Asymptotically Optimum Radar Detection in Compound-Gaussian Clutter

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An asymptotically optimum receiver designed for detecting coherent pulse trains in compound-Gaussian clutter is introduced and assessed. The proposed receiver assumes knowledge of the structure of the clutter covariance matrix, but does not require that of its amplitude probability density function (apdf). Performance is analytically evaluated, showing that the loss, as measured with respect to the corresponding optimum structure, is kept within a few dBs even for a relatively small number of integrated pulses and that it largely outperforms the matched-filter detector under all instances of practical interest. Interestingly, the proposed detector achieves constant false alarm rate (CFAR), regardless of the clutter envelope distribution and, consequently, its power.

Manuscript received February 25, 1993; revised December 7, 1993 and April 6, 1994.

IEEE Log No. T-AES/31/2/09751.

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I. INTRODUCTION

A substantial bulk of work is now available in the literature about detection in non-Gaussian noise. Starting with the experimental evidence that the Gaussian assumption is no longer met in many situations of practical interest, much effort has been directed towards the study of conventional detectors under non-Gaussian clutter as well as the design and the analysis of new optimized structures. Both strategies entail preliminary statistical inference on measured data so as to work out a model for clutter returns, to be applied for analysis and synthesis purposes.

A general agreement has been reached about the validity of the so-called *compound-Gaussian* model for radar clutter. The baseband equivalent of clutter returns can be deemed as the product of two mutually independent processes: a complex, zero-mean, possibly correlated Gaussian process, also referred to as *speckle*, times a real, nonnegative, *spiky* component, which exhibits much longer decorrelation time than the former [1–3]. Otherwise stated, the measured amplitude probability density function (apdf) is a Rayleighian process whose mean square value is itself a slowly varying random process, carrying the information on the *texture* of the illuminated patch. Mathematically, such a scattering mechanism is very accurately described, for observation times on the order of the coherent processing interval (CPI) of radar systems, by means of the spherically invariant random processes (SIRPs), wherein the spiky component is a random constant, rather than a process, so that the overall clutter correlation coincides, except for a scale factor, with that of the speckle [4]. A further advantage of this model is that it is fully compatible with some widely reported properties of radar clutter, primarily with the invariance of its apdf under some linear, even time-varying transformations, such as moving target indicator (MTI) techniques and discrete Fourier transform (DFT) processing [5, 6].

Performance analysis of the conventional radar detectors subject to compound-Gaussian disturbance showed that they suffer a remarkable degradation as the actual clutter apdf deviates from the Rayleigh law [7, 8]. Better performance can be achieved by means of *ad hoc* processors, namely detection structures properly optimized with reference to specific instances of clutter apdf and covariance matrix. The problem of optimized detection in K-distributed clutter is handled in [9], showing that those receivers may yield noticeable improvement over the conventional ones. Nonetheless, not only does implementing optimized detectors require knowledge of clutter statistics, which in practice must be estimated from observables, but their performance assessment has highlighted that mismatch between design and input clutter apdfs may result in unacceptable performance degradation.

A new detector which assumes knowledge of the structure of the clutter covariance matrix, but is otherwise independent of its apdf, is introduced and assessed. We evaluate the asymptotics of the multivariate probability density function (pdf) of the clutter, showing that in the case where the target echo contains an *unknown* complex parameter, the generalized likelihood ratio test (GLRT) tends, for an increasingly high number of integrated pulses, to a test statistic independent of the clutter distribution. A canonical detector can thus be obtained by adopting it as a test statistic even for a finite number of integrated pulses.

Relevant to our scope, the corresponding detector turns out to achieve constant false alarm probability, regardless of the distribution of the input noise and, consequently, of the clutter power. As to the detection performance, we show that the loss incurred by the distribution-free detector is kept within a few dBs with respect to the corresponding GLRT strategy, even for a relatively small number of integrated pulses, and that it largely outperforms the matched-filter envelope detector under all instances of practical interest.

In Section II, after briefly reviewing the theory of radar detection in SIRPs, we present the derivation of the aforementioned distribution-free structure. In Section III we handle the performance assessment of this structure, also in comparison with the conventional envelope detector, while conclusions and hints for future research are reported in Section IV.

II. RADAR DETECTION IN COMPOUND-GAUSSIAN NOISE

In this section we summarize, for the reader's ease, some relevant results on detection in compound-Gaussian noise and subsequently derive a distribution-free detection structure.

A. Clutter Model

First, we recall that the compound-Gaussian model for clutter returns mathematically corresponds to assuming that a vector of size N of clutter samples is a spherically invariant random vector (SIRV). Precisely, if we denote by $c(t)$ the (zero-mean) complex envelope of the clutter return and by $\mathbf{c} = \mathbf{c}_I + j\mathbf{c}_Q$ the (column) vector of N samples from such a process, then $\mathbf{c}$ is written in the form

$$\mathbf{c} = s\mathbf{g} \tag{1}$$

where $\mathbf{g} = \mathbf{g}_I + j\mathbf{g}_Q$ is a complex Gaussian vector, s is a nonnegative real random variate with pdf $f(s)$, and s and $\mathbf{g}$ are independent. Without loss of generality, we assume that s has unitary mean square value. A thorough discussion of the validity of this model can be found in [3, 4]. We just highlight here that SIRVs are easily seen from (1) to be *closed* under linear transformations as a consequence of the *scaling* property of linear systems and of the *closure* property of Gaussian vectors. Thus, the cited invariance of the clutter apdf under both MTI and DFT processing is fully justified once the model (1) applies to time intervals on the order of the system CPI.

Relevant to the goal of designing *ad hoc* detection structures, any SIRV can be specified based upon a first- and second-order characterization only. More precisely, let us assume the bandpass clutter be a wide sense stationary stochastic process: the N-order pdf of $\mathbf{c}$ can then be cast in the form

$$f_{\mathbf{c}}(\mathbf{c}) = \frac{1}{\pi^N |\mathbf{M}|} h_N[\mathbf{c}\mathbf{M}^{-1}\mathbf{c}] \tag{2}$$

where $\mathbf{M}$ is the N by N covariance matrix of the vector $\mathbf{c}$, namely

$$\mathbf{M} = E[\mathbf{c}\mathbf{c}] \tag{3}$$

where denotes the conjugate transpose operator, and $h_N(\cdot)$ is defined as:

$$h_N(x) = \int_0^\infty s^{-2N} \exp\left(-\frac{x}{s^2}\right) f(s)\, ds. \tag{4}$$

The most common clutter apdfs, namely the Weibull and the K-distribution, are compatible with the model (1). Precisely, the Weibull apdf:

$$f_A(u) = abu^{b-1}\exp(-au^b) \qquad u \geq 0, \quad a,b > 0 \tag{5}$$

is amenable to a compound-Gaussian representation in the range $0 < b \leq 2$ of its shape parameter [4], while the K-distribution

$$f_A(u) = \frac{a^{\nu+1}u^\nu}{2^{\nu-1}\Gamma(\nu)} K_{\nu-1}(au) \qquad u \geq 0, \quad a,\nu > 0 \tag{6}$$

where $\Gamma(\cdot)$ is the Eulerian gamma function and $K_\nu(\cdot)$ is the modified second-kind Bessel function, can be regarded, for any value of its shape parameter ν, as the apdf of a compound-Gaussian process [4]. For both distributions a is a scale parameter related to the common variance σ^2 of the clutter quadrature components.

B. Optimized Detection in Compound-Gaussian Clutter

The problem of detecting a radar signal in clutter-dominated environment can be posed in terms of the following binary hypothesis test

$$\begin{cases} H_1 : \mathbf{z} = \alpha\mathbf{u} + \mathbf{c} \\ H_0 : \mathbf{z} = \mathbf{c} \end{cases} \tag{7}$$

where $\mathbf{z}$, $\mathbf{u}$, and $\mathbf{c}$ denote the N-dimensional complex vectors of the samples from the baseband equivalent of the received signal, the transmitted signal and the clutter, respectively, while α is a complex parameter, which can be either a known or an unknown constant, accounting for both channel and target effects.

The quoted closure property of SIRVs allows one to apply the whitening approach to detect signals in correlated disturbance with known covariance matrix $\mathbf{M} = 2\sigma^2\mathbf{\Sigma}$. More precisely, if we denote by $\mathbf{\Sigma}^{-1}$ the (nonnegative definite) inverse of the normalized clutter covariance matrix $\mathbf{\Sigma}$, which henceforth can be factorized as $\mathbf{\Sigma}^{-1} = \mathbf{AA}$, then the problem of detecting the signal $\mathbf{u}$ in correlated noise $\mathbf{c}$ is equivalent to that of detecting its filtered version, $\mathbf{p} = \mathbf{Au}$, in uncorrelated noise $\mathbf{n} = \mathbf{Ac}$. Formally, the test (7) turns out to be equivalent to the test

$$\begin{cases} H_1 : \mathbf{r} = \alpha\mathbf{p} + \mathbf{n} \\ H_0 : \mathbf{r} = \mathbf{n} \end{cases} \tag{8}$$

wherein the vector $\mathbf{r}$ is tied to the raw data as $\mathbf{r} = \mathbf{Az}$. Notice also that the whitening filter preserves the actual noise power $2\sigma^2$ and requires knowledge of the structure of the clutter covariance matrix only.

The optimum (in the Neyman–Pearson sense) solution to the hypothesis testing problem (8) is the likelihood ratio test (LRT), which, based on representation (4), can be expressed, for the case of completely known wanted target echo, as

$$\frac{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}-\alpha\mathbf{p}\|^2}{2\sigma^2 s^2}\right] f(s)\,ds}{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}\|^2}{2\sigma^2 s^2}\right] f(s)\,ds} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{9}$$

It can be shown that no uniformly most powerful (UMP) test exists for the case at hand and that the resulting detector is a generalization of the conventional minimum-distance receiver. It is thoroughly investigated in [9] with reference to K-distributed clutter.

In case of no a priori knowledge at all about the target parameter, namely if α is modeled as an unknown, nonfluctuating quantity, the GLRT is a suitable means for circumventing such an a priori uncertainty [10]. The GLRT yields for the case at hand

$$\max_\alpha \frac{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}-\alpha\mathbf{p}\|^2}{2\sigma^2 s^2}\right] f(s)\,ds}{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}\|^2}{2\sigma^2 s^2}\right] f(s)\,ds} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{10}$$

The left hand side (LHS) of (10) can be easily shown to be maximum at:

$$\alpha = \frac{\mathbf{pr}}{\|\mathbf{p}\|^2} = \frac{\mathbf{r}\cdot\mathbf{p}}{\|\mathbf{p}\|^2} \tag{11}$$

where $\cdot$ denotes the usual dot product between complex vectors. Hence, the test (10) reduces to

$$\frac{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}\|^2 - \dfrac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}}{2\sigma^2 s^2}\right] f(s)\,ds}{\displaystyle\int_0^\infty \frac{1}{s^{2N}} \exp\left[-\frac{\|\mathbf{r}\|^2}{2\sigma^2 s^2}\right] f(s)\,ds} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{12}$$

Also this detector has been analyzed in [9] with reference to K-distributed clutter.

C. Distribution-Free Detection

The main drawback of detectors (9) and (12) is that their implementation requires knowledge of the clutter distribution up to its shape and scale parameters. Now we focus on the more appealing case of signals with unknown parameters, showing that proper development of the test statistic (12) leads to a distribution-free structure.

First notice that the function $h_N(\cdot)$ of (4) can be recast in the form:

$$h_N(x) = \frac{\Gamma(N)}{2\sqrt{N}} x^{-N+1/2} \int_0^\infty \frac{1}{\sqrt{y}} f\left(\sqrt{\frac{x}{Ny}}\right) \Phi_N(y)\,dy \tag{13}$$

where the function $\Phi_N(y) = N^N \Gamma^{-1}(N) y^{N-1} e^{-Ny}$ can be interpreted as the pdf of a "normalized" *Erlang* variate, namely of the ratio of an Erlang random variable with N degrees of freedom to N. As N diverges, such a variate is well known to converge in mean square to its statistical average, thus implying that $\Phi_N(y)$ converges in distribution to the Dirac's delta function $\delta(y-1)$. Therefore, the function $h_N(\cdot)$ in (13) admits the asymptotic development

$$h_N(x) \sim \frac{\Gamma(N)}{2\sqrt{N}} x^{-N+1/2} f\left(\sqrt{\frac{x}{N}}\right) \qquad N \gg 1. \tag{14}$$

Substituting the approximation (14) into the GLRT (12) yields, after some algebra:

$$\frac{\|\mathbf{r}\|^2}{\|\mathbf{r}\|^2 - \dfrac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}} \left(\frac{f\left(\dfrac{\sqrt{\|\mathbf{r}\|^2 - \dfrac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}}}{\sigma\sqrt{2N}}\right)}{f\left(\dfrac{\|\mathbf{r}\|}{\sigma\sqrt{2N}}\right)} \right)^{1/(N-1/2)} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{15}$$

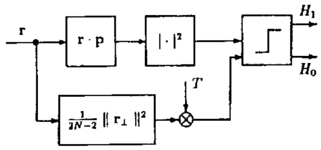

Fig. 1. Distribution-free detector for signals with unknown parameters in compound-Gaussian clutter.

For high N and a well-behaved $f(s)$, the rightmost ratio on the LHS of (15) tends to unity and the test can be rewritten as

$$\frac{\|\mathbf{r}\|^2}{\|\mathbf{r}\|^2 - \dfrac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{16}$$

or, equivalently

$$\frac{\|\mathbf{r}_\perp\|^2 + \dfrac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}}{\|\mathbf{r}_\perp\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{17}$$

where $\mathbf{r}_\perp$ denotes the component of $\mathbf{r}$ orthogonal to the direction of $\mathbf{p}$. Finally, straightforward algebraic manipulations yield

$$\frac{|\mathbf{r}\cdot\mathbf{p}|^2}{\dfrac{1}{2(N-1)}\|\mathbf{r}_\perp\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{18}$$

For future reference, it is useful to rewrite the above test in terms of the raw data and of the unfiltered useful signal. Substituting $\mathbf{r} = \mathbf{A}\mathbf{z}$ and $\mathbf{p} = \mathbf{A}\mathbf{u}$ in (16) and using the relationship $(1/2\sigma^2)\mathbf{A}\mathbf{A} = \mathbf{M}^{-1}$ yields, after some algebra

$$\frac{1}{1 - \dfrac{|\mathbf{u}\mathbf{M}^{-1}\mathbf{z}|^2}{(\mathbf{z}\mathbf{M}^{-1}\mathbf{z})(\mathbf{u}\mathbf{M}^{-1}\mathbf{u})}} \underset{H_0}{\overset{H_1}{\gtrless}} T$$

or, equivalently

$$\frac{|\mathbf{u}\mathbf{M}^{-1}\mathbf{z}|^2}{(\mathbf{z}\mathbf{M}^{-1}\mathbf{z})(\mathbf{u}\mathbf{M}^{-1}\mathbf{u})} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{19}$$

Notice that the same symbol T is used in (16)–(19) for the appropriate modifications of the original threshold in (15).

The block diagram of the detector implementing the test (18) is depicted in Fig. 1. The received vector is passed through two parallel channels. The upper branch evaluates the numerator of the test statistic in (18), namely performs the conventional incoherent processing, while the lower branch normalizes such a statistic to the estimated clutter power. In fact, $\mathbf{r}_\perp$ does not contain the wanted target echo and, hence, is a vector of $N - 1$ uncorrelated noise samples which allows the power of the disturbance to be estimated.

Due to the representation (1) of the clutter vector, the LHS of (18), under H_0, can be written as

$$\frac{|\mathbf{r}\cdot\mathbf{i}_p|^2}{\|\mathbf{r}_\perp\|^2} = \frac{|\mathbf{g}\cdot\mathbf{i}_p|^2}{\|\mathbf{g}_\perp\|^2} \tag{20}$$

where $\mathbf{i}_p$ is the versor parallel to the direction of $\mathbf{p}$. As a consequence, the statistic (20) is functionally and statistically independent of both the pdf $f(s)$ of the modulating variate and the clutter power σ^2. Otherwise stated, the introduced receiver not only is asymptotically optimum and *canonical* with respect to the clutter apdf, but it also ensures the constant false alarm rate (CFAR) property.

Notice that detector (18) operates as a cell-averaging CFAR (CA-CFAR) detector and, in fact, the test statistic (20) is equivalent to that of a plain CA-CFAR operating in Gaussian environment. However, it is worth underlining that the proposed detector estimates the noise power by means of time samples from the range cell under test, while the aforementioned CA-CFAR detector resorts to spatial samples from range cells adjacent to that being tested. As a consequence detector (18), unlike a CA-CFAR, achieves one and the same probability of false alarm (P_{fa}) also in compound-Gaussian clutter: in the former case, in fact, the spiky component remains unchanged on time intervals in the order of the system CPI, while in the latter it is a narrowband random process, whose correlation is related to the correlation between clutter samples in close spatial proximity with each other [1].

Similar developments apply to the case of completely known target echo. More precisely, substituting (14) in (9) yields the asymptotically optimum test

$$\frac{\Re\{\alpha\mathbf{r}\cdot\mathbf{p}\} - \frac{1}{2}|\alpha|^2\|\mathbf{p}\|^2}{\dfrac{1}{2N}\|\mathbf{r}\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{21}$$

where $\Re(z)$ denotes the real part of the complex number z. Although knowledge of the clutter apdf is not required to evaluate the test statistic in (21), it is necessary to set the threshold according to the desired P_{fa}, thus confirming that detector (21) is useless in real situations.

A possible criticism could be raised against receiver (18) in that it assumes knowledge of the structure of the clutter covariance matrix. A viable modification of the proposed strategy which circumvents this drawback amounts to using an estimate $\hat{\mathbf{M}}$ of the covariance matrix for the implementation of the whitening filter. More precisely, such an estimate can be achieved by resorting to a sample covariance matrix based on samples from range cells surrounding the one under test (*secondary* data), under the hypothesis that range cells in spatial proximity share the same correlation properties [11, 12]. The resulting test, rewritten in terms of the estimated covariance matrix is

$$\frac{|\mathbf{u}\hat{\mathbf{M}}^{-1}\mathbf{z}|^2}{(\mathbf{z}\hat{\mathbf{M}}^{-1}\mathbf{z})(\mathbf{u}\hat{\mathbf{M}}^{-1}\mathbf{u})} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{22}$$

Such a structure is superficially similar to that developed by Kelly in [11] with reference to Gaussian clutter, but a deeper insight highlights that its operation, in addition to its synthesis approach, is substantially different. In fact notice that, for increasingly high number of secondary data, K say, the estimate tends to the true covariance matrix with probability one [13], whence the detector (22) reduces to the detector

$$\frac{|\mathbf{u}\mathbf{M}^{-1}\mathbf{z}|^2}{(\mathbf{z}\mathbf{M}^{-1}\mathbf{z})(\mathbf{u}\mathbf{M}^{-1}\mathbf{u})} \underset{H_0}{\overset{H_1}{\gtrless}} T.$$

This is just the test (19), namely the asymptotically optimum test written with reference to the raw data $\mathbf{z}$ and the unfiltered useful signal $\mathbf{u}$; Kelly's receiver, instead, reduces, for infinite K, to the conventional matched-filter detector. Under this standpoint, the detector in [11], subject to Gaussian clutter, is superior to receiver (19). If, instead, the hypothesis of Gaussian disturbance is relaxed, the matched filter is no longer optimum, while detector (19) is asymptotically optimum. In fact, as N diverges, the test (19) approaches the optimum detector for completely known useful signal, as a consequence of the consistency of the maximum likelihood estimates [9]. Later in this work, we handle the comparative performance analysis of the two detection schemes for finite sample size, showing that the detector (19) outperforms the conventional one in a wide range of detection probabilities, even for moderately high number N of integrated pulses.

For finite K, the detector (22) can be shown to be CFAR with respect to the level and the structure of the clutter covariance matrix; the proof goes through exactly as in [11]. However, Monte Carlo simulation has confirmed that the detector (22) is no longer CFAR with regard to the clutter apdf, in fact, the spiky component of the clutter, as observed in the cells forming the secondary data set, can no longer be thought of as a random variate; rather, it is a narrowband stochastic process, whose spectral characteristics turn out to depend on the spatial correlation of the clutter [2].

Therefore, inasmuch as one is concerned with distribution-free detection in situations where the clutter apdf may vary on a time scale much shorter than its correlation properties, the best strategy appears to implement the test (19), where the covariance matrix is estimated *off line*. Equivalently, upon estimation of the clutter covariance matrix, one can resort to the whitening approach and hence implement the test (18). In the companion situation where the clutter covariance is itself a rapidly varying measure, one could resort to the detector (22), with the understanding that proper threshold setting requires a priori knowledge of the clutter apdf or on-line tracking of its variations.

III. PERFORMANCE PREDICTION

In the following, as a consequence of all the above considerations, we focus only on the receiver structure (18) for signals with unknown parameters. We recall that detector (18) is perfectly equivalent to the corresponding GLRT test for infinite N, but no finite-size sample optimality can be claimed. On the other hand, its structure does not depend upon the pdf of the impinging disturbance: thus a distribution-free detector can be obtained by simply adopting this test even for finite N. Such a choice, however, inevitably entails some detection loss, as measured with respect to the GLRT detector (12): it is thus necessary to assess the performance of the distribution-free structure, so as to verify that this loss is kept within acceptable limits.

As already stated the distribution of the statistic (20) is one and the same independent of both the pdf $f(s)$ of the modulating variate and the clutter power σ^2. In fact, it is the ratio between two mutually independent central chi-square variates and hence is proportional to an F-distributed variate with degrees of freedom 2 and $2(N-1)$ [14, p. 246 ff.]. As a consequence the false alarm probability can be cast in the form

$$P_{\mathrm{fa}} = (1+t)^{-N+1} \tag{23}$$

where $t = T/(2(N-1)\|\mathbf{p}\|^2)$ and T, in turn, is the threshold in (18). Notice that such a formula is perfectly equal to the false alarm rate achieved by a plain CA-CFAR operating in Gaussian environment.

Consider now the evaluation of the detection probability. Under H_1, the test statistic to be implemented (18) can be written as

$$\frac{|s\mathbf{g}\cdot\mathbf{i}_p + \alpha\|\mathbf{p}\||}{\|\mathbf{r}_\perp\|} \tag{24}$$

and hence, given s, is the ratio between a Rice-distributed variate and an independent central chi with $2(N-1)$ degrees of freedom. The distribution of such a ratio is reported in [15]. Thus, the detection probability can be obtained by averaging s out of the conditional complementary cumulative distribution function (cdf), i.e.

$$P_{\mathrm{d}} = 1 - \int_0^\infty \left[\frac{t}{1+t} e^{-N\gamma/s^2(1+t)} \sum_{j=0}^{N-2} \frac{1}{j!}\left(\frac{N\gamma t}{s^2(1+t)}\right)^j \times \sum_{l=j}^{N-2} \binom{l}{j} \left(\frac{1}{1+t}\right)^l\right] f(s)\,ds \tag{25}$$

where γ

$$\gamma = \frac{|\alpha|^2\|\mathbf{p}\|^2}{2\sigma^2 N} \tag{26}$$

is the signal-to-noise ratio.

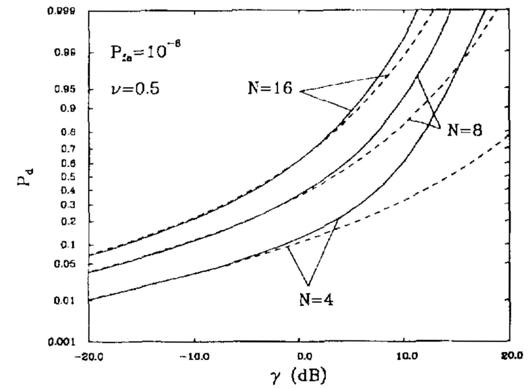

Fig. 2. Detection performance of optimized (——) and distribution-free detector (- - -) for signals with unknown parameters in K-distributed clutter, N as parameter.

In Fig. 2 the relationship (25) is plotted on a normal probability paper versus γ for K-distributed clutter with $\nu = 0.5$ or, equivalently, Weibull clutter with $b = 1$, $P_{\rm fa} = 10^{-6}$ and some values of N. In order to verify from a quantitative standpoint the goodness of the approximation leading to the test (18), the performance of the test (12) is reported also. The figure shows that inasmuch as the condition $N \gg 1$ is met, the two detectors yield practically equivalent performance; on the other hand, even for moderate N (i.e., $N \geq 8$), the loss is kept within a few dBs except for close-to-one detection probabilities. The performance is instead substantially poorer for low N, which stems from the fact that the test statistic (18) is only asymptotically sufficient. We stress here that such a loss is largely compensated for by the aforementioned constancy of the false alarm rate with respect to the clutter distributional parameters.

Relationship (25) highlights that the performance depends on the signal-to-noise ratio only, consequently, it is independent of the signal pattern; nor does it depend on the argument of the complex gain α. At the synthesis stage and in the above derivations, α has been modeled as a nonrandom unknown parameter, so as to achieve a nonparametric detection structure; in real situations, though, the wanted target echo may exhibit fluctuations, so that, at the analysis stage, a more complete model for such an echo would be desirable. A possible choice is the so-called chi-model for target amplitude, wherein the modulus of α is assumed to fluctuate according to a chi-distributed variate with $2m$ degrees of freedom, namely

$$f_{|\alpha|}(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)(\overline{|\alpha|^2})^m} \exp\left(-\frac{mx^2}{\overline{|\alpha|^2}}\right) \qquad m = 1,2\ldots; \qquad (27)$$

while its phase is assumed uniformly distributed in $(-\pi,\pi)$. The model (27) subsumes most target amplitude fluctuation instances of relevant practical interest: the case of Rayleigh-distributed amplitude, corresponding to $m = 1$, the Rayleigh-plus-dominant ($m = 2$), the nonfluctuating ($m = \infty$).

In order to work out a closed-form expression for the system performance subject to fluctuating amplitude, it is convenient to interpret the relationship (25) as a conditional probability, given α. On the other hand, this expression depends on α only through the quantity γ (26), which in turn is proportional to a chi-square variate with $2m$ degrees of freedom. Introducing (27) and (26) into (25) and carrying out the integration with respect to $|\alpha|$ yields

$$P_{\rm d} = 1 - \int_0^\infty \left[\frac{t}{1+t}\sum_{j=0}^{N-2} \frac{m^m\Gamma(m+j)\left(\frac{N\overline{\gamma}t}{s^2(1+t)}\right)^j}{\Gamma(m)j!\left[\frac{N\overline{\gamma}}{s^2(1+t)}+m\right]^{m+j}} \times \sum_{l=j}^{N-2}\binom{l}{j}\left(\frac{1}{1+t}\right)^l\right] f(s)\,ds \qquad (28)$$

where now $\overline{\gamma}$ denotes the average signal-to-noise ratio, namely

$$\overline{\gamma} = \frac{\overline{|\alpha|^2}\|\mathbf{p}\|^2}{2\sigma^2 N}.$$

This relationship can be shown to reduce to (25) in the limit $m \to \infty$, namely as the fluctuation becomes more and more constrained. An important special case is $m = 1$, corresponding to Rayleigh-fluctuating amplitude, which yields:

$$P_{\rm d} = \int_0^\infty \left(1 + \frac{t}{\frac{N\overline{\gamma}}{s^2}+1}\right)^{-N+1} f(s)\,ds. \qquad (29)$$

Notice that if the noise is Gaussian (i.e., for $f(s) = \delta(s-1)$) this equation reduces to the well-known formula for the detection probability of a CA-CFAR with $N-1$ taps.

In Fig. 3 the detection performance of the receiver (18) is reported for K-distributed clutter with varying shape parameter and for the two limiting cases of Rayleigh-distributed and nonfluctuating target amplitude. The influence of the fluctuation law is more thoroughly analyzed in Fig. 4 where, for fixed ν, a set of curves, indexed by the shape parameter m of the target apdf, is shown. The former figure demonstrates that, for the nonfluctuating case, the clutter shape parameter is quite influential on the performance. Conversely, in the presence of Rayleigh-fluctuating targets, the performance is primarily influenced by the fluctuation law in the region of high signal-to-noise ratios, while being mainly ruled by the clutter spikyness for low signal-to-noise ratios. On the other hand, Fig. 4 shows that $m = 1$ and $m = \infty$ represent the

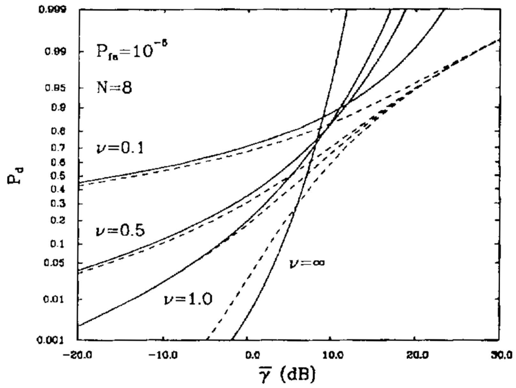

Fig. 3. Detection performance of distribution-free detector in presence of steady (——) and Rayleighian fluctuating (- - -) targets embedded in K-distributed clutter, ν as parameter.

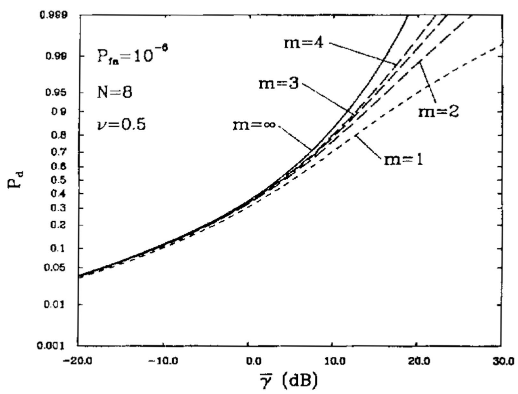

Fig. 4. Detection performance of distribution-free detector in presence of chi-fluctuating targets embedded in K-distributed clutter and several values of chi-distribution shape parameter m.

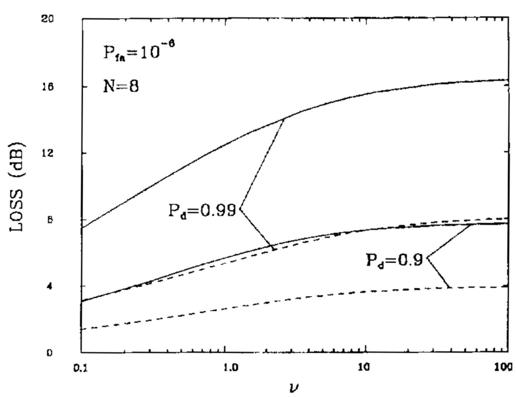

Fig. 5. Fluctuation loss of distribution-free detector versus shape parameter of impinging K-distributed noise, some values of P_d and of fluctuation-law shape parameter m (—— $m = 1$, - - - $m = 2$).

worst and the best case, respectively, so that the curves for intermediate values of m end up with laying between these limit curves. For low signal-to-noise ratios, hence, the actual value of m is *irrelevant* for performance prediction purposes. Such a behavior is somewhat confirmed by the plots of Fig. 5, where

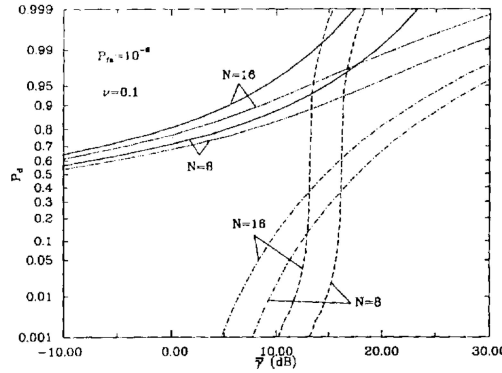

Fig. 6. Detection performance of distribution-free and conventional envelope detectors in presence of steady (distribution-free ——, conventional - - -) and Rayleighian fluctuating (distribution-free · · · · ·, conventional — · — · —) targets embedded in uncorrelated K-distributed clutter, $\nu = 0.1$, $P_{fa} = 10^{-6}$, $N = 8, 16$.

the fluctuation loss, defined as the incremental signal-to-noise ratio necessary for achieving the same detection performance as for nonfluctuating targets, is represented versus the shape parameter ν of the K-distributed clutter for $m = 1$ and $m = 2$; the curves are indexed by the detection probability P_d. As expected, the spikier the clutter, the smaller the fluctuation loss, while lower values of P_d result in "flatter" loss curves.

In Fig. 6 the detector (18) is compared with the matched-filter detector which is tantamount to Kelly's detector in the limit $K \to \infty$. From a practical point of view this comparison is of interest when a good enough estimate of the clutter covariance matrix is available. The operating characteristics of the two receivers (ROCs) refer to both steady and Rayleighian fluctuating targets embedded in uncorrelated K-distributed clutter with $\nu = 0.1$; notice that these ROCs differ from those in correlated clutter only for a detection gain introduced by the whitening transformation [9]. The ROCs of the envelope detector exhibit a marked threshold effect, namely an abrupt transition with increasing $\bar{\gamma}$ from almost undetectability to close-to-one detection probabilities. On the contrary, the ROCs of the detector (18) are quite smooth, at least for relatively low values of the clutter shape parameter. The curves show that the proposed detector largely outperforms the conventional one in the range of interest for P_d ($P_d \leq 0.99$), also in the presence of nonfluctuating targets, provided that the number of integrated pulses is high enough ($N \geq 16$, $\nu = 0.1$). In addition, it should be noticed that a plain conventional detector, unlike detector (18), does not ensure the CFAR property with respect to the clutter power. Otherwise stated, a practical implementation of the envelope detector requires one to account for an additional loss with respect to our receiver which we have not considered in the previous comparison.

IV. CONCLUSIONS

In this paper we have handled the design and the assessment of a robust radar detector in a non-Gaussian environment, namely a detector capable of ensuring fair performance even in the presence of incomplete a priori knowledge about the clutter statistics. Starting upon the experimental evidence that non-Gaussian radar clutter can be modeled as a compound-Gaussian process and developing some previously known results, we derived an asymptotically optimum canonical detector assuming target signal with unknown amplitude and phase. The synthesized receiver is similar to the conventional optimum detector for Gaussian noise, but for a normalization factor appearing in the test statistic and attempting to estimate the average power of the clutter. Relevant to our goal, the detector achieves, upon proper threshold setting, one and the same false alarm rate independent of the clutter marginal distribution. Otherwise stated, the detection threshold can be set once and for all, based solely upon the design false alarm rate, implying that this detector is CFAR with respect to the clutter apdf and, consequently, its power.

Closed-form formulas for both the false alarm and the detection probability are given for the proposed detector. In regard to the detection performance, we contrasted it with the corresponding GLRT receiver. The results show that, for finite sample size, the loss is kept within a few dBs; conversely, as the sample size diverges, the loss tends to zero as a consequence of the asymptotic optimality of the detector. The proposed receiver has also been compared with the matched-filter envelope detector showing that it ensures a satisfactory improvement of performance for relatively high number of integrated pulses.

The proposed approach proves valuable when the clutter distribution is not known in advance. Even so, some criticism might be raised against this detector, in that it assumes perfect knowledge of the structure of the clutter covariance matrix, to be used for designing the whitening filter. A possible next step is to study completely adaptive detection structures, which estimate the clutter covariance matrix, while retaining some appealing characteristics, such as the CFAR property and the asymptotic optimality.

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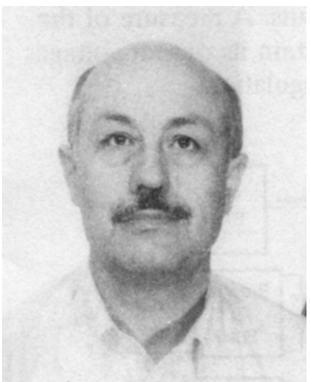

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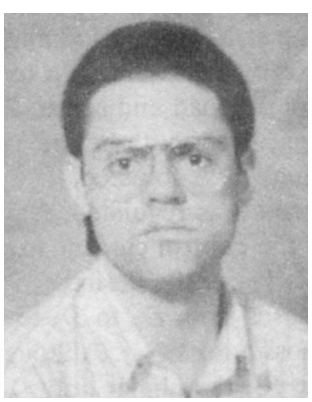

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