Adaptive Matched Filter Detection in Spherically Invariant Noise

Ernesto Conte, Marco Lops, and Giuseppe Ricci

Abstract— **This work addresses radar detection of coherent pulse trains embedded in spherically invariant noise with unknown statistics. Starting upon a newly proposed detector, which assumes knowledge of the structure of the clutter covariance matrix, we substitute the actual matrix by a proper estimate based on a set of secondary data vectors. Interestingly, the resulting detector achieves constant false alarm rate with respect to the texture component of the clutter, and incurs an acceptable loss with respect to the case of known covariance matrix.**

I. Introduction

EXPERIMENTAL data indicate that clutter in high-resolution radars can be generally modeled as a compound-Gaussian process which, when observed on sufficiently short time intervals, degenerates into a spherically invariant random process (SIRP). As expected, receivers optimized to detect radar signals in SIRP outperform those designed under the assumption of Gaussian disturbance, under practically all instances of target radar cross-section fluctuation. However, their implementation requires knowledge of the clutter statistics up to the relevant distributional parameters, which is clearly unrealistic in practical situations [1].

Thus, it is of primary concern to come up with *canonical* structures, namely, independent of both the clutter amplitude probability density function (apdf) and its autocorrelation function (acf). A first step toward this task is the receiver introduced in [2]: Such a detector is asymptotically optimum as the number of integrated pulses N diverges and ensures constant false alarm rate (CFAR) with respect to the clutter distribution, but still requires knowledge of the structure of the clutter covariance matrix.

In this letter, we introduce an *adaptive* detection strategy, wherein an estimate of the structure of the clutter covariance matrix, based on K "secondary data vectors," is substituted in the detection structure introduced in [2] in place of the true matrix; the resulting detector is still canonical with respect to the texture component of the clutter, while its performance is only marginally affected by the correlation properties of the data in the cell under test.

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II. Noise Model

In the following, we assume a clutter-dominated environment, wherein the clutter is modeled as a SIRP. Thus, N samples from the baseband equivalent of the clutter returns form a spherically invariant random vector, i.e.,

$$\mathbf{c} = s\mathbf{g}$$

where the Gaussian vector $\mathbf{g}$ is assumed to have the circular property associated with the inphase and the quadrature components of a wide-sense stationary bandpass random process and, without loss of generality, the texture component s is assumed to have unit second moment.

Moreover, we suppose that a bunch of K range gates surrounding the cell under test shares its spectral properties. Thus, denoting by $\mathbf{c}_k$ the vector whose entries are samples of the clutter from the kth of those cells, we have

$$\mathbf{M} = E[\mathbf{c}\mathbf{c}^\dagger] = E[\mathbf{c}_k\mathbf{c}_k^\dagger], \quad k = 1, \cdots, K$$

with $\dagger$ denoting the conjugate transpose operator. Nevertheless, the mean power of the clutter from these different resolution cells, as averaged over the ensemble of small-scale realizations, is still a stochastic, possibly correlated, process [3] that cannot be approximated with a random constant: We suppose that clutter returns from the kth of those range cells can be deemed as

$$\mathbf{c}_k = s_k\mathbf{g}_k, \quad k = 1, \cdots, K$$

where $\{\mathbf{g}, \mathbf{g}_1, \cdots, \mathbf{g}_K\}$ is a sequence of independent, identically distributed Gaussian vectors while $\{s, s_1, \cdots, s_K\}$ is a set of samples drawn from a possibly correlated process that determines the spatial correlation of the clutter texture. Although an accurate model for the spatial correlation has not been proposed yet, Ward *et al.* have shown that the spectral properties of the texture component significantly depend upon the grazing angle and the viewing direction [3].

III. System Design

Assuming knowledge of the clutter covariance matrix, the problem of detecting a radar signal (known to within a multiplicative constant, say, α) embedded in SIRP can be solved based on the generalized likelihood ratio test (GLRT). As $N \rightarrow \infty$, the GLRT leads to the following test to decide between the signal-plus-noise (H_1) and the noise-only hypotheses (H_0) [2]:

$$\frac{|\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{r}|^2}{(\mathbf{r}^\dagger \mathbf{M}^{-1}\mathbf{r})(\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{1}$$

where $\mathbf{r}$ and $\mathbf{p}$ represent the N-dimensional complex vectors of the samples from the baseband equivalents of the received and the transmitted signal, respectively. The statistic on the left-hand side of (1) is easily seen to consist of a plain matched filter plus a normalization factor. This detector will be referred to as the normalized matched filter (NMF) in the following.

The structure of the above detector is independent of the clutter apdf. It requires, instead, knowledge of the covariance matrix, but for a multiplicative factor. Additionally, under the null hypothesis, both the spiky component s and the clutter power $2\sigma^2$, say, factor out from the test statistic, whence the threshold level can be set once and for all, e.g., assuming Gaussian clutter with unit power; in other words, the detector is also CFAR, in that, upon threshold setting, the FAR is kept constant independent of the clutter apdf and, in particular, of its power [2].

As the structure of the clutter covariance matrix is unknown, a possible strategy is to adopt the test (1), with the unknown matrix $\mathbf{M}$ replaced by a suitable estimate. To this end, a set of "secondary data" may be collected from range cells surrounding and including that being tested [4]. Precisely, we assume that the receiver has access to N returns from each of K ($K > N$) secondary cells, namely to a set of KN-dimensional vectors, $\mathbf{r}_1, \cdots, \mathbf{r}_K$, which supposedly are signal free. The most natural choice for estimating $\mathbf{M}$ would be to consider the sample covariance matrix, i.e.,

$$\widehat{\mathbf{M}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{r}_k \mathbf{r}_k^\dagger . \tag{2}$$

As shown in [5], this choice would lead to a threshold level independent of the actual clutter covariance matrix. On the other hand, the non-Gaussian nature of the clutter would, in this case, induce a dependence of the threshold level on the statistics of the texture component, as can be easily checked by direct substitution of (2) in (1) and recalling that $\mathbf{r}_k = s_k \mathbf{g}_k$, $k = 1, \ldots, K$. As a consequence, it appears meaningful to ensure the CFAR property with respect to the texture statistics, namely to the joint statistical properties of the sequence $s, s_1, \cdots, s_K$. To this end, we exploit the fact that the returns from the kth range cell possess one and the same spiky component s_k. In fact, such circumstance suggests the following estimate for the structure $\mathbf{\Sigma} = \frac{1}{2\sigma^2}\mathbf{M}$ of the covariance matrix [5]:

$$\widehat{\mathbf{\Sigma}} = \frac{1}{K}\sum_{k=1}^{K} \frac{\mathbf{r}_k \mathbf{r}_k^\dagger}{\frac{1}{N} \parallel \mathbf{r}_k \parallel^2} \tag{3}$$

which can also be interpreted as the sample covariance matrix of the normalized data

$$\frac{\mathbf{r}_k}{\sqrt{\frac{1}{N} \parallel \mathbf{r}_k \parallel^2}}, \quad k = 1, \cdots, K$$

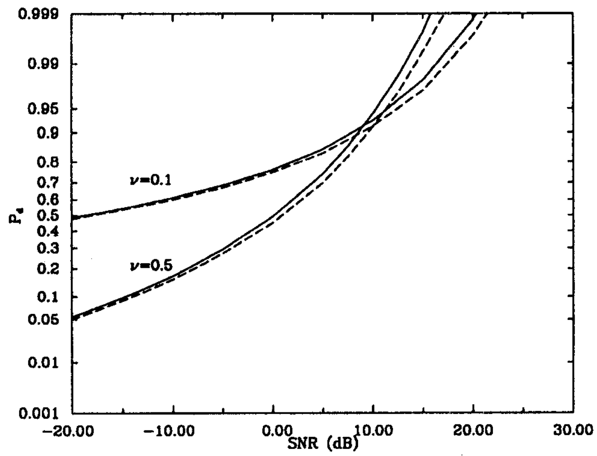

Fig. 1. Operating characteristics of the NMF and the ANMF for $N = 16$ and $K = 64$, $P_{fa} = 10^{-4}$, $\rho_G = 0.6$, $\rho_S = 0$, and some values of ν: ——— NMF, - - - - - ANMF.

obtained by dividing the original secondary data through their estimated root-mean-square value. Since $\frac{1}{N} \parallel \mathbf{r}_k \parallel^2$ converges in mean square, for increasing N, to the local clutter power $2s_k^2\sigma^2$, we expect (3) to be an asymptotically unbiased and consistent estimator of $\mathbf{\Sigma}$ as K and N become increasingly large. The resulting detector will be referred to as adaptive normalized matched filter (ANMF).

A major characteristic of the ANMF is that it is canonical with respect to the statistics of the texture; in fact, under H_0, all of the spiky components of the received clutter echoes factor out from the test statistic, implying that the threshold level can be set once and for all, independent of the model being actually in force for the texture.

The pdf of the test statistic under H_0, though, turns out to depend on the actual structure of the clutter covariance matrix, whence *a priori* knowledge of $\mathbf{\Sigma}$ is apparently necessary for setting the detection threshold: This point is addressed by computer simulations in the next section.

IV. Performance Assessment

In the following, we refer to one of the most widely accepted models for the clutter apdf, the K-distribution, i.e.,

$$f(u) = \frac{a^{\nu+1}u^\nu}{2^{\nu-1}\Gamma(\nu)} K_{\nu-1}(au), \quad u \geq 0, \quad a, \nu > 0$$

where $\Gamma(\cdot)$ is the Eulerian gamma function, $K_\nu(\cdot)$ is the modified second-kind Bessel function, and a and ν are a scale and a shape parameter, respectively. Moreover, we assume that noise returns from a given range-cell are exponentially correlated while the transmitted signal is a coherent pulse train with zero Doppler shift. Finally, since closed-form expressions of the test statistic of the ANMF are not available, we assess its performance resorting to Monte Carlo simulation.

The operating characteristics of the ANMF, namely curves of its probability of detection (P_d) versus the signal-to-noise ratio (SNR)

$$\text{SNR} = \frac{|\alpha|^2 \parallel \mathbf{p} \parallel^2}{2\sigma^2 N}$$

are plotted in Fig. 1 for $N = 16$ and $K = 64$; probability of false alarm (P_{fa}) equal to 10^{-4}; one-lag correlation coefficient (ρ_G) equal to 0.6; a cell-to-cell independent spiky component ($\rho_S = 0$); and some values of ν. In the same figure, the operating characteristics of the NMF receiver, representative of perfect knowledge of the clutter covariance matrix, are plotted too. The comparison shows that the ANMF detector incurs a negligible loss.

We have also assessed the impact on P_{fa} of a mismatch between ρ_G and its estimate, $\widehat{\rho}_G$ say. Preliminary results indicate that the difference between the nominal and the actual P_{fa} is not significant, whence CFAR is substantially achieved if the threshold is set based on $\widehat{\rho}_G$.

V. Conclusions

This paper addresses adaptive radar detection in the presence of compound-Gaussian clutter with unknown statistics. In particular, we propose a suitable estimate of the structure of the covariance matrix, based on secondary data vectors, to be substituted in the limiting form (1) of the GLRT in place of the actual matrix. The resulting detector is CFAR with respect to the texture statistics; additionally, even though it is not theoretically invariant with respect to the clutter spectral properties, it is quite robust with respect to variations in the clutter temporal correlation. Finally, this receiver guarantees a negligible loss with respect to the NMF even for relatively small values of N and K.

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