Radar detection of signals with unknown parameters in K-distributed clutter

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Indexing terms: Radar, Detection, Clutter

Abstract: The detection of signals with unknown parameters in correlated K-distributed noise, using the generalised Neyman–Pearson strategy is considered. The a priori uncertainty on the signal is removed by performing a maximum likelihood estimate of the unknown parameters. The resulting receivers can be regarded as a generalisation of the conventional detector, but for a zero-memory nonlinearity depending on the amplitude probability density function of the noise as well as on the number of integrated pulses. It is shown that the performance for uncorrelated observations is unaffected by the specific signal pattern, but depends only on the signal-to-noise ratio; moreover, the effect of the clutter correlation on the performance can be accounted for simply by a detection gain. A performance assessment, carried out by computer simulation, shows that the proposed receivers significantly outperform conventional ones as the noise amplitude probability density function markedly deviates from the Rayleigh law. It also shows that the generalised Neyman–Pearson strategy is a suitable means of circumventing the uncertainty on wanted target echos since the operating characteristics of the receivers for the case of signals with unknown parameters closely follow those of the receiver for a completely known signal.

1 Introduction

The theory of radar detection in clutter is well established for the case in which the baseband equivalent of the clutter is a complex Gaussian process, which implies that the amplitude is a Rayleigh variate [1, Chaps. 10, 11]. In such cases the Neyman–Pearson test of optimum detection, in the sense of maximum detection rate for constrained false-alarm rate, can be simply implemented in all instances of practical interest: coherent or incoherent detection, totally or partially known target signal, partial characterisation of the disturbance [1, Chaps. 10, 11]. The latter case, namely that of unknown clutter power, requires additional constant false-alarm rate (CFAR) procedures.

Paper 7832F (E15), first received 26th July and in revised form 4th December 1990

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There are situations, however, where this Gaussian model is not applicable, since the assumption that the received clutter results from a large number of independent and identically distributed elementary scatterers does not hold. In fact, if only a limited number of such scatterers actually contribute to the received clutter echo, as is the case for high resolution and/or low grazing angles, then the measured amplitude probability density function (APDF) exhibits large deviations from the Rayleigh distribution [2, Chap. 13 and references] and is better fitted by families of APDFs, such as the Weibull [3, 4] and the K-distribution [5–8]. Receivers designed under the Gaussian assumption, i.e. conventional receivers, are no longer optimum for such non-Gaussian interference, and their performance may degrade with respect to that predicted according to the Gaussian model.

The problem of radar detection in non-Gaussian clutter — and, in particular, in K-distributed clutter — has already received some attention, at least at the analysis stage. The problem of predicting the performance of both fixed-threshold and ideal CFAR detectors in compound K-distributed noise and the effect of a binary post-detection integration are commanded in Reference 6. This analysis is extended in Reference 9 to account for the presence of thermal noise also, and in Reference 10 the classical cell-averaging CFAR procedure in compound K-distributed plus thermal noise is analysed. The effect of the spatial correlation on the performance is studied in Reference 7, and the performance of the classical noncoherent detector in K-distributed clutter is thoroughly analysed in Reference 11. Finally, a general approach to theoretical performance prediction for both fixed and adaptive threshold detectors is provided in Reference 12.

In contrast, little advance has been made with the task of designing detectors, in the previously cited Neyman–Pearson sense, for given non-Gaussian statistics of the disturbance. Since it lacks the higher order statistical characterisation of clutter as a complex non-Gaussian process, this task is particularly hard when coherent detection is to be considered and/or the correlation of clutter is to be accounted for. As we show here, this difficulty can be overcome by resorting to the theory of spherically invariant random processes (SIRPs), which provides joint PDFs of arbitrarily high order, allowing for independent specification of the APDF and the covariance functions of the clutter [13, 14]. In this context, the spatial correlation of the clutter is not explicitly accounted for, since, as long as one deals with the detection of a target in noise with *known* statistics, it does not play any role. If, conversely, the clutter is known only to belong to the K-distribution family, but no information is available on its statistics, the spatial correlation will be of primary concern, since the *shape* of the APDF turns out to depend upon such correlation [7].

Based on such theory, the design of an optimum detector of a known signal in K-distributed clutter was considered in Reference 15, and a performance assessment was carried out in comparison with the conventional receiver, to show that significant improvements may be achieved by this *ad hoc* design, especially in the case of a marked discrepancy of the actual clutter APDF from the Rayleigh law. In this paper, we extend that work, by introducing and assessing detectors which implement the so-called generalised log-likelihood ratio test (GLLRT) for signals with unknown parameters, to encompass other common instances of target characterisation. More specifically, we consider wanted target echos with known amplitude and unknown phase, and wanted target echos with unknown amplitude and unknown phase.

2 The K-APDF and the SIRP clutter model

The K-APDF, as proposed on physical grounds by Jakeman and Pusey [5], is

$$f_A(r) = \frac{4}{\Gamma(\nu)}\left(\frac{\nu}{\sigma^2}\right)^{\nu+1} r^{\nu} K_{\nu-1}\left(\frac{2\nu r}{\sigma^2}\right) \quad r \geqslant 0 \tag{1}$$

where $\Gamma(\cdot)$ is the Eulerian function, $K_\nu(\cdot)$ is the modified second-kind Bessel function of order ν, σ^2 is the common power of the quadrature components and ν is a shape parameter, ranging from 0 to $+\infty$. The Rayleigh distribution belongs to the family of eqns. 1 as the limiting case $\nu \rightarrow +\infty$; a value of ν in the low range indicates discrepancies from the Rayleigh APDF, mainly in the high-amplitude tail of the distribution.

The assumption underlying the K-distribution model is that the received echo results from a number M of elementary scatterers, which are still K-distributed with one and the same shape parameter: large deviations from the Rayleigh law can be due either to a small number or to a marked spikyness of contributing scatterers or, obviously, to a combination of the two. These properties, and the reported fit to experimental data [6–8], make the K-APDF an excellent candidate as the APDF of the clutter. One additional advantage of the K-APDF is that it is admissible as the APDF of a SIRP [13]. The following short review of relevant properties and interpretations of a SIRP explains this advantage.

Let $x(k) = x_c(k) + jx_q(k)$, $k = \ldots, -1, 0, 1, \ldots$, be a general discrete-time complex process representing the baseband equivalent of a radio-frequency signal. Its Nth-order statistical characterisation amounts to assigning the PDF of the complex N-dimensional (row) vector $\boldsymbol{x} = \boldsymbol{x}_c + j\boldsymbol{x}_q$, namely the joint PDF of the $2N$ entries of the real vector $(\boldsymbol{x}_c, \boldsymbol{x}_q)$. The sequence $x(k)$ is a complex SIRP if, for any N, its Nth order PDF can be cast in the form

$$f_{\boldsymbol{x}}(\boldsymbol{x}) = f_{\boldsymbol{x}_c, \boldsymbol{x}_q}(\boldsymbol{x}_c, \boldsymbol{x}_q) = \frac{1}{(2\pi)^N\sqrt{(|\boldsymbol{M}|)}} h_{2N}(\|\boldsymbol{x} - \boldsymbol{\mu}\|_{\boldsymbol{M}}^2) \tag{2}$$

where $\boldsymbol{\mu}$ is the mean $E[\boldsymbol{x}_c] + jE[\boldsymbol{x}_q] = \boldsymbol{\mu}_c + j\boldsymbol{\mu}_q$, $\boldsymbol{M}$ is the $2N$-dimensional covariance matrix $E[(\boldsymbol{x}_c, \boldsymbol{x}_q)^T(\boldsymbol{x}_c, \boldsymbol{x}_q)]$, $h_{2N}(\cdot)$ is a suitable function depending on N and

$$\|\boldsymbol{x} - \boldsymbol{\mu}\|_{\boldsymbol{M}} = \sqrt{[(\boldsymbol{x}_c - \boldsymbol{\mu}_c, \boldsymbol{x}_q - \boldsymbol{\mu}_q)\boldsymbol{M}^{-1}(\boldsymbol{x}_c - \boldsymbol{\mu}_c, \boldsymbol{x}_q - \boldsymbol{\mu}_q)^T]} \tag{3}$$

is the norm of $\boldsymbol{x} - \boldsymbol{\mu}$ defined by the definite positive matrix $\boldsymbol{M}^{-1}$.

Notice that, for white observations

$$f_{\boldsymbol{x}}(\boldsymbol{x}) = (2\pi)^{-N} h_{2N}(\|\boldsymbol{x} - \boldsymbol{\mu}\|^2) \tag{4}$$

i.e. the norm $\|\boldsymbol{x} - \boldsymbol{\mu}\|_{\boldsymbol{M}}$ reduces to the usual Euclidean norm.

Such a process can be deemed as the product of a complex Gaussian process $g(k)$ times a real non-negative *modulating* variate s, independent of $g(k)$, namely

$$x(k) = sg(k) \tag{5}$$

and hence can be modelled as shown in Fig. 1 (Yao's representation theorem [16]). Here, $w(k)$ is a white

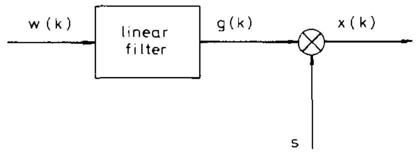

Fig. 1 *SIRP model for a complex non-Gaussian correlated sequence*

Gaussian process, and the linear filter shapes the correlation functions of $g(k)$ as specified by $\boldsymbol{M}$, whereas the multiplier independently induces the non-Rayleigh APDF through the PDF of s, say $f(s)$. Thus a complete specification of the process amounts to assigning the first and second order moments — as for Gaussian processes — plus the auxiliary density $f(s)$ [13, 14]. Note that, if we feed the process, eqn. 5, to a linear system, the output process is still conditionally Gaussian, as a consequence of the closure property of Gaussian processes, and the PDF of the modulating variate s remains unchanged. Thus the SIRPs are closed with respect to linear transformations.

The model of eqn. 5 has a physical interpretation in the light of the composite scattering theory, according to which the predetection clutter can be viewed as a Gaussian random process, resulting from diffusion by a large number of elementary scatterers, modulated by a highly correlated process which accounts for the gross reflectivity characteristics of the illuminated patch. If this compound process is observed by windows much shorter than the average decorrelation time of the modulating process, so that the latter is constant inside any window, then the model of eqn. 5 applies with the modulating process degenerating into the modulating variate s, changing from window to window. With this approximation the correlation properties of the compound process coincide with those of the underlying Gaussian process apart from a possible scale factor, namely the mean square value of s, which we will assume hereafter, without loss of generality, to be unity. Besides its physical meaning, the model of Fig. 1 lends itself readily to computer simulation procedures, allowing for independent control of the APDF, through the PDF $f(s)$, and of the correlation properties, which can be accommodated by suitable design of the linear filter.

The special case of a correlated complex SIRP with K-APDF is a zero-mean process, characterised by the Nth order PDF [13]:

$$f_{\boldsymbol{x}}(\boldsymbol{x}) = \frac{[\sqrt{(2\nu)}\|\boldsymbol{x}\|_{\boldsymbol{M}}]^{\nu-N}}{2^{\nu-1}\Gamma(\nu)\pi^N|\boldsymbol{M}|^{1/2}} K_{\nu-N}[\sqrt{(2\nu)}\|\boldsymbol{x}\|_{\boldsymbol{M}}] \tag{6}$$

and by a generalised-chi $f(s)$, namely

$$f(s) = \frac{2\nu^\nu}{\Gamma(\nu)} s^{2\nu-1} \exp(-\nu s^2) \quad s \geqslant 0 \tag{7}$$

This specific SIRP model for the received clutter echo has been validated in References 6–8.

Summing up, the characterisation of clutter as a complex SIRP is convenient in view of the synthesis of optimum receivers since

(*a*) The clutter model is compatible with the assumption of K-APDF eqn. 1

(*b*) arbitrary correlation properties can be forced upon the clutter quadrature components

(*c*) multivariate PDFs of the clutter of any order can be derived.

3 Detection in K-distributed clutter

The problem of detecting a known signal embedded in an additive disturbance (clutter) can be stated in terms of the following hypotheses test

$$\begin{cases} H_0: \boldsymbol{z} = \boldsymbol{c} \\ H_1: \boldsymbol{z} = \boldsymbol{v} + \boldsymbol{c} \end{cases} \tag{8}$$

where $\boldsymbol{z} = \boldsymbol{z}_c + j\boldsymbol{z}_q$, $\boldsymbol{v} = \boldsymbol{v}_c + j\boldsymbol{v}_q$ and $\boldsymbol{c} = \boldsymbol{c}_c + j\boldsymbol{c}_q$ are complex vectors whose components are samples from the baseband equivalent of the received signal, the target signal and the clutter, respectively. Since, in general, the clutter signal is a non-white zero-mean process, $\boldsymbol{c}$ has a nondiagonal covariance matrix $\boldsymbol{M}$. The theorem of reversibility [19, p. 289] ensures, however, that there is no penalty in performance if the vector $\boldsymbol{c}$ is whitened by transforming the received vector $\boldsymbol{z}$ through

$$(\boldsymbol{r}_c, \boldsymbol{r}_q) = (\boldsymbol{z}_c, \boldsymbol{z}_q)\boldsymbol{E}\boldsymbol{D}^{-1/2} \tag{9}$$

where $\boldsymbol{E}$ is the matrix of the eigenvectors of $\boldsymbol{M}$, and $\boldsymbol{D}$ is the diagonal matrix of its eigenvalues [17, pp. 32–36]. Thus, the problem of detecting the complex signal $\boldsymbol{v} = \boldsymbol{v}_c + j\boldsymbol{v}_q$ in correlated noise is equivalent to that of detecting its filtered version $\boldsymbol{u} = \boldsymbol{u}_c + j\boldsymbol{u}_q$ in uncorrelated noise, say $\boldsymbol{n}$, with

$$(\boldsymbol{u}_c, \boldsymbol{u}_q) = (\boldsymbol{v}_c, \boldsymbol{v}_q)\boldsymbol{E}\boldsymbol{D}^{-1/2} \tag{10}$$

and leads to the hypotheses test

$$\begin{cases} H_0: \boldsymbol{r} = \boldsymbol{n} \\ H_1: \boldsymbol{r} = \boldsymbol{u} + \boldsymbol{n} \end{cases} \tag{11}$$

The structure of the optimum detector for problem 11 depends upon the degree of the a priori knowledge about the signal. In what follows we account for possible uncertainties in the phase θ and the amplitude A of the signal. For example, the wanted target echo may be a coherent pulse train with unknown initial phase and unknown attenuation A uniformly affecting all pulses of the train. Thus, it will be convenient to elicit the possible unknown parameters by writing $\boldsymbol{u} = Ae^{j\theta}\boldsymbol{p}$.

Optimum detection, in the Neyman–Pearson sense, of a completely known signal in the presence of noise with PDF $f_n(\cdot)$ is accomplished by the log-likelihood ratio test (LLRT):

$$\Lambda = \log\left[\frac{f_n(\boldsymbol{r} - Ae^{i\theta}\boldsymbol{p})}{f_n(\boldsymbol{r})}\right] \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{12}$$

This is obviously the most favourable among all instances of a priori knowledge about the signal: the operating characteristics of the test 12 are unbeatable by any other test to detect partially known signals, given the noise model $f_n(\cdot)$. This bound will be referred to as the 'perfect measurement bound'.

If the wanted target echo is partially known, then the detection problem is to discriminate a *composite* hypothesis H_1 against a simple alternative H_0. A distribution-free solution to the problem is the GLLRT [18, Chap. 9]:

$$\Lambda = \max_{\Omega} \log\left[\frac{f_n(\boldsymbol{r} - A\boldsymbol{p}e^{j\theta})}{f_n(\boldsymbol{r})}\right] \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{13}$$

where Ω is the space of the unknown parameters; more precisely it is the space of the phase θ in the case of unknown phase and the space of the pair (A, θ) in the case of unknown amplitude and phase. This test is equivalent to the folowing one:

$$\Lambda = \log\left[\frac{f_n(\boldsymbol{r} - \widehat{Ae^{j\theta}}\boldsymbol{p})}{f_n(\boldsymbol{r})}\right] \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{14}$$

i.e. it can be obtained from the LLRT by replacing the unknown parameters by their maxmum likelihood (ML) estimates, denoted by the symbol $\widehat{(\cdot)}$.

The operating characteristics of the test 14 are tied to the accuracy of the ML estimators. Little can be anticipated in general about this accuracy, except for the following asymptotical properties [19, p. 71]:

(*a*) ML estimates are consistent, i.e. they converge in probability to the true value as the sample size increases to infinity

(*b*) ML estimates are asymptotically efficient, i.e. they aproach the Cramer-Rao bound as the sample size or the signal-to-noise ratio increases to infinity.

A more canonical approach could be followed if the signal parameters were modelled as random variates, distributed according to a known PDF. In this case the optimum test, in the Neyman–Pearson sense, can be obtained by averaging the conditional likelihood ratio $L(\boldsymbol{r}|\boldsymbol{a})$ with respect to the distribution of the signal parameters. Hence, letting $p(\boldsymbol{a})$ be the multivariate PDF of the parameters with values in Ω, the Neyman–Pearson test is

$$\int_{\Omega} L(\boldsymbol{r}|\boldsymbol{a})p(\boldsymbol{a})d\boldsymbol{a} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{15}$$

Although the two approaches are different, they yield similar tests when $p(\boldsymbol{a})$ is smooth, since, in such a case, the most significant contribution to the integral in expr. 15 still comes from the region of Ω where $L(\boldsymbol{r}|\boldsymbol{a})$ is maximum. We stress that the former approach is preferable in our setup since it is independent of the distribution of the parameters.

3.1 Detection of a known signal in K-distributed noise

From now on we assume that the noise vector has the K-APDF of eqn. 1 and obeys the SIRP model, whence its multivariate PDF is given by eqn. 6. Owing to the above properties of SIRPs with respect to linear transformations, the whitening transformation of eqn. 9 does not affect the K-APDF and the SIRP model apart from the diagonalisation of the covariance matrix $\boldsymbol{M}$. Therefore, the covariance matrix $\boldsymbol{M}$ reduces to the identity matrix of order $2N$ and the Nth order PDF of the noise becomes

$$f_x(\boldsymbol{x}) = \frac{[\sqrt{(2\nu)}\|\boldsymbol{x}\|]^{\nu-N}}{2^{\nu-1}\Gamma(\nu)\pi^N} K_{\nu-N}[\sqrt{(2\nu)}\|\boldsymbol{x}\|] \tag{16}$$

Substituting this into expr. 12, the Neyman–Pearson test can be given the form [15]

$$g_\nu(\|\boldsymbol{r}-\boldsymbol{u}\|) - g_\nu(\|\boldsymbol{r}\|) \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{17}$$

where

$$g_\nu(x) = \log\,[x^{\nu-N} K_{\nu-N}(\sqrt{(2\nu)}x)] \tag{18}$$

The block diagram of the optimum processor implementing this test is shown in Fig. 2. The distances $q_0 = \|\boldsymbol{r}\|$

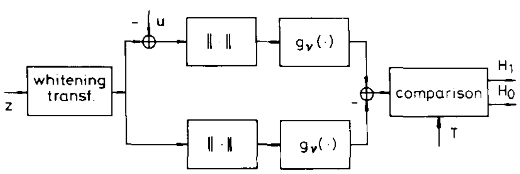

Fig. 2 *Optimum coherent detector in K-distributed clutter*

and $q_1 = \|\boldsymbol{r}-\boldsymbol{u}\|$ are computed and warped through the zero-memory nonlinearity (ZMNL) (eqn. 18); then the difference between the warped distances is compared to the detection threshold T. We stress that the ZMNL, and hence the receiver structure, depends upon the number N of integrated pulses.

A relevant feature of this detection scheme is that the PDF of the test statistic depends upon the signal-to-noise ratio (SNR)

$$\text{SNR} = \frac{\|\boldsymbol{u}\|^2}{E[\|\boldsymbol{n}\|^2]} = \frac{A^2\|\boldsymbol{p}\|^2}{2N} \tag{19}$$

but otherwise it is independent of the signal pattern $\boldsymbol{p}$.

Since any noise in the SIRP class is conditionally Gaussian given the modulating variate s (see eqn. 5), it is enough to prove the statement with reference to Gaussian white noise.

Let us decompose $\boldsymbol{r}$ into its component along $\boldsymbol{p}$ and into the othogonal component, namely

$$\boldsymbol{r} = \boldsymbol{r}_\perp + \frac{\boldsymbol{r}\cdot\boldsymbol{p}}{\|\boldsymbol{p}\|^2}\boldsymbol{p} = \boldsymbol{r}_\perp + r_p\frac{\boldsymbol{p}}{\|\boldsymbol{p}\|} \tag{20}$$

Here $(\cdot)$ denotes the dot product between the complex row vectors $\boldsymbol{r}$ and $\boldsymbol{p}$. As $\boldsymbol{r}_\perp$ contains noise only, it does *not* depend upon the signal under both hypotheses. Then, we can write

$$q_1^2 = \|\boldsymbol{r}_\perp\|^2 + |r_p - A\|\boldsymbol{p}\|e^{j\theta}|^2$$
$$q_0^2 = \|\boldsymbol{r}_\perp\|^2 + |r_p|^2 \tag{21}$$

Conditioned upon $\|\boldsymbol{r}_\perp\|^2 = y$, we have, under the hypothesis H_0

$$H_0: \begin{cases} q_1^2 = y + |n_p - A\|\boldsymbol{p}\|e^{j\theta}|^2 \\ q_0^2 = y + |n_p|^2 \end{cases} \tag{22}$$

and, under the hypothesis H_1

$$H_1: \begin{cases} q_1^2 = y + |n_p|^2 \\ q_0^2 = y + |n_p + A\|\boldsymbol{p}\|\,e^{j\theta}|^2 \end{cases} \tag{23}$$

where obviously n_p denotes the projection of the noise along the signal. The modulus $|n_p|$ does not depend upon the signal. Moreover, we have $|n_p \pm A\|\boldsymbol{p}\|\,e^{j\theta}| = |n_p e^{-j\theta} \pm A\|\boldsymbol{p}\|\,| = |n_p \pm A\|\boldsymbol{p}\|\,|$, since, owing to the circular invariance of the nose, the phase shift θ can be absorbed in the uniformly distributed phase of n_p. Thus, both under H_0 and under H_1 the joint distribution of q_0 and q_1, and hence the test statistic expr. 17, does not depend on the signal pattern but only on the energy of the received signal, namely on the SNR (eqn. 19).

Two more remarks are in order. First, the distribution of the test statistic is independent of the phase θ under both hypotheses. Secondly, the distribution of the test statistic depends upon the norm of the wanted target echo, not only under the H_1 but also under the H_0 hypothesis. Therefore, unlike the conventional detector, the threshold setting to achieve a given false alarm rate must account for both the useful target strength and the average noise power.

In keeping with the fact that the K-APDF (eqn. 1) reduces to a Rayleigh distribution as $\nu \to +\infty$, the receiver implementing the test of expr. 17 reduces to the conventional receiver as $\nu \to +\infty$. In fact, the limiting expression for the nonlinearity in expr. 17, after suitable normalisation and neglecting additive constants, is

$$g_\infty(x) = -\frac{x^2}{2} \tag{24}$$

which leads to the so-called minimum-distance decision rule.

3.2 Detection of unknown phase signals in K-distributed noise

The detection of known-amplitude, unknown-phase wanted-target echo is handled via the GLLRT (expr. 13), with Ω the space of the unknown phase θ. The complex N-dimensional vector is conditionally Gaussian, given the modulating variate s, which implies that $(\boldsymbol{r}_c, \boldsymbol{r}_q)$ is a real $2N$-dimensional conditionally Gaussian vector, distributed as $\mathcal{N}(\boldsymbol{0}, s^2\boldsymbol{I})$ under H_0, and as $\mathcal{N}((\boldsymbol{u}_c, \boldsymbol{u}_q), s^2\boldsymbol{I})$ under H_1. Thus, the GLLRT is written as

$$\log \frac{\displaystyle\max_{\theta\in(-\pi,\pi)} \int_0^{+\infty} \frac{1}{s^{2N}} \exp\left[-\frac{\|\boldsymbol{r} - Ae^{j\theta}\boldsymbol{p}\|^2}{2s^2}\right] f(s)\,ds}{\displaystyle\int_0^{+\infty} \frac{1}{s^{2N}} \exp\left[-\frac{\|\boldsymbol{r}\|^2}{2s^2}\right] f(s)\,ds} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{25}$$

with $f(s)$ as given in eqn. 7. As the exponent in the numerator of expr. 25 can be written as

$$\|\boldsymbol{r} - Ae^{j\theta}\boldsymbol{p}\|^2 = \|\boldsymbol{r}\|^2 + A^2\|\boldsymbol{p}\|^2 - 2A\,|\boldsymbol{r}\cdot\boldsymbol{p}|\cos(\phi - \theta) \tag{26}$$

with ϕ the phase of the dot product $\boldsymbol{r}\cdot\boldsymbol{p}$, the ML estimate $\hat{\theta}$ is clearly given by ϕ. Substituting into expr. 25 leads to the test

$$g_\nu(\sqrt{(\|\boldsymbol{r}\|^2 + A^2\|\boldsymbol{p}\|^2 - 2A\,|\boldsymbol{r}\cdot\boldsymbol{p}|)}) - g_\nu(\|\boldsymbol{r}\|) \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{27}$$

with g_ν given in expr. 17.

The GLLRT detector implementing this test is depicted in Fig. 3. Note that the receiver structure is the

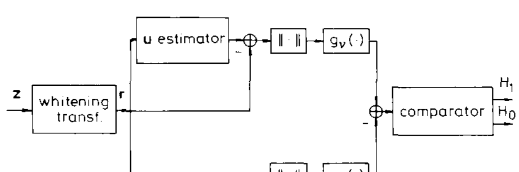

Fig. 3 *GLLRT detector for target with unknown parameters in K-distributed noise*

same as in Fig. 2, but with $\boldsymbol{u}$ replaced by its estimate $\hat{\boldsymbol{u}} = A \exp(j\hat{\theta})\boldsymbol{p}$. Thus the receiver can still be regarded as a generalisation of the minimum-distance detector, but now q_1 represents the distance between the received signal and the estimate of the wanted target echo.

In this case too, the test statistic, under both hypotheses, depends upon SNR only. In fact, based on the decomposition of eqn. 20, we have the same expression of q_0^2 as in eqns. 21, and we have

$$q_1^2 = \|\boldsymbol{r}_\perp\|^2 + |r_p - A\|\boldsymbol{p}\|e^{j\hat{\theta}}|^2 = \|\boldsymbol{r}_\perp\|^2 + (|r_p| - A\|\boldsymbol{p}\|)^2 \quad (28)$$

where we have used the fact that $\angle r_p = \hat{\theta}$. Thus, the same arguments and remarks as in Section 3.1 apply here.

In the case $\nu \to +\infty$, as $g_\nu(\cdot)$ assumes its limiting form (eqn. 24), the test of expr. 27 reduces to

$$|\boldsymbol{r} \cdot \boldsymbol{p}| \underset{H_0}{\overset{H_1}{\gtrless}} T \quad (29)$$

namely to the conventional decision test based on the envelope at the output of a matched filter. Had we modelled the initial phase as a random variate, uniformly distributed in $(-\pi, \pi)$, rather than an unknown parameter, we would have ended up with the same decision test as expr. 29. Thus, the solution to the detection problem for Gaussian noise is the same, whether the phase is considered to be uniform random variate or an unknown parameter.

Instead, for non-Gaussian noise, and in particular for K-distributed noise, the two approaches yield different tests, even if the phase is uniform. However, from the discussion following expr. 15, if θ is uniformly distributed in $(-\pi, \pi)$, the test of expr. 27 is approximately equivalent to the test resulting from the average expr. 15.

3.3 Detection of unknown amplitude, unknown phase signals in K-distributed noise

When both A and θ are unknown parameters the GLLRT of expr. 13 can be written as

$$\log \frac{\max\limits_{(\theta, A)} \int_0^{+\infty} \frac{1}{s^{2N}} \exp\left[-\frac{\|\boldsymbol{r} - \alpha\boldsymbol{p}\|^2}{2s^2}\right] f(s)\, ds}{\int_0^{+\infty} \frac{1}{s^{2N}} \exp\left[-\frac{\|\boldsymbol{r}\|^2}{2s^2}\right] f(s)\, ds} \underset{H_0}{\overset{H_1}{\gtrless}} T \quad (30)$$

where

$$\alpha = Ae^{j\theta} \quad (31)$$

Resorting to the decomposition of eqn. 20, the norm in the numerator is written as

$$\|\boldsymbol{r} - \alpha\boldsymbol{p}\|^2 = \|\boldsymbol{r}_\perp\|^2 + |r_p - \alpha\|\boldsymbol{p}\||^2 \quad (32)$$

and attains its minimum value for

$$\hat{\alpha} = \frac{r_p}{\|\boldsymbol{p}\|} = \frac{\boldsymbol{r} \cdot \boldsymbol{p}}{\|\boldsymbol{p}\|^2} \quad (33)$$

which represents the ML estimate of α. Substituting eqn. 33 into expr. 30 and taking the logarithm leads to the test

$$g_\nu\left[\sqrt{\left(\|r\|^2 - \frac{|\boldsymbol{r} \cdot \boldsymbol{p}|^2}{\|\boldsymbol{p}\|^2}\right)}\right] - g_\nu(\|\boldsymbol{r}\|) \underset{H_0}{\overset{H_1}{\gtrless}} T \quad (34)$$

Hence, the GLLRT detector performs an estimate of the actual target signal, namely

$$\hat{\boldsymbol{u}} = \hat{\alpha}\boldsymbol{p} \quad (35)$$

and therefore it is the same as that outlined in Fig. 3. In particular, notice that the nonlinearity is one and the same for all cases.

In this case too, by similar arguments as for the previous cases, the PDF of the test statistic is seen to depend upon the SNR only. In fact, the square distance q_1^2 between the received and the estimated signal can be written as

$$q_1^2 = \left\|\boldsymbol{r} - \frac{\boldsymbol{r} \cdot \boldsymbol{p}}{\|\boldsymbol{p}\|^2}\boldsymbol{p}\right\|^2 = \|\boldsymbol{r}_\perp\|^2 \quad (36)$$

while the norm q_0^2 (eqn. 21) can be written as

$$q_0^2 = \|\boldsymbol{r}_\perp\|^2 + |r_p|^2 = q_1^2 + |r_p|^2 \quad (37)$$

The claimed property then follows from the fact that $\boldsymbol{r}_\perp$ does not contain the wanted target echo, while the PDF of $|r_p|^2$ is independent of the signal pattern under both hypotheses.

Unlike the previous two cases, the threshold setting for the test of expr. 34 to achieve a given false alarm rate is independent of the SNR since, under H_0, both q_1 and q_0 depend on noise only.

Owing to the asymptotical properties of 'unbiasedness' and efficiency of ML estimates, the performance of the GLLRT detector approaches the perfect measurement bound as the size N of the integrated sample increases to infinity. For finite N, it can be shown that

(*a*) $E[\hat{\alpha} | H_0] = 0$, $E[\hat{\alpha} | H_1] = \alpha$, i.e. $\hat{\alpha}$ is unbiased under both hypotheses for any N

(*b*) $\text{Var}[\hat{\alpha} | H_0] = \text{Var}[\hat{\alpha} | H_1] = 2/\|\boldsymbol{p}\|^2$, i.e. $\hat{\alpha}$ is consistent, since $\|\boldsymbol{p}\|^2$ is proportional to N

(*c*) if the predetection noise is Gaussian, $\hat{\alpha}$ is an *efficient* estimate, and in fact attains the Carmer–Rao bound [19, p. 66].

Note that, as $\nu \to +\infty$, as the noise converges to the Gaussian distribution, detector of expr. 34 uses $g_\infty(\cdot)$ (eqn. 24) and hence reduces to the conventional envelope detector:

$$\frac{|\boldsymbol{r} \cdot \boldsymbol{p}|^2}{\|\boldsymbol{p}\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} T \quad (38)$$

Had we modelled A and θ as random variates, the former with given PDF $p(A)$ and the latter uniformly distributed in $(-\pi, \pi)$, we would have obtained the same result as in expr. 38 apart from a scale factor which can be included in the threshold. This behaviour of the GLLRT was also observed when dealing with a target with unknown phase in Gaussian noise, so that the a priori uncertainty in the signal amplitude has no influence on the structure of the detector. One possible justification is that the envelope detector performs the LLRT whenever the signal phase is uniformly distributed in $(-\pi, \pi)$, independently of the amplitude distribution [19]. On the other hand, performing the LLRT in the case of uniformly distributed phase is perfectly equivalent to performing the GLLRT in the case of unknown (nonrandom) initial phase. Therefore, since the signal amplitude — be it a random or deterministic constant — does not influence the structure of the LLRT receiver, a similar behaviour is expected for the GLLRT detector.

4 Performance assessment

This Section is devoted to the assessment of the performances of the detectors introduced above. In particular, we investigate to what extent they depend upon the clutter shape parameter and on the number of integrated pulses. We also evaluate the loss due to the a priori uncertainty in the wanted target echo by direct comparison with the perfect measurement bound. Finally, we compare the proposed receivers with the corresponding conventional envelope detector.

The analysis of the GLLRT detectors (exprs. 27 and 34) is carried out with reference to the case of uncorrelated clutter. In this situation, as shown in the previous Section, the PDF of the test statistic depends only upon the SNR under both hypotheses. Therefore, owing to the closure of SIRPs under linear transformations, the performance in correlated clutter can be read off the receiver operating characteristics (ROCs) in uncorrelated clutter by simply interpreting the abscissas as the signal-to-noise ratios at the output of the whitening transformation.

Since a closed-form expression for the PDF of the test statistic is not available, the performance is obtained by computer simulations. We adopted the Monte-Carlo counting procedure for estimating the detection probabilities. We resorted to an extrapolative procedure based on extreme value theory [20] for setting the threshold T corresponding to a fixed false-alarm probability (P_{fa}) in the range (10^{-4}, 10^{-8}). This procedure requires the generation and processing of only about 10^5 clutter patterns to obtain reasonably accurate estimates of T.

The ROCs of the GLLRT detector for a signal with unknown phase and for a signal with unknown amplitude and phase are reported in Figs. 4 and 5 for several

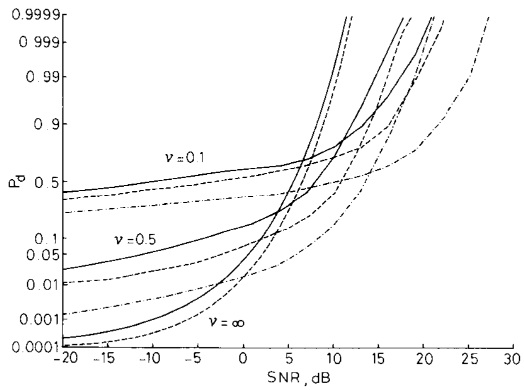

Fig. 4 *Operating characteristics of GLLRT in K-distributed noise for $N = 2$ and v as a parameter*

——— known signal
– – – – known amplitude, unknown phase signal
—·—· unknown amplitude, unknown phase signal
$P_{fa} = 10^{-4}$

values of the shape parameter v, and for $N = 2$ and $N = 4$ integrated pulses, respectively. In these Figures, the ROCs for a perfectly known target signal are also shown for comparison.

The shape parameter is seen to affect significantly the performances. In fact, for $v = \infty$, i.e. for Gaussian clutter, ROCs exhibit a marked *threshold effect*: that is, a sharp transition with increasing SNR from almost undetectability to close-to-one detection rate. As v decreases, the threshold effect is progressively smoothed, so that the ROCs cross each other. Thus, in spiky clutter (low v), the detectability of weak signals is enhanced, at the price of a certain loss of detectability of strong signals.

The influence of the number N of integrated pulses on the performance can be studied through the plots of Fig. 6. As N increases, since the integration gain is a non-decreasing function of N, the performances for all of the three target models improve noticeably.

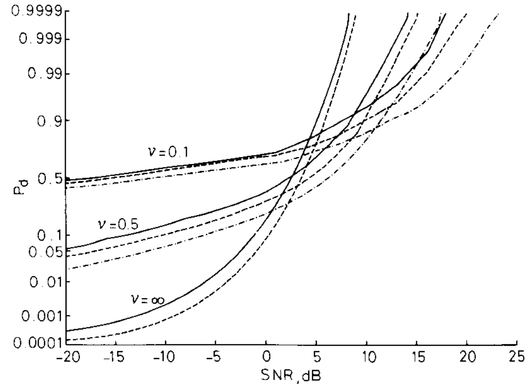

Fig. 5 *Operating characteristics of GLLRT in K-distributed noise for $N = 4$ and v as a parameter*

——— known signal
– – – – known amplitude, unknown phase signal
—·—· unknown amplitude, unknown phase signal
$P_{fa} = 10^{-4}$

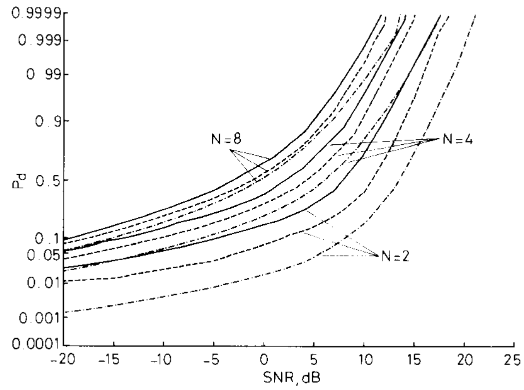

Fig. 6 *Operating characteristics of GLLRT in K-distributed noise for $v = 0.5$ and N as a parameter*

——— known signal
– – – – known amplitude, unknown phase signal
—·—· unknown amplitude, unknown phase signal
$P_{fa} = 10^{-4}$

The plots of Figs. 4, 5 and 6 all refer to one and same P_{fa}. However, with changing P_{fa}, the effects of the shape parameter and of the number of integrated pulses on the performances remain the same.

The loss due to the a priori uncertainty on the wanted target echo can also be assessed from Figs. 4 to 6 as the horizontal displacement between the ROCs for a completely known signal and the corresponding ROCs for signals with unknown parameters. The detection loss can, in fact, be defined as the incremental SNR required to obtain, for a certain signal model with unknown parameters, the same detection rate as for a completely known signal. This detection loss is affected by signal strength. It is in fact significant for weak signals, but otherwise it can be considered as negligible. This is more true as N increases, as a consequence of the consistency and unbiasedness of the ML estimates. In any case, the noticeable detection loss in the region of weak signals is due to the flatness of the ROCs for low SNRs, but the decrease of P_d for a given SNR is not very significant.

Relative to one another, the detection loss for the case of unknown phase and that for the case of both unknown phase and unknown amplitude also depend upon the shape parameter v. In particular, for very low v, the uncertainty in the amplitude contributes significantly to the loss, whereas for moderate to high values of v losses are essentially the same whether the amplitude is known or unknown. This might be explained by the fact that, as v increases, the GLLRT detectors converge towards the same conventional envelope detector. Therefore, since the performances are not affected by the actual signal phase, but only by the SNR, the detection losses for the two target models tend to coincide as v increases to infinity.

Overall, the detection loss is negligible in most cases of interest; this is an indirect confirmation that the strategy of estimating the unknown parameters is suitable means of circumventing the uncertainty of such parameters.

For a comparison between the GLLRT detectors and the conventional envelope detector we refer to Fig. 7,

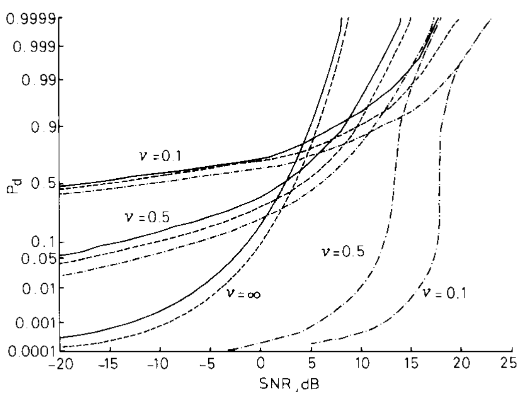

Fig. 7 *Performance comparison between GLLRT and conventional detectors for N = 4 and v as a parameter*

——— optimum detector for completely known signal
- - - - GLLRT detector for signal with unknown phase
· -·- GLLRT detector for signal with unknown amplitude and phase
·····—· conventional envelope detector

where several values of v are considered. Obviously, as v increases, the performances tend to be close to one another since, in fact, the two receivers tend to become equivalent (see Section 3). However, as v decreases, the newly proposed detectors offer a definite advantage over the conventional receiver. For very low values of v, the advantage extends over the whole range of P_d's of interest; otherwise it is significant only in detecting weak signals. Also note that the threshold effect is much sharper for the conventional detector than for the GLLRT detectors, especially as v decreases.

As pointed out above, the ROCs of Figs. 4–7 can be used for evaluating the performance in correlated clutter, provided that the relationship between the SNR at the output and at the input of the whitening tansformation is known. More precisely, the effective SNR in decibels, say SNR_o, to enter with in Figs. 4–7 is

$$SNR_o = SNR_i + G \tag{39}$$

where SNR_i denotes the signal-to-noise ratio prior to the whitening transformation and G is the SNR gain in decibels introduced by the transformation (detection gain).

For the sake of simplicity, we refer to the case of wide-sense stationary clutter, i.e. we assume that the quadrature components of the received disturbance have the same autocorrelation and zero crosscorrelation. Moreover, we assume that the wanted target echo, prior to the whitening transformation, is a coherent train, i.e.

$$v_k = A \exp(j\theta) \exp[j2\pi F_d(k-1)] \quad k = 1, \ldots, N \tag{40}$$

where F_d is the target Doppler shift normalised to the pulse repetition frequency (PRF). In Figs. 8 and 9 we

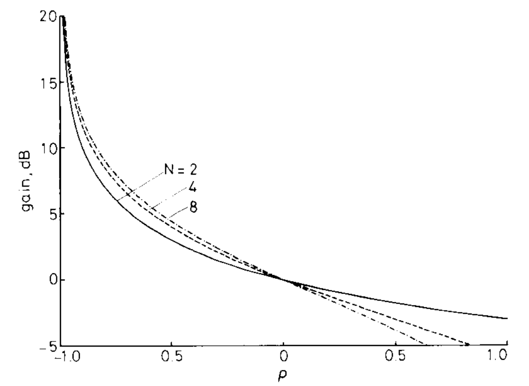

Fig. 8 *Detection gain for a coherent train in exponentially correlated clutter against the one-lag correlation coefficient with N as a parameter*

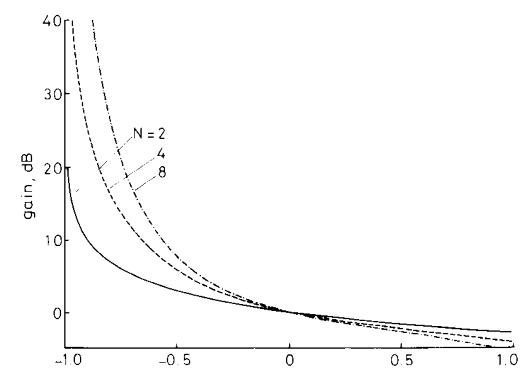

Fig. 9 *Detection gain for a coherent train in clutter with Gaussian-shaped correlation against the one-lag correlation coefficient with N as a parameter*

report the plots of the detection gain G for $N = 2$, $N = 4$ and $N = 8$ against the one-lag correlation coefficient ρ with $F_d = 0$. Fig. 8 refers to exponential autocorrelation, while Fig. 9 refers to Gaussian-shaped autocorrelation. In both cases, we observe that the whitening transformation produces a loss if ρ is positive and a gain if ρ is negative. The amount of gain or loss increases with the absolute value of ρ as well as with the number of integrated pulses.

This behaviour is explained by considering the power spectra of signal and clutter. The spectrum of the zero-Doppler target signal is a train of spectral lines spaced apart by the PRF. For $\rho > 0$, the clutter power spectrum also consists of spectral lines spaced apart by the PRF; the shape of each line depends upon the autocorrelation of the clutter, whereas its width decreases with ρ, resulting in a flat spectrum for $\rho = 0$. For $\rho < 0$ the clutter spectrum has the same shape as before, but with a shift of 0.5 PRF. Therefore, for $\rho > 0$, the spectra of the clutter and of the useful echo overlap, resulting in a 'masking effect' which worsens as ρ increases. For $\rho < 0$, on the other hand, the spectral lines of the signal and the clutter are separated by 0.5 PRF; thus, as the absolute value of ρ increases, the signal can be better detected. In other words, whatever the autocorrelation function of the clutter, assuming a zero-Doppler target signal immersed in a zero-Doppler correlated clutter with $\rho < 0$ is exactly equivalent to assuming that the clutter possesses a Doppler frequency of 0.5 PRF with a positive one-lag correlation coefficient $|\rho|$.

5 Conclusions

We have shown that optimum detection of signals in K-distributed clutter is feasible once the SIRP model for the clutter is adopted. The main addition to the conventional receiver (that for Rayleigh clutter) is an easily computable zero-memory nonlinearity, which is required to warp the signal space metric according to the amplutude PDF of the clutter. This result is consistent with the known results for detection in Rayleigh clutter, since the nonlinearity has no effect as the K-distribution converges with increasing ν to the Rayleigh one.

Other esults presented here also turn out to be natural, though not obvious, extensions of known facts regarding detection in Rayleigh clutter. For example, the operating characteristics in uncorrelated clutter can be given in terms of the signal-to-noise ratio only, as the actual signal pattern is unimportant. The case of correlated clutter can be handled by whitening transformations at the design stage, and hence by a detection gain at the analysis stage.

One remarkable difference with respect to detection in Rayleigh clutter is that one of the two approaches to handle the possible uncertainty about the distribution of the unknown parameters, is no longer feasible. In fact, unlike the case of Rayleigh disturbance, there is no test with respect to signal amplitude which is the most powerful in all cases, even when the phase is assumed to be uniform. Thus the average likelihood ratio test, and hence the receiver structure, depends upon the amplitude distribution. The other approach to circumvent the uncertainty in the signal parameters, namely to introduce their estimates in the likelihood ratio test, is instead distribution-free by nature and, therefore, is more preferable in our set-up. The resulting detector is also robust against the actual phase signal, in the sense that its performance is independent of whether the signal phase is modelled as a random variate with arbitrary PDF or as an unknown nonrandom parameter.

Regarding the disturbance, the receiver structure, and in particular the zero-memory nonlinearity, is dependent upon the shape parameter of the K-distribution. In some cases, this parameter may be assumed to be known at least approximately. For example, the values of the shape parameter for sea clutter have been expressed as functions of the grazing angle, the across-range resolution, the polarisation and a suitable aspect factor [21]. In particular, the impact of the spatial correlation on the actual shape parameter of the received disturbance for varying across-range resolution has been analysed in Reference 7 with respect to sea clutter. Hence, such correlation might be accounted for both at a design stage, by keying the zero-memory nonlinearity to the actual value of ν, and at an analysis stage, by entering the ROCs with the predicted value of the shape parameter.

Moreover, from the performance analysis of the previous section, if can be informed that the actual value of ν does not critically affect the performance, so that even a rough knowledge of the shape parameter may suffice to select the zero-memory nonlinearity. Thus, one might conceive an adaptive processor, namely one capable of switching among a small number of zero-memory nonlinearities according to either previously known or estimated values of the shape parameter.

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