largest eigenvalues, $\boldsymbol{U}_{n,a}$ is a $2NM \times (2NM-2K+1)$ matrix containing on its columns the remaining eigenvectors of the matrix $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}$, and, finally, $\boldsymbol{\Lambda}_{n,a}$ is a diagonal matrix of order $(2NM-2K+1)$ whose entries equal the noise level $2\mathcal{N}_0$. It is easy to show that the column span of $\boldsymbol{U}_{s,a}$ defines

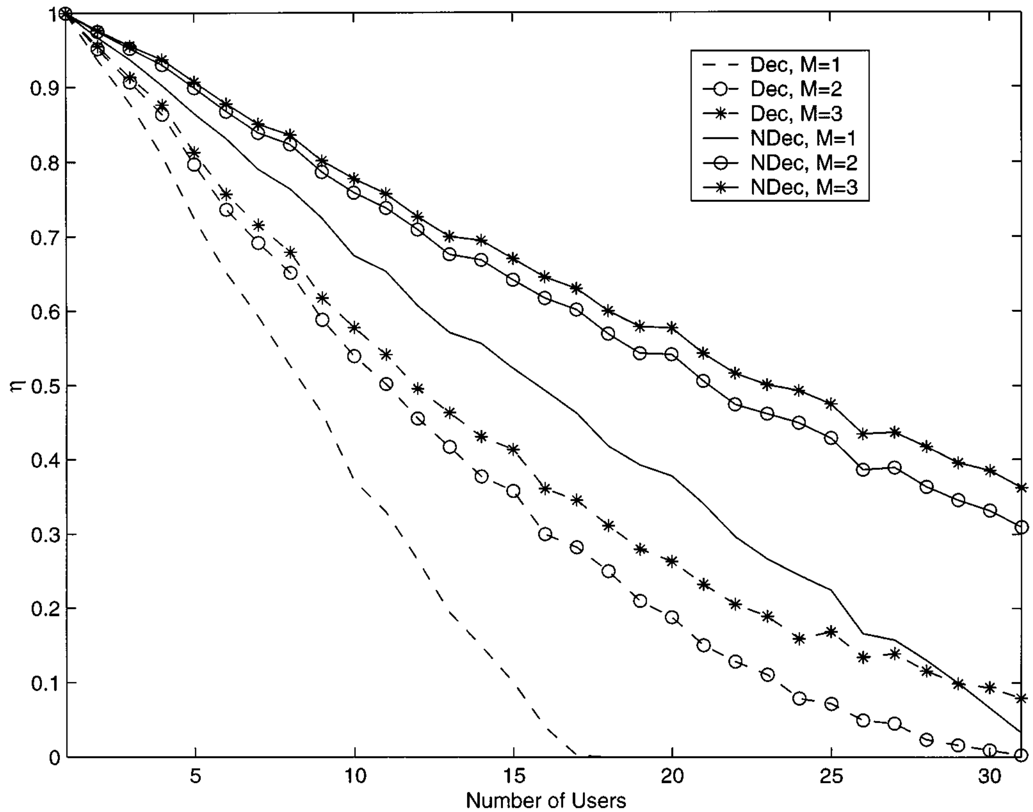

Fig. 3. Near–far resistance of the classical decorrelating detector and of the newly proposed multiuser receivers versus the users number and for several values of M.

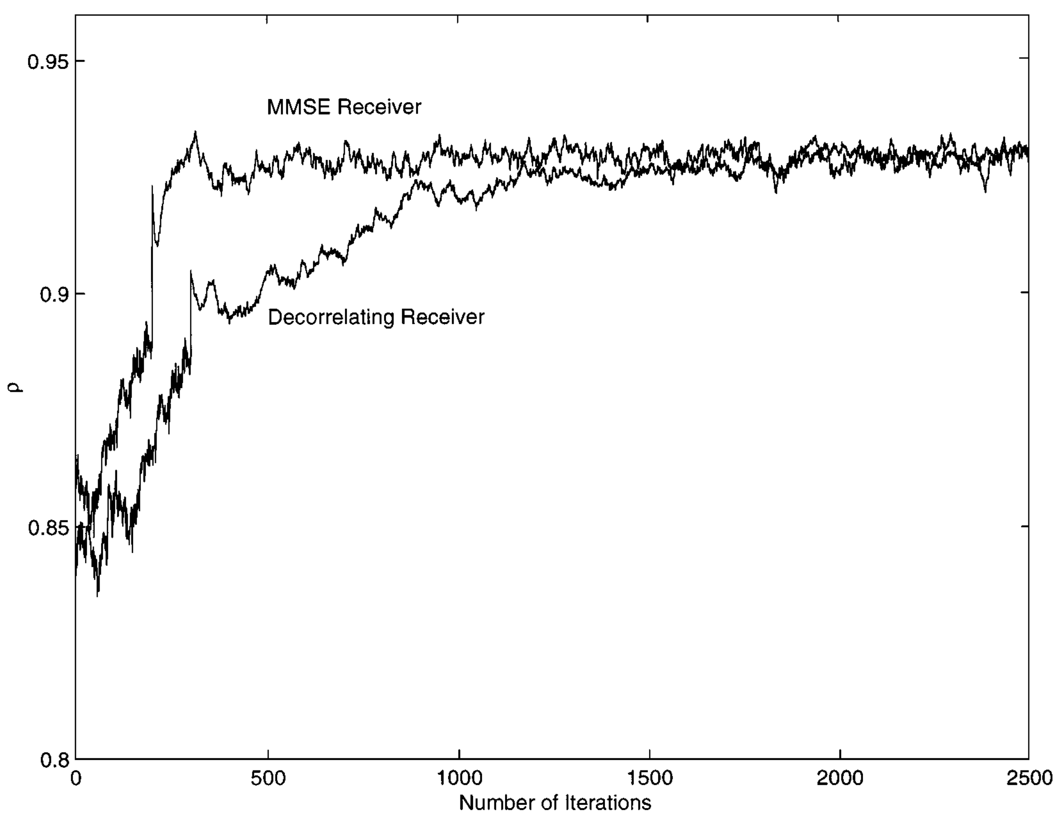

Fig. 4. Convergence dynamics of the PASTd algorithm for the tracking of the newly proposed multiuser detectors.

the *signal space*, while its orthogonal complement, termed as the *noise subspace*, is defined by the column span of $\boldsymbol{U}_{n,a}$. Since processing the component of the observables $\boldsymbol{r}_a(p)$ lying in the noise subspace would not add any relevant information, it is obviously understood that any linear receiver is to be contained in the signal space: accordingly, the newly proposed receivers $\tilde{\boldsymbol{d}}_{0,\mathrm{DEC}}$ and $\tilde{\boldsymbol{d}}_{0,\mathrm{MMSE}}$ can be written in terms of signal space parameters. More precisely, following [13], we have

$$\tilde{\boldsymbol{d}}_{0a,\mathrm{DEC}} = \frac{\boldsymbol{U}_{s,a}\left(\boldsymbol{\Lambda}_{s,a} - 2\mathcal{N}_0\boldsymbol{I}_{2K-1}\right)^{-1}\boldsymbol{U}_{s,a}^H\boldsymbol{s}_{0a}^0}{\boldsymbol{s}_{0a}^{0H}\boldsymbol{U}_{s,a}\left(\boldsymbol{\Lambda}_{s,a} - 2\mathcal{N}_0\boldsymbol{I}_{2K-1}\right)^{-1}\boldsymbol{U}_{s,a}^H\boldsymbol{s}_{0a}^0}$$

$$\tilde{\boldsymbol{d}}_{0a,\mathrm{MMSE}} = \frac{\boldsymbol{U}_{s,a}\boldsymbol{\Lambda}_{s,a}^{-1}\boldsymbol{U}_{s,a}^H\boldsymbol{s}_{0a}^0}{\boldsymbol{s}_{0a}^{0H}\boldsymbol{U}_{s,a}\boldsymbol{\Lambda}_{s,a}^{-1}\boldsymbol{U}_{s,a}^H\boldsymbol{s}_{0a}^0}. \tag{24}$$

The newly proposed receivers can now be implemented in a blind adaptive fashion by means of any subspace-tracking algorithm operating on the augmented vector sequence $\boldsymbol{r}_a(\cdot)$ and capable of tracking the $2K-1$ most dominant eigenvalues (and corresponding eigenvectors) of the covariance matrix $\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}$. We do not dwell any longer on such an issue: indeed, the open literature is rich in subspace-tracking algorithms, and giving further details on such an issue would not add much conceptual value to our work. Here, our actual aim has been just pointing out that the newly proposed detection structures can be expressed in terms of signal space parameters and that the tools of subspace tracking theory may be applied for their blind adaptive realization.

However, in order to experimentally validate the applicability of the subspace approach to the case at hand, we have selected the very popular PASTd algorithm [14] (whose complexity is linear in the dimensionality of the input vector $\boldsymbol{r}_a(\cdot)$) and have reported in Fig. 4 the convergence dynamics for both the decorrelating and the MMSE receivers. More precisely, we considered a synchronous system with $K = 10$ users and $\gamma_0 = 10$ dB. The plots are the result of an average over 100 independent runs; additionally, in order to speed up convergence, the initial estimates of the eigenvectors and eigenvalues of the signal space have been obtained by applying an SVD to a 50-samples estimate of the observables covariance matrix. On the vertical axis, the normalized correlation between the estimated receiver and the actual one is represented, while on the horizontal axis, the number of iterations (i.e., signaling intervals) is reported. It is seen that the PASTd algorithm is capable of tracking the desired solutions, whence it may be concluded that the newly proposed receivers are amenable to a recursive blind implementation exhibiting satisfactory performance.

VI. Conclusion

In this correspondence, we have dealt with the problem of linear multiuser detection in asynchronous DS/CDMA systems employing BPSK modulation. Based on the constrained minimization of a generalized MOE, we derived two novel linear multiuser detectors, which may be regarded as a generalization of the well-known decorrelating and MMSE receivers.

The effectiveness of the newly proposed detectors has been validated through both theoretical consideration and numerical results. In particular, such an analysis confirms that the new structures outperform the "classical" decorrelating and MMSE receivers, at the price, however, of a slight increase in complexity. Additionally, it has also been shown that both the RLS algorithm and any subspace-tracking algorithm can be applied in order to come up with blind adaptive versions of the new receivers.

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