Widely Linear Reception Strategies for Layered Space-Time Wireless Communications

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***Abstract*—A new class of receivers based on widely linear data processing has been recently proposed for data detection in communication systems affected by improper complex noise. In this paper, it is shown that this detection strategy may be applied to wireless communication systems employing multiple transmit and receive antennas and adopting a noncircular modulation. Improved versions of the linear decorrelating and minimum mean square error (mmse) receivers, and of the nonlinear nulling and cancellation (V-BLAST) receiver are, thus, developed and analyzed. In particular, we show that the improved receivers outperform the conventional ones both in terms of the error probability and of the capacity to cope with the power disparities that the fading channel may induce on the data streams transmitted by different antennas. Moreover, the improved receivers exhibit satisfactory performance also in systems with a number of transmit antennas exceeding the number of receive antennas. Finally, we also consider the situation in which the propagation channel is not perfectly known to the receiver, and show that the performance of the improved receivers is less sensitive to the channel estimation errors than the conventional receivers.**

***Index Terms*—Data detection, multiantenna systems, widely linear communications, wireless communications.**

I. Introduction

OF late there has been a growing interest in the design of wireless communication systems equipped with multiple transmit and receive antennas. Indeed, recent studies have shown that the capacity of a wireless communication system with multiple antennas grows with a law approximately linear in the minimum between the number of receive and transmit antennas, whereby the use of multiple antennas can be useful to design communication systems with high spectral efficiency.

Motivated by these considerations, many papers have appeared in the open literature presenting theoretical findings and system design guidelines for multiantenna communications [1]–[5]. In particular, we cite here the popular Bell Labs Layered Space Time Architecture (BLAST), which was invented by Foschini. In its vertical version (V-BLAST), the BLAST system is able to exploit the spatial dimension transmitting parallel data streams simultaneously and separating them at the receiver according to their distinct spatial signatures.

For a layered space-time narrowband communication system, the received signal model is similar to that modeling the received signal in a single-antenna synchronous direct-sequence code division multiple access (DS/CDMA) system,[1] whereby multiuser detection concepts can be applied to design data detectors for multiantenna systems. In the context of multiuser detection, novel linear detection schemes have been introduced and analyzed in [6]–[9] for DS/CDMA systems employing a BPSK modulation. Indeed, in this situation the baseband equivalent of the multiaccess interference (MAI) can be modeled as an improper random complex process.[2] It is shown in [11] and [12] that, for a communication system affected by improper random complex disturbance (including improper intersymbol interference and improper multiple access interference), classical linear detection structures are outperformed by alternative linear structures, which are named "widely linear," and that process independently the received data and their conjugate. The papers [6]–[9], thus, apply the concept of widely linear filtering to come up with improved versions of the classical multiuser detectors. Interestingly, these new receivers, although derived for BPSK modulation, can be straightforwardly extended to other signaling strategies such as multilevel amplitude-shift-keying (ASK), Gaussian minimum-shift-keying (GMSK), and offset quadrature-amplitude-modulation (QAM) [13]–[15]. Moreover, as shown in [16], there are situations in which the widely linear receivers with real data symbols achieve a spectral efficiency larger than linear receivers with complex modulations. As a consequence, the restriction to noncircular constellations (i.e., constellations such that the statistical average of the squared symbols is not zero) is not really a limitation and its adoption coupled with widely linear receivers may bring remarkable advantages over linear receivers with complex circular constellations not only in terms of error probability, but also in terms of spectral efficiency.

In this paper, we apply the concepts of widely linear filtering in order to obtain a family of improved receivers for multiantenna systems. The contributions of this paper may be summarized as follows.

- We derive improved versions of the linear decorrelating and minimum mean square error (mmse) receivers, and, also, an improved version of the nonlinear nulling and

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[1]In particular, the number of transmit and receive antennas play the role of the number of users and the processing gain, respectively.

[2]A complex random process $z(t)$ is said to be proper if its pseudoautocorrelation function $R_z(t,u) = E[z(t)z(u)]$ ($E[\cdot]$ denotes the statistical expectation) is zero $\forall t, u$ while is said to be improper when $R_z(t,u)$ is nonzero [10].

cancellation receiver used in the V-BLAST system. Remarkably, it is shown that the improved receivers exhibit satisfactory performance also when the number of transmit antennas exceeds the number of receive antennas.

- We provide a theoretical study showing that there is a nonnegligible probability that, due to the channel randomness, the information streams transmitted by a given antenna may be received with a power much smaller than that of the information streams transmitted by the remaining antennas. Motivated by this finding, we import to wireless multiantenna systems the asymptotic efficiency and near-far resistance concepts, and introduce a new performance measure, that we nickname the "fading unbalance resistance," and that gives a measure of the ability of a receiver to achieve satisfactory performance in the situation that the data streams from the several transmit antennas are received, due to the channel randomness, with very unbalanced powers. We thus compute the conditional and average fading unbalance resistance (FUR) for both the classical and improved linear receivers, showing that the new receivers bring a substantial performance advantage with respect to the conventional ones, and, again, that they may operate with a number of transmit antennas larger than the number of receive antennas.
- We evaluate the system performance in the situation, of primary practical interest, that the channel coefficients are not perfectly estimated. Also in this situation results show that the improved detectors are less sensitive than conventional receivers to the channel estimation errors.
- An usual criticism against the use of widely linear receivers is the fact that they can be used only in conjunction with noncircular (i.e., improper) constellations, and that they are not useful in detecting circular constellations such as M-ary PSK and QAM. In order to investigate on this point, we show that widely linear reception of an M-ary noncircular constellation such as ASK outperforms linear reception of M-ary circular constellations such as PSK and QAM. As a consequence, we confute conventional wisdom that widely linear filtering is not useful in situations where high spectral efficiency and good performance is to be achieved.

The rest of this paper is organized as follows. In Section II, the system model is briefly outlined, while Section III is devoted to the design of the improved detectors. Sections IV–VI contain the performance analysis of the improved detectors; in particular, in Section IV we focus on the conditional performance (i.e., given the realization of the channel gains) and we carry out a performance comparison between widely linear reception of M-ary ASK and linear reception of M-ary QAM and PSK. In Section V we focus on the unconditional performance (i.e., averaged on the channel statistics). Section VI analyzes the receivers' performance in the presence of channel estimation errors. Finally, concluding remarks are given in Section VII.

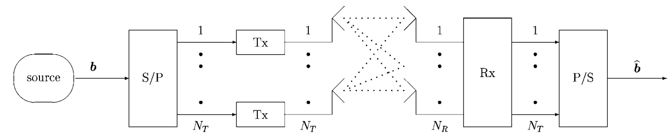

Fig. 1. Layered space–time wireless communication system diagram.

II. SYSTEM MODEL

Consider an uncoded single-user wireless communication system using a BPSK modulation format[3] (see Fig. 1). Denote by N_T the number of transmitting antennas, and by $N_R \geq N_T$ the number of receiving antennas[4]. At the transmitter, a data frame of length L is demultiplexed into N_T substreams of length L/N_T (we assume that L/N_T is an integer), to be transmitted one by each antenna at a data rate N_T times slower in the same frequency band. Moreover, we assume that the propagation channel introduces a flat fading, and we denote by $h_{n,m}$ the complex gain accounting for the channel propagation effects from the mth transmit antenna to the nth receive antenna. In keeping with the model in [3] and [4], we model this gain as a standard complex Gaussian random variate, and assume that $h_{n,m}$ and $h_{n',m'}$ are statistically independent if either $n \neq n'$ or $m \neq m'$ (rich scattering environment). Moreover, the channel coherence time is assumed to exceed the data frame duration, so that the channel gains can be assumed to be constant during the transmission of the entire L-bits frame. After matched filtering and sampling, the samples observed on the N_R receiving antennas in the pth signaling interval are stacked in the following N_R-dimensional column vector:

$$\boldsymbol{r}(p) = \boldsymbol{H}\boldsymbol{A}\boldsymbol{b}(p) + \boldsymbol{w}(p) \qquad p = 0, \ldots, L/N_T - 1. \tag{1}$$

In (1), $\boldsymbol{b}(p) = [b_1(p), b_2(p), \ldots, b_{N_T}(p)]^T$, is an N_T-dimensional column vector[5] with $(\cdot)^T$ denoting transposition, while the $N_R \times N_T$-dimensional matrix $\boldsymbol{H}$, containing the normalized spatial signatures, is defined as

$$\boldsymbol{H} = \left[\frac{\boldsymbol{h}_1}{\|\boldsymbol{h}_1\|}, \frac{\boldsymbol{h}_2}{\|\boldsymbol{h}_2\|}, \ldots, \frac{\boldsymbol{h}_{N_T}}{\|\boldsymbol{h}_{N_T}\|}\right] \tag{2}$$

wherein the channel coefficients $h_{n,i}$ for $n = 1, \ldots, N_R$ have been stacked in the N_R-dimensional column vector $\boldsymbol{h}_i$ for $i = 1, \ldots, N_T$. Finally, $\boldsymbol{A}$ is the following $N_T \times N_T$-dimensional matrix:

$$\boldsymbol{A} = \text{diag}(A_1\|\boldsymbol{h}_1\|, \ldots, A_{N_T}\|\boldsymbol{h}_{N_T}\|) \tag{3}$$

with $A_1, \ldots, A_{N_T}$ scalar constants tied to the energy of the signals transmitted on each antenna (we assume $A_1 = \ldots =$

[3]For the sake of simplicity, BPSK modulation is considered here. Note however that the proposed techniques can be straightforwardly extended to a number of modulation schemes including GMSK, M-ary ASK and offset QAM (with $M > 2$). Moreover, in the following we will show simulation results showing the symbol error probability corresponding to a 16-ASK modulation.

[4]We assume for the moment that $N_R \geq N_T$, but this assumption will be relaxed when deriving the improved receivers.

[5]$b_m(p) \in \{-1, +1\}$ is the symbol transmitted on the mth antenna in the pth symbol interval.

$A_{N_T} = A$), and $\boldsymbol{w}(p)$ is the vector of the thermal noise samples. We assume that the entries of $\boldsymbol{w}(p)$ are complex zero-mean Gaussian random variates, and that the noise is spatially white, i.e., $E[\boldsymbol{w}(p)\boldsymbol{w}^H(p)] = 2\mathcal{N}_0 \boldsymbol{I}_{N_R}$, wherein $(\cdot)^H$ denotes conjugate transposition, while $\boldsymbol{I}_{N_R}$ is the identity matrix of order N_R. Note also that, denoting by $\boldsymbol{r}_R(p)$ and $\boldsymbol{r}_I(p)$ the real part and the coefficient of the imaginary part of the vector $\boldsymbol{r}(p)$, respectively, we have that

$$E\left[\boldsymbol{r}_R(p)\boldsymbol{r}_I^T(p)\right] = \boldsymbol{H}_R \boldsymbol{A}\boldsymbol{A}^T \boldsymbol{H}_I^T \tag{4}$$

with $\boldsymbol{H}_R$ and $\boldsymbol{H}_I$ the real part and the coefficient of the imaginary part of the matrix $\boldsymbol{H}$, respectively. Since the crosscorrelation between the real and imaginary part of the data $\boldsymbol{r}(p)$ is nonzero, (4) reveals that, conditioned on the channel realizations, the observables $\boldsymbol{r}(p)$ are an improper random process, thus implying that the same techniques developed in [6] can be applied here to obtain improved detection procedures. It is also easily seen that, if the information symbols have a circular symmetry (as happens, for instance, if an M-ary phase shift keying modulation format is employed), the cross-correlation matrix in (4) vanishes and no performance advantage can be gained through the use of the improved receivers. As a final remark, we note that since the entries of $\boldsymbol{HA}$ are statistically independent and $N_R \geq N_T$, the columns of $\boldsymbol{H}$ are linearly independent with probability (w.p.) 1.

III. Design of the Improved Detectors

In what follows, we focus on the design of the improved detectors; in particular, we briefly adapt to the multiantenna scenario the improved decorrelating and mmse receivers, which were developed in [6] with reference to a DS/CDMA system, and then introduce an improved version of the nulling and cancellation receiver, which, we recall, is a nonlinear receiver. We assume for the moment that the channel matrix $\boldsymbol{H}$ is known at the receiver.

To begin with, consider the following linear decision rule[6]:

$$\hat{\boldsymbol{b}}(p) = \mathrm{sign}(\mathcal{R}\{\boldsymbol{D}^H \boldsymbol{r}(p)\}) \tag{5}$$

where $\mathrm{sign}(\cdot)$ denotes the signum function, and $\boldsymbol{D}$ is a $N_R \times N_T$-dimensional matrix to be determined according to some optimization criterion. The improved linear receivers are based on the consideration that, given (5), the real part of the test statistic $\boldsymbol{D}^H\boldsymbol{r}(p)$ determines the system performance rather than the whole test statistic. As a consequence, improved versions of the mmse and decorrelating receivers can be derived by considering the following optimization problems:

$$\boldsymbol{D} = \arg \min_{\boldsymbol{X} \in \mathcal{C}^{N_R \times N_T}} \mathrm{E}[||\mathcal{R}\{\boldsymbol{b}(p) - \boldsymbol{X}^H \boldsymbol{r}(p)\}||^2] \tag{6}$$

and

$$\begin{cases} \boldsymbol{D} = \arg\min_{\boldsymbol{X} \in \mathcal{C}^{N_R \times N_T}} \mathrm{E}[||\mathcal{R}\{\boldsymbol{X}^H(\boldsymbol{r}(p) - \boldsymbol{w}(p))\}||^2] \\ \text{subject to} : \mathcal{R}\left\{\boldsymbol{x}_i^H \boldsymbol{h}_i\right\} = 1 \quad \forall i = 1, \ldots, N_T \end{cases} \tag{7}$$

[6] $\mathcal{R}\{\cdot\}$ denotes real part.

respectively. In these problems, the interference contribution to the real part only of the test statistic is minimized (see also [6] for further details). Letting $\tilde{\boldsymbol{D}}_a = [(\boldsymbol{D}_R)^T \quad (\boldsymbol{D}_I)^T]^T$, with $(\cdot)_R$ and $(\cdot)_I$ denoting the real and the coefficient of the imaginary parts, respectively, and denoting by $\boldsymbol{r}_a(p)$ the $2N_R$-dimensional real column vector whose first N_R entries are the real part of $\boldsymbol{r}(p)$ and the last N_R ones are the coefficient of the imaginary parts of $\boldsymbol{r}(p)$, it can be shown that the solution to (6) and (7) can be expressed as

$$\tilde{\boldsymbol{D}}_{a,\mathrm{mmse}} = \boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a}{}^{-1} \boldsymbol{H}_a \boldsymbol{A} \tag{8}$$

and

$$\tilde{\boldsymbol{D}}_{a,\mathrm{DEC}}^H = \overline{\boldsymbol{H}}_a^+ \tag{9}$$

respectively, wherein $(\cdot)^+$ denotes generalized Moore-Penrose pseudoinversion, $\overline{\boldsymbol{H}} = \boldsymbol{HA}$, and

$$\begin{aligned} \boldsymbol{H}_a &= [(\boldsymbol{H}_R)^T \quad (\boldsymbol{H}_I)^T]^T, \\ \overline{\boldsymbol{H}}_a &= [(\bar{\boldsymbol{H}}_R)^T \quad (\bar{\boldsymbol{H}}_I)^T]^T, \\ \boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a} &= \mathrm{E}[\boldsymbol{r}_a(p)\boldsymbol{r}_a^H(p)]. \end{aligned} \tag{10}$$

The same concepts that led to the improved mmse and decorrelating detector can be used to develop an improved version of the conventional nulling and cancellation (N&C) receiver, which was introduced in [3], [4]. Basically, this receiver performs a sequential detection of the bits transmitted by each transmit antenna, subtracting the contribution from the already detected bits and zero-forcing the not yet detected bits. Accordingly, an improved version of the nulling and cancellation receiver can be obtained by performing an improved zero-forcing of the undetected bits. In particular, the improved N&C receiver adopts the improved decorrelating rule to null out the undetected bits; it implements the following steps:

1) set $m = 1, \boldsymbol{r}_{a_m}(p) = \boldsymbol{r}_a(p)$ and $\overline{\boldsymbol{H}}_{a_m} = \overline{\boldsymbol{H}}_a/A$;
2) set $\boldsymbol{Q}_m = \overline{\boldsymbol{H}}_{a_m}^+$;
3) set $l_m = \arg\min_{j=1,\ldots,N_T\, j \notin \{l_1,\cdots,l_{m-1}\}} ||(\boldsymbol{Q}_m)_{j,:}||^2$ where $(\boldsymbol{Q}_m)_{j,:}$ denotes the jth row of $\boldsymbol{Q}_m$;
4) set $\boldsymbol{p}_{l_m} = [(\boldsymbol{Q}_m)_{l_m,:}]^T$;
5) set $\hat{b}_{l_m}(p) = \mathrm{sign}(\boldsymbol{p}_{l_m}^T \boldsymbol{r}_{a_m}(p))$;
6) set $\boldsymbol{r}_{a_{m+1}}(p) = \boldsymbol{r}_{a_m}(p) - \hat{b}_{l_m}(p)(\overline{\boldsymbol{H}}_{a_m})_{:,l_m} A$;
7) let $\overline{\boldsymbol{H}}_{a_{m+1}}$ be equal to $\overline{\boldsymbol{H}}_{a_m}$ with the l_mth column set to zero;
8) set $m = m + 1$;
9) repeat steps 2 through 8 until $m = N_T + 1$.

In order to give an insight into the performance of the proposed receivers, in Fig. 2 the bit error probability (BER) is shown versus the average signal-to-noise ratio (SNR), for the classical detectors and for the improved detectors under the ideal assumption that the channel is perfectly known to the receiver. The curves shown have been obtained through computer simulations, and are the result of an average over 10^6 independent channel realizations. We considered both the situations that $N_T = N_R = 4$ (plot on the left) and that $N_T = 6$ and $N_R = 4$ (plot on the right). For the sake of comparison, the figure also shows the BER corresponding to the minimum error probability maximum likelihood (ML) receiver, whose

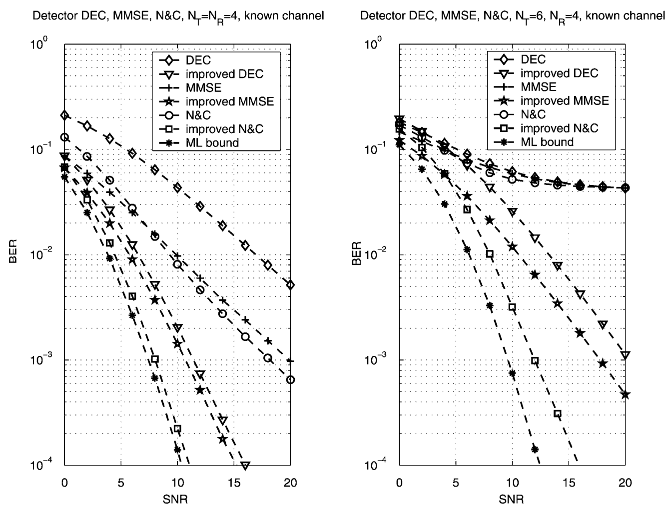

Fig. 2. System BER versus the SNR[dB].

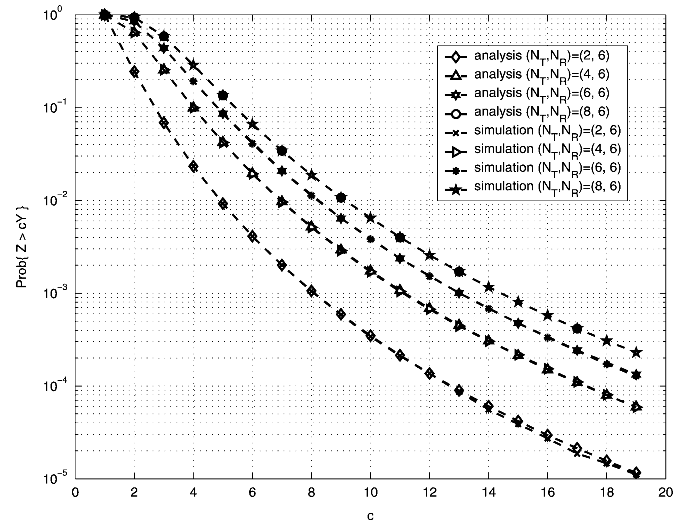

Fig. 3. $\mathrm{Prob}\{Z > cY\}$ versus the constant c.

computational complexity is exponential in the number of transmit antennas. Interestingly, it is seen that in both situations the improved receivers provide a huge performance gain with respect to the conventional receivers. In particular, for an error probability equal to 10^{-3}, the new receivers can achieve an SNR gain larger than 6 dB for $N_T = N_R = 4$, while, for $N_R < N_T$ the performance of the conventional receivers is affected by an error-floor larger than 10^{-2}. The improved receivers, conversely, exhibit a satisfactory performance in this scenario too, with the improved N&C receiver performing a couple of dB's worse than the optimum ML receiver.

IV. Performance Analysis Conditioned on the Channel Realization

A. *New Performance Measures for MIMO Wireless Links*

In this section we analyze the performance of the linear detectors conditioned on a certain realization of the matrix $\boldsymbol{H}$. First of all, it is worth noting that, since the channel matrix $\tilde{\boldsymbol{H}} = \overline{\boldsymbol{H}}/A$ is made of independent entries, even though the antenna transmit power is the same across the N_T transmit antennas, the data symbols are received with unbalanced powers. As a consequence, there may be a nonzero probability that some of the columns of the channel matrix $\tilde{\boldsymbol{H}}$ may have a norm much larger than the remaining columns. This situation is quite similar to the well-known near-far problem that occurs in CDMA cellular system, wherein users close to the base-station may mask the signal transmitted by far users. In order to shed some light on this phenomenon, let us denote by Y_i the squared norm of the ith column of the matrix $\tilde{\boldsymbol{H}}$, i.e.,

$$Y_i = \|\boldsymbol{h}_i\|^2 = \sum_{j=1}^{N_R} |h_{j,i}|^2. \qquad (11)$$

It is easy to show that Y_i is a chi-square random variate with $2N_R$ degrees of freedom, thus implying that its pdf is expressed as

$$p_{Y_i}(y) = \frac{y^{N_R-1}}{(N_R-1)!}e^{-y} \quad \text{with} \quad y \geq 0. \qquad (12)$$

Letting now $Z = \max(Y_1, \ldots, Y_{N_T})$, and $Y = \min(Y_1, \ldots, Y_{N_T})$, it is easily understood that the probability that the norms of the columns of the channel matrix assume unbalanced realizations is measured by the following probability:

$$\mathrm{Prob}\{\max(Y_1, \ldots, Y_{N_T}) > c \times \min(Y_1, \ldots, Y_{N_T})\} = \mathrm{Prob}\{Z > cY\} \qquad (13)$$

with $c > 1$. Indeed, (13) measures the probability that the maximum of the squared norms of the columns of the channel matrix $\tilde{\boldsymbol{H}}$ exceeds by c times the minimum of such squared norms. Upon some straightforward, but not trivial, algebra, it is easily found that

$$\mathrm{Prob}\{Z > cY\} = 1 - \int_0^{+\infty} N_T p_{Y_1}(\nu) \times [F_{Y_1}(c\nu) - F_{Y_1}(\nu)]^{N_T-1} d\nu \qquad (14)$$

with

$$F_{Y_1}(y) = 1 - e^{-y} \sum_{k=0}^{N_R-1} \frac{y^k}{k!} \quad y \geq 0 \qquad (15)$$

the cumulative distribution function of Y_1. In Fig. 3, the probability (14) is reported versus the constant c and for several values of the pairs (N_T, N_R). It is seen that this probability may grow as large as $0.1 - 0.5$ for $c \in [2, 6]$ and is in between 10^{-2} and 10^{-3} for $c = 10$. As a consequence, the analysis shows that the problem of power disparities across received symbols has a nonnegligible probability of occurrence and is to be properly taken into account. It is thus of interest to come up with new performance measures, that, similarly to the asymptotic efficiency and the near-far resistance in CDMA systems, give an insight into the ability of the considered receivers to cope with deep fades causing huge power disparities between the transmitted data symbols. In what follows we, thus, extend to MIMO wireless links the concepts of asymptotic efficiency and near-far resistance. In particular, in our context we define the "fading unbalance resistance" that mimics the role of the near-far resistance in CDMA systems. Inspired by the asymptotic efficiency definition given for CDMA systems in [17, p. 121] and [18], we define in the following the MIMO asymptotic efficiency for multiantenna systems. Denote with $P_j(\lambda)$ the bit error probability of the bit $b_j(p)$ transmitted by the jth antenna when the power spectral density of the Gaussian noise is $2\lambda^2 = 2\mathcal{N}_0$ and define $e_j(\lambda)$ such that $P_j(\lambda) = (1/2)\mathrm{erfc}(\sqrt{e_j(\lambda)}/(\sqrt{2}\lambda))$, with $\mathrm{erfc}(\cdot)$ the complementary error function. Then $e_j(\lambda)$, termed the effective energy, represents the energy required, in the single-input multiple-output case, to achieve a bit error rate $P_j(\lambda)$ in the same white Gaussian noise and with the same fading coefficients $h(1,j), \ldots, h(N_R, j)$. The asymptotic efficiency for high SNR is, thus, defined as [17, p. 121]

$$\mu_j \triangleq \sup_t \left\{0 \le t \le 1 \colon \lim_{\lambda \to 0} \frac{2P_j(\lambda)}{\mathrm{erfc}((\sqrt{t\boldsymbol{A}(j,j)})/(\sqrt{2}\lambda))} < \infty \right\}. \tag{16}$$

Starting upon (16), and based on [19], [20], we can then define a fading unbalance resistance (FUR) that is a performance measure that mimics the role of the near-far resistance in CDMA systems. The FUR measures the receiver ability to cope with situations in which the transmitted symbols are received with large power disparities by virtue of the hostile channel matrix realization. Thus, the FUR of the conventional linear receivers in detecting the symbols from the antenna 1 can be defined as the asymptotic efficiency minimized over the received energies of all the other antennas, i.e., as

$$\delta^{\mathrm{c}} \triangleq \inf_{\boldsymbol{A}(j,j)>0, j\neq 1} \mu_1. \tag{17}$$

Neglecting, for the sake of brevity, the mathematical derivation which, by the way, follow quite closely the ones reported in [17], [20], it can be shown that the conventional linear receivers FUR in detecting the symbols from the antenna 1 can be written as

$$\delta^{\mathrm{c}} = \frac{1}{(\boldsymbol{H}^H\boldsymbol{H})^{-1}_{1,1}}. \tag{18}$$

Conversely, the FUR of the improved linear detectors can be shown to be written as [6]

$$\delta = \frac{1}{(\mathcal{R}\{\boldsymbol{H}^H\boldsymbol{H}\}^{-1})_{1,1}}. \tag{19}$$

An alternative expression for δ^{c} and δ can be found according to the results of [17, p. 197]. In particular, the FUR for the conventional and improved decorrelating and mmse linear receivers can be, respectively, expressed as

$$\delta^c = \frac{\|\boldsymbol{h}_1^{\perp}\|^2}{\|\boldsymbol{h}_1\|^2}$$

and

$$\delta = \frac{\|\boldsymbol{h}_{1,a}^{\perp}\|^2}{\|\boldsymbol{h}_{1,a}\|^2} \tag{20}$$

with $\boldsymbol{h}_1^{\perp} = \boldsymbol{H}^{+}\boldsymbol{h}_1$ and $\boldsymbol{h}_{1,a}^{\perp} = \boldsymbol{H}_a^{+}\boldsymbol{h}_{1,a}$. Based on (20), algebraic considerations show that the FUR of the improved linear detectors δ is not smaller than that of the conventional linear detectors δ^{c} [7].

In order to establish a relationship between the bit error probability of the conventional and the improved decorrelating detectors we can let $\boldsymbol{z}(p) = \sum_{i=2}^{N_T} \boldsymbol{H}(:,i)\boldsymbol{A}(i,i)b_i(p)$ denote the contribution of the multiple antenna spatial interference, and let $\boldsymbol{M}_{\boldsymbol{zz}} = \mathrm{E}[\boldsymbol{z}(p)\boldsymbol{z}(p)^H]$ and $\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a} = \mathrm{E}[\boldsymbol{z}_a(p)\boldsymbol{z}_a(p)^H]$. Indeed, denoting by $\boldsymbol{h}_{1a}^{\perp}$ the projection of the $2N_R$-dimensional spatial signature $\boldsymbol{h}_{1a}$ onto the orthogonal complement (in $\mathcal{R}^{2N_R}$) of the range span of the matrix $\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a}$, and by $\boldsymbol{h}_1^{\perp}$ the projection of the original signature $\boldsymbol{h}_1$ onto the orthogonal complement (in $\mathcal{C}^{N_R}$) of the range span of the matrix $\boldsymbol{M}_{\boldsymbol{zz}}$, the error probabilities $P^c_{\mathrm{DEC}}(e)$ and $P_{\mathrm{DEC}}(e)$ of the conventional and improved decorrelators are easily written as

$$P^c_{\mathrm{DEC}}(e) = \frac{1}{2}\mathrm{erfc}\left(\frac{A\|\boldsymbol{h}_1^{\perp}\|}{\sqrt{2\mathcal{N}_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{A\|\boldsymbol{h}_1\|\sqrt{\delta^c}}{\sqrt{2\mathcal{N}_0}}\right)$$

and

$$P_{\mathrm{DEC}}(e) = \frac{1}{2}\mathrm{erfc}\left(\frac{A\|\boldsymbol{h}_{1,a}^{\perp}\|}{\sqrt{2\mathcal{N}_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{A\|\boldsymbol{h}_{1,a}\|\sqrt{\delta}}{\sqrt{2\mathcal{N}_0}}\right). \tag{21}$$

Since we have that $\delta \ge \delta^c$, it is easily shown that $P_{\mathrm{DEC}}(e) \le P^c_{\mathrm{DEC}}(e)$.

With regard to the mmse linear receivers, the conventional and improved detectors can be compared based on the output mean square error (mse). For the conventional mmse receiver we have

$$\mathrm{mse} = E[\|\boldsymbol{b}(p) - \boldsymbol{D}^H\boldsymbol{r}(p)\|^2] \tag{22}$$

substituting $\boldsymbol{D}^H = \boldsymbol{A}^H\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^H + 2\mathcal{N}_0\boldsymbol{I}_{N_R})^{-1}$ into (22), we obtain

$$\mathrm{mse} = N_T - \mathrm{trace}\{\boldsymbol{A}^H\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^H + 2\mathcal{N}\boldsymbol{I}_{N_R})^{-1}\boldsymbol{H}\boldsymbol{A}\}. \tag{23}$$

The mse achieved by the improved mmse receiver is instead written as

$$\mathrm{mse}_{\mathrm{imp}} = E\left[\left\|\boldsymbol{b}(p) - \tilde{\boldsymbol{D}}_a^H\boldsymbol{r}_a(p)\right\|^2\right] \tag{24}$$

and, based on (1) and (8) we have

$$\mathrm{mse}_{\mathrm{imp}} = N_T - \mathrm{trace}\left\{\boldsymbol{A}^H\boldsymbol{H}_a^H(\boldsymbol{M}_{\boldsymbol{r}_a\boldsymbol{r}_a})^{-1}\boldsymbol{H}_a\boldsymbol{A}\right\}. \tag{25}$$

In Fig. 4, we report the mse and the $\mathrm{mse}_{\mathrm{imp}}$, averaged on the channel realizations, versus $E_b/\mathcal{N}_0$, for a system with $N_T = 4$ and $N_R = 6$. It can be noted that the improved mmse receiver allows, for a mse of 10^{-1}, a gain of about 6 dB with respect to the conventional mmse receiver. For the sake of fairness, it should be however noted that, since the considered constellation is real, the actual performance is governed by the mean square value of the real part of the error, thus, implying that the considered mse is an indirect measure of the system performance.

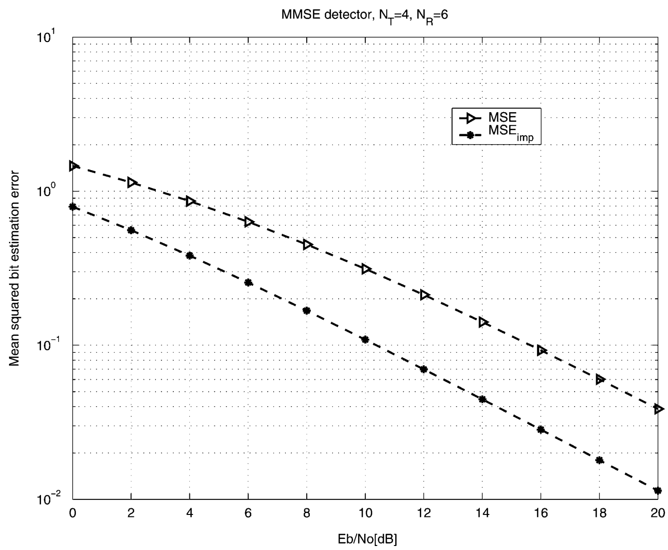

Fig. 4. Mean squared error versus $E_b/\mathcal{N}$[dB] for the improved and conventional mmse detectors.

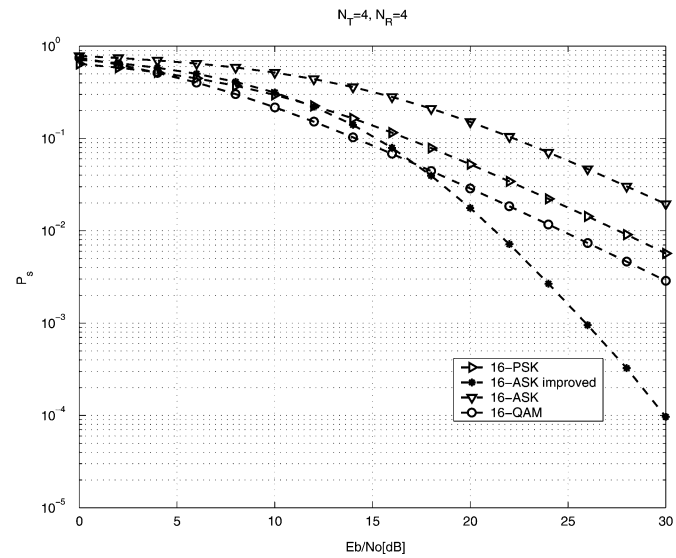

Fig. 5. Symbol error probability for the 16-ASK, 16-PSK, and 16-QAM modulation schemes versus $E_b/\mathcal{N}_0$[dB] for the mmse detector with $N_T = 4$ and $N_R = 4$.

B. Noncircular Versus Circular Constellations

A criticism that is usually made against widely linear reception structures is that their applicability is confined to real signal constellations, thus implying that they are useless in applications where achieving high spectral efficiency is a must, and complex signal constellations such as PSK and QAM are to be employed. In this subsection we confute this claim, showing that adopting an M-ary real modulation and a widely linear receiver provides better performance than using an M-ary complex modulation with a linear receiver. First of all, we note that in [16] the author has shown that, for a large DS/CDMA system, i.e., a system wherein both the processing gain and the number of users grow large with their ratio fixed, there may be situations wherein widely linear reception of real constellations achieves larger spectral efficiency than linear reception of complex constellations. Based on the already discussed analogy between synchronous DS/CDMA systems and multiantenna wireless communication systems, this result could be imported to our context with moderate efforts considering a system wherein both the number of transmit antennas and the number of receive antennas go to infinity with their ratio fixed (see also [21], wherein the capacity of large multiantenna systems is studied). Here, however, for the sake of simplicity, we just resort to a simulation study and compare the error probability achieved by an improved mmse reception of a 16-ASK constellation with the probability achieved by conventional linear reception of a 16-PSK and a 16-QAM constellation. The results reported in Figs. 5 and 6, are for two different values of the pair (N_T, N_R). The results are really astonishing; for the case $N_T = N_R = 4$, it is seen that improved mmse reception of 16-ASK exhibits a performance gain with respect to conventional mmse reception of 16-QAM of about 3 dB at a symbol error probability of 10^{-2}, and of about 8 dB at a symbol error probability of 10^{-3}. The results of Fig. 6 are even more impressive. Indeed, it is seen that while conventional mmse receiver is not able to operate with a number of transmit antennas larger than the number of receive antennas, the new improved receivers exhibit a quite satisfactory performance. Other results, not reported here for the sake of brevity, show that the improved reception strategies are able to operate with a number of transmit antennas up to twice the number of receive antennas. Conventional receivers, instead, provide satisfactory performance only for $N_T \leq N_R$. In Fig. 7, we finally report the symbol error probability versus $E_b/\mathcal{N}_0$[dB] for the N&C detector in the case $N_T = N_R = 4$. It can be noted that the improved N&C reception of 16-ASK gives a performance gain with respect to conventional 16-QAM of about 5 dB at a symbol error probability of 10^{-2}.

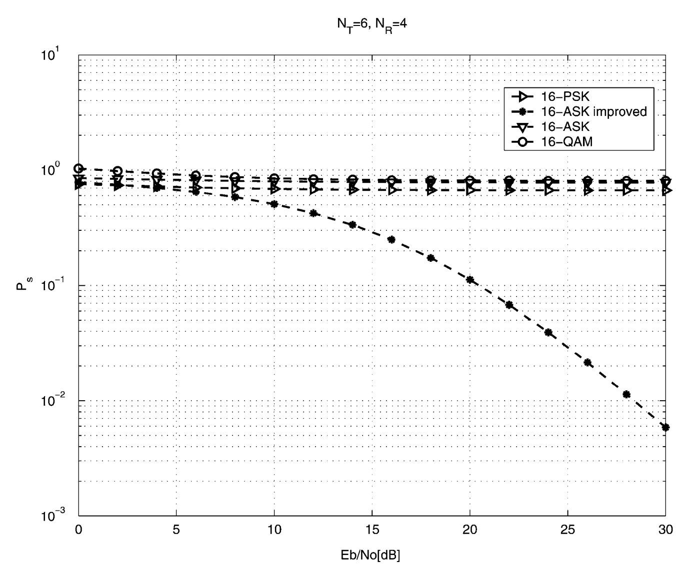

Fig. 6. Symbol error probability for the 16-ASK, 16-PSK, and 16-QAM modulation schemes versus $E_b/\mathcal{N}_0$[dB] for the mmse detector with $N_T = 6$ and $N_R = 4$.

V. Performance Analysis With Random Spatial Signatures

While the previous section has focused on the conditional performance assessment, in what follows we derive a closed-

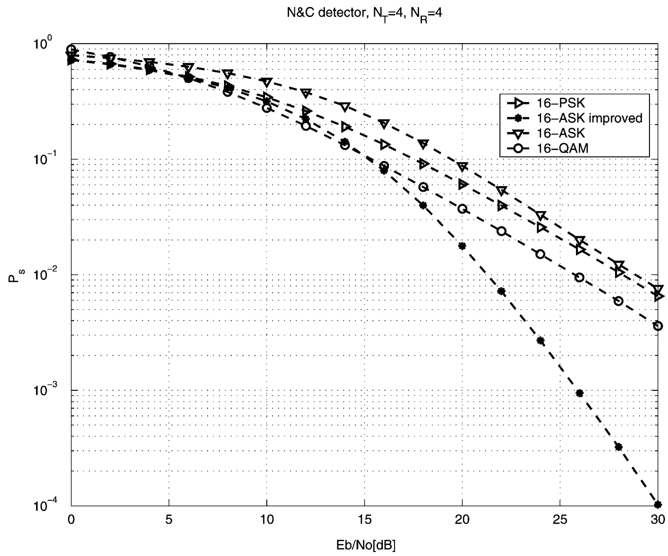

Fig. 7. Symbol error probability for the 16-ASK, 16-PSK, and 16-QAM modulation schemes versus $E_b/\mathcal{N}_0$[dB] for the N&C detector with $N_T = 4$ and $N_R = 4$.

form formula for the average FUR for both the conventional and improved linear receivers. We consider the relevant case of Rayleigh fading, i.e., the channel coefficients $h_{n,m}$ are statistically independent zero-mean complex Gaussian random variates with variance $2\sigma^2$. To begin with, note that the correlation matrix $\boldsymbol{H}^H\boldsymbol{H}$ can be partitioned as follows:

$$\boldsymbol{H}^H\boldsymbol{H} = \begin{bmatrix} 1 & \boldsymbol{\rho}_c^H \\ \boldsymbol{\rho}_c & \boldsymbol{R}_c \end{bmatrix} \tag{26}$$

wherein, for $N_R \geq N_T$, the $(N_T-1)\times(N_T-1)$-dimensional matrix $\boldsymbol{R}_c$ is nonsingular w.p. 1 and $\boldsymbol{\rho}_c$ is the following N_T-1-dimensional column vector:

$$\boldsymbol{\rho}_c = \left[\frac{\boldsymbol{h}_1^T\boldsymbol{h}_2^*}{\|\boldsymbol{h}_1\|\|\boldsymbol{h}_2\|}, \ldots, \frac{\boldsymbol{h}_1^T\boldsymbol{h}_{N_T}^*}{\|\boldsymbol{h}_1\|\|\boldsymbol{h}_{N_T}\|}\right]^T. \tag{27}$$

The entries of the N_R-dimensional column vector $\boldsymbol{h}_i$ can be written as

$$h_{j,i} = |h_{j,i}|e^{j\phi_{j,i}}, \ \forall\, j = 1,\ldots N_R, i = 1,\ldots N_T$$

with

$$|h_{j,i}| = \sqrt{\mathcal{R}\{h_{j,i}\}^2 + \mathcal{I}\{h_{j,i}\}^2}.$$

The real and imaginary parts of $h_{j,i}, \mathcal{R}\{h_{j,i}\}$ and $\mathcal{I}\{h_{j,i}\}$, respectively, are zero-mean statistically independent Gaussian random variates, each having variance σ^2. Accordingly $|h_{j,i}|$ is a Rayleigh distributed random variate with the following probability density function

$$p_{|h_{j,i}|}(\gamma_{j,i}) = \frac{\gamma_{j,i}}{\sigma^2}e^{-\gamma_{j,i}^2/(2\sigma^2)} \tag{28}$$

while $\phi_{j,i}$ is a random variate uniformly distributed between 0 and 2π, for $j = 1,\ldots N_R, i = 1,\ldots N_T$. Using the standard formula for the inverse of a block-partitioned matrix [17, p. 196], the system FUR (18) can be written as

$$\delta^c = 1 - \boldsymbol{\rho}_c^H\boldsymbol{R}_c^{-1}\boldsymbol{\rho}_c = 1 - \sum_{l,m=1}^{N_T-1}\rho_c^*(l)\left(\boldsymbol{R}_c^{-1}\right)_{l,m}\rho_c(m). \tag{29}$$

The product $\rho_c^*(l)\rho_c(m)$ between the lth and the mth entry of the vectors $\boldsymbol{\rho}_c^H$ and $\boldsymbol{\rho}_c$ respectively, is expressed as

$$\rho_c^*(l)\rho_c(m) = \frac{\left(\boldsymbol{h}_1^H\boldsymbol{h}_{l+1}\right)\left(\boldsymbol{h}_{m+1}^H\boldsymbol{h}_1\right)}{\|\boldsymbol{h}_1\|^2\,\|\boldsymbol{h}_{l+1}\|\,\|\boldsymbol{h}_{m+1}\|} \tag{30}$$

and averaging this equation with respect to $\phi_{j,1}$ for $j = 1,\ldots N_R$, i.e., with respect to the phases of the spatial signature of the first antenna we obtain

$$\begin{aligned}&\mathrm{E}[\rho_c^*(l)\rho_c(m)\,|\,|h_{1,1}|,|h_{2,1}|,\ldots,|h_{N_R,1}|,\boldsymbol{h}_2,\ldots,\boldsymbol{h}_{N_T}]\\&= \frac{\left(\sum_{n=1}^{N_R}|h_{n,1}|^2|h_{n,l+1}||h_{n,m+1}|e^{j(\phi_{n,l+1}-\phi_{n,m+1})}\right)}{\|\boldsymbol{h}_1\|^2\|\boldsymbol{h}_{l+1}\|\|\boldsymbol{h}_{m+1}\|}.\end{aligned} \tag{31}$$

In order to average the previous expression with respect to $|h_{j,1}|$ for $j = 1,\ldots N_R$, the following definite integral is to be computed:

$$\int_0^\infty\!\!\cdots\!\int_0^\infty \frac{\gamma_{j,1}^2}{\sum_{p=1}^{N_R}\gamma_{p,1}^2}\prod_{k=1}^{N_R} \times\left[\frac{\gamma_{k,1}}{\sigma^2}e^{-\frac{\gamma_{k,1}^2}{2\sigma^2}}\right]d\gamma_{1,1}\ldots d\gamma_{N_R,1}. \tag{32}$$

Since, as shown in the Appendix, this integral is equal to $1/N_R$, it can be shown that

$$\begin{aligned}\mathrm{E}[\rho_c^*(l)\rho_c(m)|\boldsymbol{h}_2,\ldots,\boldsymbol{h}_{N_T}] &= \frac{1}{N_R\|\boldsymbol{h}_{l+1}\|\|\boldsymbol{h}_{m+1}\|}\\&\quad\times\left(\sum_{n=1}^{N_R}|h_{n,l+1}||h_{n,m+1}|e^{j(\phi_{n,l+1}-\phi_{n,m+1})}\right)\\&= \frac{1}{N_R}(\boldsymbol{R}_c)_{(m,l)}.\end{aligned} \tag{33}$$

Substituting (33) into (29), we finally obtain a closed-form expression for the average FUR of the conventional linear detectors, i.e.

$$\begin{aligned}\mathrm{E}[\delta^c] &= 1 - \frac{1}{N_R}\mathrm{E}\left[\sum_{l,m=1}^{N_T-1}(\boldsymbol{R}_c)_{m,l}\left(\boldsymbol{R}_c^{-1}\right)_{l,m}\right]\\&= 1 - \frac{N_T-1}{N_R}.\end{aligned} \tag{34}$$

Let us now concentrate on the average of the FUR (19) of the improved detectors. First of all, note that

$$\begin{aligned}\delta &= 1 - \boldsymbol{\rho}_I^H \boldsymbol{R}_{Ia}^{-1} \boldsymbol{\rho}_I \\ &= 1 - \sum_{l,m=1}^{N_T-1} \rho_I^*(l) \left(\boldsymbol{R}_{Ia}^{-1}\right)_{l,m} \rho_I(m) \end{aligned} \tag{35}$$

wherein $\boldsymbol{\rho}_I$ is a nonsingular $N_T - 1$-dimensional column vector, $\boldsymbol{R}_{Ia}$ is an $N_T - 1 \times N_T - 1$-dimensional matrix and

$$\rho_I^*(l)\rho_I(m) = \mathcal{R}\left\{ \frac{\boldsymbol{h}_1^H \boldsymbol{h}_{l+1}}{\|\boldsymbol{h}_1\| \|\boldsymbol{h}_{l+1}\|} \right\} \times \mathcal{R}\left\{ \frac{\boldsymbol{h}_1^H \boldsymbol{h}_{m+1}}{\|\boldsymbol{h}_1\| \|\boldsymbol{h}_{m+1}\|} \right\}. \tag{36}$$

According to this equation we can write

$$\begin{aligned}&\mathrm{E}[\rho_I^*(l)\rho_I(m) | \boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_{N_T}] \\ &= \frac{1}{2\|\boldsymbol{h}_1\|^2 \|\boldsymbol{h}_{l+1}\| \|\boldsymbol{h}_{m+1}\|} \\ &\times \left\{ \sum_{n=1}^{N_R} \left[|h_{n,1}|^2 |h_{n,l+1}| |h_{n,m+1}| (\cos(\phi_{n,l+1} - \phi_{n,m+1}) \right.\right. \\ &+ \cos(\phi_{n,l+1} + \phi_{n,m+1} - 2\phi_{n,1})) \\ &+ \sum_{p=1, p\neq n}^{N_R} |h_{n,1}| |h_{n,l+1}| |h_{p,1}| |h_{p,m+1}| \\ &\times (\cos(\phi_{n,l+1} - \phi_{n,1} + \phi_{p,m+1} - \phi_{p,1}) \\ &+ \left.\left. \cos(\phi_{n,l+1} - \phi_{n,1} - \phi_{p,m+1} + \phi_{p,1}))\right] \right\}. \end{aligned} \tag{37}$$

Averaging (37) with respect to the phases of the spatial signature of the desired antenna yields

$$\begin{aligned}&\mathrm{E}[\rho_I^*(l)\rho_I(m) | |h_{1,1}|, |h_{2,1}|, \ldots, |h_{N_R,1}|, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_{N_T}] \\ &= \frac{\left(\sum_{n=1}^{N_R} |h_{n,1}|^2 |h_{n,l+1}| |h_{n,m+1}| \cos(\phi_{n,l+1} - \phi_{n,m+1})\right)}{2\|\boldsymbol{h}_1\|^2 \|\boldsymbol{h}_{l+1}\| \|\boldsymbol{h}_{m+1}\|}. \end{aligned} \tag{38}$$

Now, since it can be shown that

$$\begin{aligned}&\mathrm{E}[\rho_I^*(l)\rho_I(m) | \boldsymbol{h}_2, \ldots, \boldsymbol{h}_{N_T}] \\ &= \frac{1}{2N_R} \mathcal{R}\left(\frac{\boldsymbol{h}_{m+1}^H \boldsymbol{h}_{l+1}}{\|\boldsymbol{h}_{m+1}\| \|\boldsymbol{h}_{l+1}\|} \right) \\ &= \frac{1}{2N_R} (\boldsymbol{R}_{Ia})_{(m,l)} \end{aligned} \tag{39}$$

we finally obtain the following expression for the FUR of the improved linear detectors:

$$\begin{aligned}\mathrm{E}[\delta] &= 1 - \frac{1}{2N_R} \mathrm{E}\left[\sum_{l,m=1}^{N_T-1} (\boldsymbol{R}_{Ia})_{m,l} \left(\boldsymbol{R}_{Ia}^{-1}\right)_{l,m} \right] \\ &= 1 - \frac{N_T - 1}{2N_R}. \end{aligned} \tag{40}$$

Comparing (34) and (40) it is seen that the improved linear receivers bring on the average a performance gain with respect to the classical ones; in particular the new receivers may support a doubled number of transmitting antennas with respect to the classical receivers. In Fig. 8 we report the average FUR versus the number of transmitting antennas N_T (here we have $N_R = 16$), for the classical decorrelating, labeled in the figure as "DEC," and for the improved decorrelating detectors, labeled in the figure as "NDEC." We also report the result of a computer-aided numerical average, showing that the analytical formulas (34) and (40) are in excellent agreement with the experimental results.

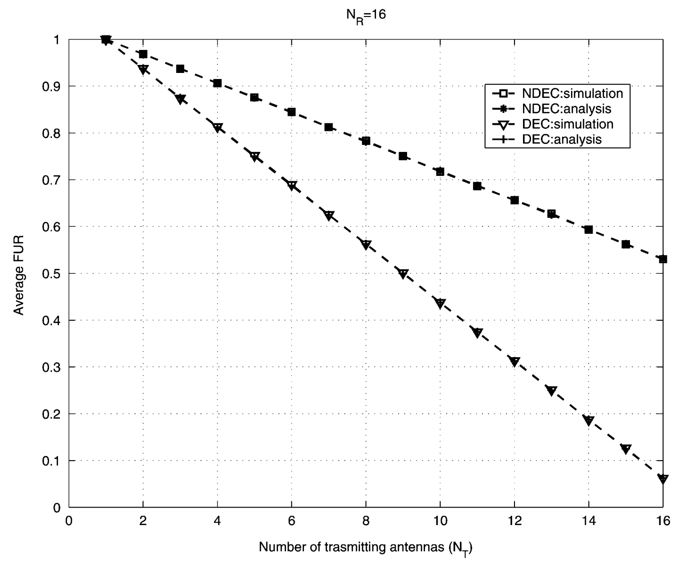

Fig. 8. Average FUR versus the number of transmitting antennas N_T.

VI. Performance Analysis With Channel Estimation Error

So far, we have assumed that the receiver has a perfect knowledge of the channel coefficients, i.e., no channel estimation error is considered. In what follows we thus evaluate the asymptotic efficiency for the linear classical and improved receivers when channel estimation errors are present; our derivations are inspired by [22], wherein the performance of the conventional mmse receiver is investigated in the presence of channel estimation errors. To begin with, note that for a given symbol interval p, and for a generic linear transformation $\boldsymbol{D}$ to be used in the decision rule (5), it can be shown that the conditional bit error probability on the jth detected symbol, given the remaining $N_T - 1$ information bits and the channel matrix $\boldsymbol{H}$, is expressed as

$$\begin{aligned}&P_{j|\boldsymbol{b}(p),\boldsymbol{H}} \\ &= \mathrm{Prob}\{\mathcal{R}\{[\boldsymbol{D}^H \boldsymbol{r}(p)]_j\} > 0 | b_j(p) = -1\} \\ &= \sum_{\boldsymbol{b}(p) \in \{-1,1\}^{N_T-1}, b_j=-1} \mathrm{Prob}\{\mathcal{R}\{[\boldsymbol{D}^H \boldsymbol{r}(p)]_j\} > 0 | \boldsymbol{b}(p)\} \\ &\times \mathrm{Prob}\{\boldsymbol{b}(p) | b_j(p) = -1\} \end{aligned} \tag{41}$$

wherein we have denoted with $b_j(p)$ the jth element of the N_T-dimensional column vector $\boldsymbol{b}(p)$, with $\mathrm{Prob}\{\boldsymbol{b}(p)|b_j(p) = -1\}$ the probability that this random vector assumes a generic realization in $\{-1,1\}^{N_T-1}$, given $b_j(p)=-1$, and, finally, $[\cdot]_j$ denotes the jth entry of the correspondent vector. Now, nontrivial algebraic manipulations lead to

$$P_{j|\boldsymbol{b}(p),\boldsymbol{H}} = \sum_{\boldsymbol{b}(p)\in(-1,1)^{N_T-1},b_j=-1} \operatorname{erfc}\left(\frac{\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,j)\boldsymbol{A}(j,j)]_j\}}{\sqrt{2\mathcal{N}_0[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}} - \frac{\sum_{k=1,k\neq j}^{N_T}\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,k)\boldsymbol{A}(k,k)b_k(p)]_j\}}{\sqrt{2\mathcal{N}_0[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}}\right) \times \left(\frac{1}{2}\right)^{N_T}. \tag{42}$$

Given (42), an upper bound for the bit error probability can be obtained by selecting the erfc-function with the smallest argument, namely we have

$$P_{j|\boldsymbol{b}(p),\boldsymbol{H}} \leq \frac{1}{2}\operatorname{erfc}\left(\frac{\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,j)\boldsymbol{A}(j,j)]_j\}}{\sqrt{2\mathcal{N}_0[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}} - \frac{\sum_{k=1,k\neq j}^{N_T}|\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,k)\boldsymbol{A}(k,k)]_j\}|}{\sqrt{2\mathcal{N}_0[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}}\right), \tag{43}$$

with $|\cdot|$ denoting the absolute value. Since, as already discussed, the asymptotic efficiency can be expressed as

$$\mu_{j|\boldsymbol{H},\boldsymbol{D}} = \sup_t \left\{0\leq t\leq 1 : \lim_{\lambda\to 0}\frac{2P_{j|\boldsymbol{H},\boldsymbol{D}}}{\operatorname{erfc}((\sqrt{t}\boldsymbol{A}(j,j))/(\sqrt{2}\lambda))}<\infty\right\} \tag{44}$$

it can be shown that in our situation we have

$$\mu^c_{j|\boldsymbol{H},\boldsymbol{D}} = \max{}^2\left\{0, \frac{\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,j)\boldsymbol{A}(j,j)]_j\}}{\sqrt{\boldsymbol{A}^2(j,j)[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}} - \frac{\sum_{k=1,k\neq j}^{N_T}|\mathcal{R}\{[\boldsymbol{D}^H\boldsymbol{H}(:,k)\boldsymbol{A}(k,k)]_j\}|}{\sqrt{\boldsymbol{A}^2(j,j)[\boldsymbol{D}^H\boldsymbol{D}]_{(j,j)}}}\right\}. \tag{45}$$

Consider now the improved linear detection rules. It can be shown that the conditional error probability is now written as

$$P_{j|\boldsymbol{b}(p),\boldsymbol{H}_a} = \sum_{\boldsymbol{b}(p)\in\{-1,1\}^{N_T-1},b_j=-1} \operatorname{erfc}\left(\frac{\left[\tilde{\boldsymbol{D}}_a^H\boldsymbol{H}_a(:,j)\boldsymbol{A}(j,j)\right]_j}{\sqrt{2\mathcal{N}_0\left[\tilde{\boldsymbol{D}}_a^H\tilde{\boldsymbol{D}}_a\right]_{(j,j)}}} - \frac{\sum_{k=1,k\neq j}^{N_T}\left[\tilde{\boldsymbol{D}}_a^H\boldsymbol{H}_a(:,k)\boldsymbol{A}(k,k)b_k(p)\right]_j}{\sqrt{2\mathcal{N}_0\left[\tilde{\boldsymbol{D}}_a^H\tilde{\boldsymbol{D}}_a\right]_{(j,j)}}}\right) \times \left(\frac{1}{2}\right)^{N_T} \tag{46}$$

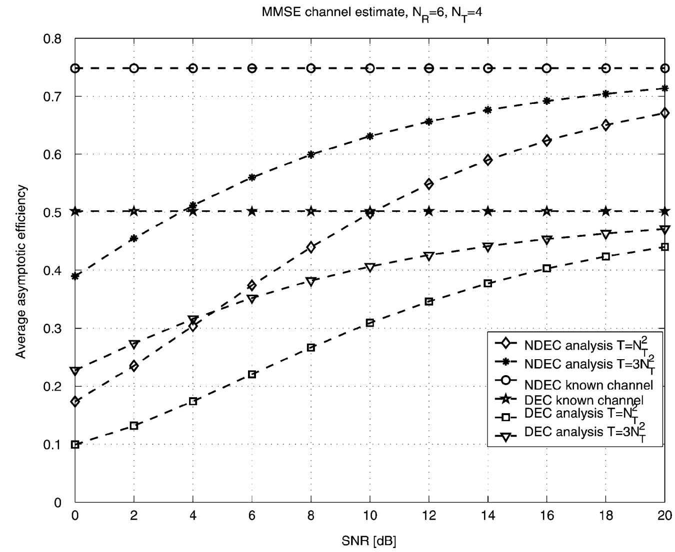

Fig. 9. Average asymptotic efficiency versus the SNR.

while the asymptotic efficiency can be expressed as

$$\mu_{j|\boldsymbol{H}_a,\tilde{\boldsymbol{D}}_a} = \max{}^2\left\{0, \frac{\left[\tilde{\boldsymbol{D}}_a^H\boldsymbol{H}_a(:,j)\boldsymbol{A}(j,j)\right]_j}{\sqrt{\boldsymbol{A}^2(j,j)\left[\tilde{\boldsymbol{D}}_a^H\tilde{\boldsymbol{D}}_a\right]_{(j,j)}}} - \frac{\sum_{k=1,k\neq j}^{N_T}\left|\left[\tilde{\boldsymbol{D}}_a^H\boldsymbol{H}_a(:,k)\boldsymbol{A}(k,k)\right]_j\right|}{\sqrt{\boldsymbol{A}^2(j,j)\left[\tilde{\boldsymbol{D}}_a^H\tilde{\boldsymbol{D}}_a\right]_{(j,j)}}}\right\}. \tag{47}$$

In Fig. 9, we report the average asymptotic efficiency versus the average SNR assuming that the channel (to be substituted in the matrix $\boldsymbol{D}$) has been estimated according to an mmse optimality criterion using T training bits. As reasonably expected, results show that increasing the length of the training sequence and for large values of the SNR the asymptotic efficiency converges to the value corresponding to the situation that the channel is perfectly known. Moreover, note that again the improved linear detection structures bring a performance improvement with respect to the conventional ones, thus implying that they are less sensitive to the channel estimation errors. For example at an SNR of 10 dB, when the channel has been estimated with $T=N_T^2$ training bits, the average asymptotic efficiency of the new proposed linear receivers (NDEC) takes on the same value as the conventional linear receivers (DEC) in the case that the channel is perfectly known. Unfortunately, we were not able to provide a theoretical justifications of this behavior. The superiority of the improved detectors in the case that the channel is not perfectly known is thus an issue that deserves further theoretical investigation and is left for future work.

VII. Conclusion

In this paper, the application of widely linear reception structures to multiantenna wireless communication systems has been studied. In particular, we have shown that widely linear detection exhibits several advantages with respect to linear reception structures. Indeed, using the new detection structures permits achieving lower symbol error probabilities, minor performance sensitivity to channel estimation errors, and improved resistance to the fading unbalance problem. Moreover, we have here confuted the usual claim that the applicability of widely linear detection to noncircular constellations is a serious limitations in situations where high spectral efficiency is an issue. Actually, our results show that widely linear detection permits operating with a number of transmit antenna larger (up to a factor of two) than the number of receive antennas, and, also that widely linear reception of an M-ary real constellation outperforms linear reception of M-ary complex constellations.

APPENDIX

In the following, we briefly show that

$$\int_0^{+\infty}\cdots\int_0^{+\infty}\frac{x_j^2}{\sum_{p=1}^{N_R}x_p^2}\left[\prod_{k=1}^{N_R}\frac{x_k}{\sigma^2}e^{-x_k^2/2\sigma^2}\right]\times dx_1dx_2\ldots dx_j\ldots dx_{N_R}=\frac{1}{N_R}. \quad (48)$$

Using the substitution $\alpha_k^2 = x_k^2/\sigma^2$ for $k = 1,\ldots,N_R$, the integral in (48) can be written as

$$\begin{aligned}&\int_0^{\infty}\cdots\int_0^{\infty}\frac{\alpha_j^2}{\sum_{p=1}^{N_R}\alpha_p^2}\prod_{k=1}^{N_R}\left[\alpha_k e^{-\frac{\alpha_k^2}{2}}d\alpha_k\right]\\ &=\prod_{k=2,k\neq j}^{N_R}\left[\int_0^{\infty}\alpha_k e^{-\frac{\alpha_k^2}{2}}\right]\int_0^{\infty}\alpha_j^3 e^{-\frac{\alpha_j^2}{2}}\\ &\quad\times\int_0^{\infty}\frac{\alpha_1}{\sum_{p=2}^{N_R}\alpha_p^2+\alpha_1^2}e^{-\frac{\alpha_1^2}{2}}d\alpha_1\ldots d\alpha_{N_R}\end{aligned} \quad (49)$$

where we have used the fact that the Jacobian $\boldsymbol{J}$ of the transformation is such that $\det(\boldsymbol{J}) = \sigma^{N_R}$. Now, letting $\xi = (\alpha_1^2)/(2)$, (49) is written as

$$\prod_{k=2,k\neq j}^{N_R}\left[\int_0^{\infty}\alpha_k e^{-\frac{\alpha_k^2}{2}}\right]\int_0^{\infty}\alpha_j^3 e^{-\frac{\alpha_j^2}{2}}\times\int_0^{\infty}\frac{e^{-\xi}}{\sum_{p=2}^{N_R}\alpha_p^2+2\xi}d\xi\ldots d\alpha_{N_R}. \quad (50)$$

Defining, now, the new variate $t = (2\xi)/(\sum_{p=2}^{N_R}\alpha_p^2) + 1$, we have

$$\begin{aligned}&\prod_{k=2,k\neq j}^{N_R}\left[\int_0^{\infty}\alpha_k e^{-\frac{\alpha_k^2}{2}}\right]\int_0^{\infty}\alpha_j^3 e^{-\frac{\alpha_j^2}{2}}\\ &\quad\times\int_0^{\infty}\frac{e^{-\xi}}{\sum_{p=2}^{N_R}\alpha_p^2+2\xi}d\xi\ldots d\alpha_{N_R}\\ &=\prod_{k=2,k\neq j}^{N_R}\left[\int_0^{\infty}\alpha_k\right]\int_0^{\infty}\alpha_j^3\frac{1}{2}\\ &\quad\times\int_1^{\infty}\frac{e^{\frac{-\sum_{p=2}^{N_R}\alpha_p^2 t}{2}}}{t}dtd\alpha_2\ldots d\alpha_{N_R}\\ &=\prod_{k=2,k\neq j}^{N_R}\left[\int_0^{\infty}\alpha_k e^{-\frac{\alpha_k^2 t}{2}}\right]\\ &\quad\times\int_0^{\infty}\alpha_j^3 e^{-\frac{\alpha_j^2 t}{2}}\frac{1}{2}\int_1^{\infty}\frac{1}{t}dtd\alpha_2\ldots d\alpha_{N_R}.\end{aligned} \quad (51)$$

Now, since

$$\int_0^{\infty}\alpha_j^3 e^{-\frac{\alpha_j^2}{2}t}d\alpha_j=\frac{2}{t^2}, \quad (52)$$

and

$$\int_0^{\infty}\alpha_l e^{-\frac{\alpha_l^2}{2}t}d\alpha_l=\frac{1}{t}\quad\forall l=2,\ldots N_R, l\neq j \quad (53)$$

we finally have

$$\int_0^{\infty}\cdots\int_0^{\infty}\frac{x_j^2}{\sum_{p=1}^{N_R}x_p^2}\prod_{k=1}^{N_R}\left[\frac{x_k}{\sigma^2}e^{-\frac{x_k^2}{2\sigma^2}}\right]\times dx_1\ldots dx_{N_R}=\int_1^{\infty}\frac{1}{t^{(N_R+1)}}dt=\frac{1}{N_R}. \quad (54)$$

Q.E.D.

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