Robust Waveform Design for MIMO Radars

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***Abstract*—The problem of robust waveform design for multiple-input, multiple-output radars equipped with widely spaced antennas is addressed here. Robust design is needed as a number of parameters may be unknown, e.g., the target scattering covariance matrix and the disturbance covariance matrix. Following a min-max approach, the code matrix is designed to minimize the worst-case cost over all possible target (or target and disturbance) covariance matrices. The same min-max solution applies to many commonly adopted performance measures, such as the average signal-to-disturbance ratio, the linear minimum mean square error in estimating the target response, the mutual information between the received signal echoes and the target response, and the approximation of the detection probability in the high- and low-signal regimes for a fixed probability of false alarm. Examples illustrating the behavior of the min-max codes are provided.**

***Index Terms*—Chernoff's bound, linear minimum mean square error, MIMO radar, min-max, minimax, mutual information, robust waveform design.**

I. Introduction

CONSIDER the $M \times L$ multiple-input multiple-output (MIMO) radar architecture of Fig. 1 and assume that a target—with extension V and V' in the transmit and receive sensor alignment direction, respectively—is present, d and d' being the sensor spacing at the transmitter and receiver. Assume that the time delays of the different paths between each transmit antenna and the target are not resolvable, which in turn implies that the bandwidth of the transmitted waveforms is smaller than $\frac{c}{(M-1)d}$, c denoting the speed of light. Moreover, assume that the Doppler spread across the different transmit-receive pairs is either negligible or known. Denoting λ the carrier wavelength, the scattering towards receive element ℓ can be modeled through a beam of angular width λ/V' and the arc illuminated at distance R'_ℓ has length $\lambda R'_\ell/V'$: as a consequence, uncorrelated scattering towards different sensors occurs whenever the spacing d' satisfies the condition $\lambda R'_\ell/V' < d'$, for $\ell = 1, \ldots, L$. Similarly at the transmitter side the uncorrelated scattering condition translates to $\lambda R_m/V < d$, $m = 1, \ldots, M$. The above scheme allows accounting for transmit and receive diversity separately: for example, if a target is close enough to

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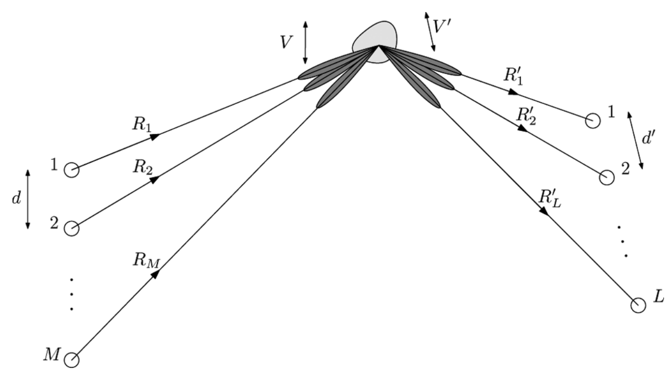

Fig. 1. Target scattering in a $M \times L$ MIMO radar.

the transmitter (i.e., $\lambda R_m/V < d$), but far away from the receiver (i.e., $\lambda R'_\ell/V' > d'$), we may have full transmit diversity, but reduced (or no) receive diversity.

The architecture in Fig. 1 has been introduced and discussed by early studies on MIMO radar, such as [1]–[4]. Under uncorrelated Gaussian scattering and disturbance, the target detectability may be enhanced by transmitting a set of orthogonal waveforms and performing disjoint energy detection at each receive antenna. In this case, the probability of a target miss vanishes as the LMth power of the signal-to-disturbance ratio (SDR) and LM can be interpreted as the number of degrees of freedom (DOF) granted by the MIMO architecture. The concept of system DOF has been further developed in [5]–[7], wherein space-time coding (STC) for MIMO radars has been introduced: relaxing the orthogonality hypothesis and assuming that the set of the admissible transmitted waveforms spans an N-dimensional linear space grants a number of DOF which is $L \min\{N, M\}$. STC is beneficial when the overall disturbance exhibits a temporal correlation and is a means for compromising between number of diversity paths and amount of energy integration along each active path, as shown in [8] and [9]. More studies involving the concept of STC are [10] and [11], both concerned with target identification and classification in white Gaussian noise, the former in the context of waveform optimization and the latter in the context of robust design for uncertain target state information.

From all of the above studies, a number of conclusions can be drawn. Fig. 1 shows that the degree of spatial correlation among the different rays depends upon the range of the target, on top of its extension and carrier frequency: as a consequence, this correlation—and ultimately the number of degrees of freedom that can be exploited—are *not* under the designer's control. Likewise, the assumptions that the overall disturbance at the receive antennas have a known (time) correlation and are spatially independent are themselves questionable, since they may be fulfilled for a particular set of range cells, but not for the whole controlled area.

These considerations pose the problem of a *robust* waveform design in the presence of prior uncertainty as to the target response and disturbance characteristics. In this context, the contribution of the paper can be summarized as follows.

- We consider a min-max waveform design under a transmit energy constraint. Lacking a consensus as to what is a valuable performance measure for MIMO radar system, we solve the min-max problem for a general class of cost functions, subsuming many of the performance figures considered so far.
- Two different scenarios are investigated. We first consider the situation that no prior information about the spatial correlation of the target scattering is available, but the disturbance covariance is known. Differently from previous works, in our model we account for both the transmit and the receive spatial correlation of the target scattering. Then, we consider the case of complete lack of prior information on both the disturbance and the target covariance.
- Our analysis shows that robust space-time code matrices must necessarily have rank M (i.e., equal to the number of transmit antennas) in order to prevent any target hiding. If the disturbance covariance matrix is known, then transmission must take place along the least interfered directions in the signal space, *and* more power must be allocated to more interfered modes. Otherwise isotropic transmission turns out to be a min-max choice.
- The same min-max solution applies to many commonly adopted performance measures, such as the signal-to-disturbance ratio, the linear minimum mean square error in estimating the unknown target response, the mutual information between the received signal echoes and the target response, and the approximation of the detection probability in the high- and low-signal regimes for a fixed probability of false alarm.
- General relationships for the achievable performance under min-max design are derived and compared with the corresponding performance that can be obtained under optimum design (i.e., with prior target/clutter information) or in the absence of STC.

The remainder of the paper is organized as follows. In the next section, we introduce the signal model for the considered MIMO radar system and state the problem of robust waveform design. Section III presents the results in the case where there is no spatial correlation among the receive antennas, while Section IV contains the corresponding analytical proofs. Section V discusses some commonly adopted cost functions which fall in the considered category. In Section VI the case where the receive antennas exhibit a spatial correlation is addressed. In Section VII some numerical examples are presented. Concluding remarks are given in Section VIII. Finally, the Appendix contains some mathematical derivations.

Notation: In what follows, $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex numbers, respectively. $\mathbb{R}^n(\mathbb{R}^{m\times n})$ and $\mathbb{C}^n(\mathbb{C}^{m\times n})$ are the sets of $n\times 1$ vectors ($m\times n$ matrices) with entries from $\mathbb{R}$ and $\mathbb{C}$, respectively. Column vectors and matrices are indicated through boldface lowercase and uppercase letters, respectively. x_i is the ith entry of $\boldsymbol{x}\in\mathbb{C}^n$ and A_{ij} is the entry (i,j) of $\boldsymbol{A}\in\mathbb{C}^{m\times n}$. $\mathrm{diag}\{a_1,\dots,a_n\}\in\mathbb{C}^{n\times n}$ is the diagonal matrix with $a_1,\dots,a_n$ on the main diagonal, $\boldsymbol{I}_m$ is the identity matrix of order m, and $\boldsymbol{O}_{m,n}$ is the $m\times |n$ zero matrix. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose, respectively. $\mathcal{M}_n$ denotes the set of Hermitian, positive semidefinite matrices from $\mathbb{C}^{n\times n}$. $\boldsymbol{A}^{-1}$ is the inverse of a non-singular matrix $\boldsymbol{A}\in\mathbb{C}^{n\times n}$. $\boldsymbol{A}^{1/2}$ is the unique positive semidefinite square root of $\boldsymbol{A}\in\mathcal{M}_n$. $\{\lambda_i(\boldsymbol{A})\}_{i=1}^n$ is the set of eigenvalues of $\boldsymbol{A}\in\mathbb{C}^{n\times n}$; if $\boldsymbol{A}$ is Hermitian the eigenvalues are sorted in descending order, i.e., $\lambda_1(\boldsymbol{A})\geq\cdots\geq\lambda_n(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})=\lambda_1(\boldsymbol{A})$, $\lambda_{\min}(\boldsymbol{A})=\lambda_n(\boldsymbol{A})$. $\mathrm{tr}(\boldsymbol{A})$ and $\mathrm{rank}(\boldsymbol{A})$ are the trace and the rank of $\boldsymbol{A}\in\mathbb{C}^{n\times n}$, respectively, while $\otimes$ denotes the Kronecker (tensor) product. A function $\phi:\mathcal{A}\subseteq\mathbb{R}^n\longrightarrow\mathbb{R}$ is symmetric if for any $\boldsymbol{x}\in\mathcal{A}$, $\pi(\boldsymbol{x})\in\mathcal{A}$ and $\phi(\boldsymbol{x})=\phi\big(\pi(\boldsymbol{x})\big)$, for every permutation π. A function $\phi:\mathcal{A}\subseteq\mathbb{R}^n\longrightarrow\mathbb{R}$ is increasing if for any $\boldsymbol{x},\boldsymbol{y}\in\mathcal{A}$, $\phi(\boldsymbol{x})\leq\phi(\boldsymbol{y})$ whenever $x_i\leq y_i$, $\forall i=1,\dots,n$. Finally, $\mathbb{E}$ denotes statistical expectation.

II. Problem Statement

Consider an $M\times L$ statistical MIMO radar whose task is to detect the presence of a target in a given range cell. Following the model of [6]–[8], the signal at the ℓth receive antenna, $\ell=1,\dots,L$, can be expressed as

$$\boldsymbol{r}_\ell=\begin{cases}\boldsymbol{A}\boldsymbol{\alpha}_\ell+\boldsymbol{w}_\ell, & \text{under } H_1\\ \boldsymbol{w}_\ell, & \text{under } H_0\end{cases}\tag{1}$$

where $\boldsymbol{r}_\ell\in\mathbb{C}^N$, N being the signal space dimension (for example, in a pulsed radar system N is the number of coded pulses transmitted by each antenna and jointly processed by the receiver); $\boldsymbol{A}\in\mathbb{C}^{N\times M}$ is the space-time code matrix, whose mth column represents the codeword transmitted by the mth transmit antenna; $\boldsymbol{\alpha}_\ell\in\mathbb{C}^M$ is the unknown random target response (the mth entry is the scattering from antenna m to antenna ℓ); finally, $\boldsymbol{w}_\ell\in\mathbb{C}^N$ is the overall disturbance (e.g., thermal noise and reverberation from the environment). In the following, we assume that $\boldsymbol{\alpha}=(\boldsymbol{\alpha}_1^T\cdots\boldsymbol{\alpha}_L^T)^T$ and $\boldsymbol{w}=(\boldsymbol{w}_1^T\cdots\boldsymbol{w}_L^T)^T$ are mutually-independent, zero-mean, random vectors with covariance matrices $\boldsymbol{I}_L\otimes\boldsymbol{R}_\alpha$ and $\boldsymbol{I}_L\otimes\boldsymbol{R}_w$, respectively.[1] For future reference, we denote by $\boldsymbol{U}_w\boldsymbol{\Lambda}_w\boldsymbol{U}_w^H$ the spectral decomposition of $\boldsymbol{R}_w$, where $\boldsymbol{U}_w\in\mathbb{C}^{N\times N}$ is unitary and $\boldsymbol{\Lambda}_w=\mathrm{diag}\big\{\lambda_1(\boldsymbol{R}_w),\dots,\lambda_N(\boldsymbol{R}_w)\big\}$, with $\lambda_{\min}(\boldsymbol{R}_w)\geq\mathcal{N}_0$, $\mathcal{N}_0$ the thermal noise floor. Also, $\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^H)$ represents the radiated energy, while the SDR under the hypothesis H_1 is defined as

$$\begin{aligned}\mathrm{SDR}&=\mathbb{E}\left[\sum_{\ell=1}^{L}\|\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{\alpha}_\ell\|^2\right]\\&=L\,\mathrm{tr}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\\&=L\sum_{i=1}^{\Delta}\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\end{aligned}\tag{2}$$

where $\Delta=\min\{M,N\}$.

[1]To simplify the exposition, no spatial receive correlation is assumed at this stage. In Section VI, the results will be extended to the case where $\mathbb{E}[\boldsymbol{\alpha}\boldsymbol{\alpha}^H]=\boldsymbol{Q}_\alpha\otimes\boldsymbol{R}_\alpha$ and $\mathbb{E}[\boldsymbol{w}\boldsymbol{w}^H]=\boldsymbol{Q}_w\otimes\boldsymbol{R}_w$, for $\boldsymbol{Q}_\alpha,\boldsymbol{Q}_w\in\mathcal{M}_L$, $\boldsymbol{Q}_w$ full-rank.

A. Equivalent MIMO Channel

Let $\boldsymbol{U\Sigma V}^H$ be the singular value decomposition of $\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha^{1/2}$, where $\boldsymbol{U} \in \mathbb{C}^{N\times N}$ and $\boldsymbol{V} \in \mathbb{C}^{M\times M}$ are unitary, and $\boldsymbol{\Sigma} \in \mathbb{R}^{N\times M}$ is a diagonal matrix with

$$\Sigma_{ii} = \begin{cases} \lambda_i^{1/2}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2}), & \text{if } i \le \Delta \\ 0, & \text{otherwise;} \end{cases}$$

also, let $\tilde{\boldsymbol{\alpha}}_\ell$ be such that $\mathbb{E}[\tilde{\boldsymbol{\alpha}}_\ell\tilde{\boldsymbol{\alpha}}_\ell^H] = \boldsymbol{I}_M$ and $\boldsymbol{\alpha}_\ell = \boldsymbol{R}_\alpha^{1/2}\tilde{\boldsymbol{\alpha}}_\ell$, $\ell = 1,\dots,L$. Then, the received signal in (1) can be equivalently represented under H_1 as

$$\tilde{\boldsymbol{r}}_\ell = \boldsymbol{R}_w^{-1/2}\boldsymbol{r}_\ell = \boldsymbol{U\Sigma V}^H\tilde{\boldsymbol{\alpha}}_\ell + \boldsymbol{R}_w^{-1/2}\boldsymbol{w}_\ell$$

and $\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$ can be interpreted as the SDR along the ith eigenmode of an equivalent MIMO channel corrupted by additive white disturbance, whose inputs are the uncorrelated entries of $\tilde{\boldsymbol{\alpha}}_\ell$ and whose impulse response is $\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha^{1/2}$ [12].

B. Cost Function and Code Design

Optimal waveform design amounts to choosing the code-matrix $\boldsymbol{A}$ so as to minimize a given cost function (or equivalently maximize the corresponding figure of merit) under a transmit energy constraint. In the sequel, we focus on the class of cost functions taking on the general form

$$f\left(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right) \tag{3}$$

where $\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$ is a Δ-dimensional vector whose ith entry is $\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$ and $f : \mathbb{R}^\Delta \to \mathbb{R}$ is decreasing and Schur-convex. The rationale behind this class of cost functions is that $\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$ represents the SDR along the ith eigenmode of an equivalent MIMO channel, which justifies the assumption that f be decreasing. As to the Schur-convexity, this includes symmetry and convexity, which are common assumptions for cost functions. For future reference, we introduce the function $f_1(x) = f\big((x\,0\cdots0)^T\big)$.

III. Robust Code Design

Robust waveform design aims at determining the code-matrix $\boldsymbol{A}$ so as to minimize the worst-case cost under all possible target covariance matrices. Also, we distinguish two relevant scenarios: known and unknown disturbance covariance matrix.

A. Known Disturbance Covariance

In this case, we are faced with the following

Problem 3.1: For given $\mathcal{E} > 0$ and $\sigma_\alpha^2 > 0$, find the code-matrix $\boldsymbol{A}$ which solves

$$\begin{aligned} \min_{A}\max_{\boldsymbol{R}_\alpha} \quad & f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big) \\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \le \mathcal{E} \\ & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}\,(\boldsymbol{R}_\alpha) \ge \sigma_\alpha^2. \end{aligned}$$

Notice that $\mathcal{E}$ is the maximum energy available for transmission, while the constraint on $\mathrm{tr}\,(\boldsymbol{R}_\alpha)$ ensures a minimum level of energy backscattered from the target (otherwise no detection would be possible).

As shown in Section IV-A, if $N < M$ there always exist targets that cannot be observed, i.e., $\mathrm{SDR} = 0$, no matter how the energy is radiated; hence, the worst case cost is always $f_1(0)$, independently of the choice of the code matrix $\boldsymbol{A}$. In order to ensure observability of all possible targets we must necessarily have $N \ge M$. In this case, a solution is

$$\boldsymbol{A} = \sqrt{\frac{\mathcal{E}}{\mathrm{tr}(\boldsymbol{\Lambda}_w^{(\pi)})}}\boldsymbol{U}_w^{(\pi)}\big(\boldsymbol{\Lambda}_w^{(\pi)}\big)^{1/2}\boldsymbol{V}^H \tag{4}$$

where π is the function defined as $\pi(i) = N - M + i$, $i = 1,\dots,M$, $\boldsymbol{U}_w^{(\pi)}$ is the submatrix of $\boldsymbol{U}_w$ formed by the columns $\pi(1),\dots,\pi(M)$, $\boldsymbol{\Lambda}_w^{(\pi)}$ is the principal submatrix of $\boldsymbol{\Lambda}_w$ formed by the rows and columns with indexes $\pi(1),\dots,\pi(M)$, and $\boldsymbol{V} \in \mathbb{C}^{M\times M}$ is an arbitrary unitary matrix.

Solution (4) deserves some comments. The min-max code matrix is rank M, otherwise all targets lying in the null space of $\boldsymbol{A}$ would be missed. If $N = M$, we can simply take $\boldsymbol{A} = \sqrt{\mathcal{E}/\mathrm{tr}(\boldsymbol{R}_w)}\boldsymbol{R}_w^{1/2}$. The permutation π establishes that the left singular vectors of the code matrix are matched to the eigenvectors of $\boldsymbol{R}_w$ corresponding to the M smallest eigenvalues: this choice ensures that the M least interfered directions in the signal space are selected for transmission. The right singular vectors, instead, cannot be matched to $\boldsymbol{R}_\alpha$, since it is unknown, and therefore $\boldsymbol{V}$ can be arbitrarily chosen. The singular values are chosen so that more energy is allocated to more interfered modes in order to equalize their SDR's. The intuition behind this choice is to make every selected direction "equivalent," so that one is prepared to every possible target. Under this coding strategy, the min-max cost is

$$f_1\left(\frac{\mathcal{E}\sigma_\alpha^2}{\sum_{i=1}^M \lambda_{N-i+1}(\boldsymbol{R}_w)}\right)$$

—which becomes $f_1\left(\mathcal{E}\sigma_\alpha^2/\mathrm{tr}(\boldsymbol{R}_w)\right)$ if $N = M$—and occur when $\mathrm{rank}(\boldsymbol{R}_\alpha) = 1$. Finally, it is important to stress that, unlike other studies, here the knowledge of the target position and RCS is not needed. Indeed, σ_α^2 is a constant that determines only the value of the SDR: in any case, the code-matrix is unaltered.

B. Unknown Disturbance Covariance

We now investigate robust STC when the disturbance covariance matrix is unknown. In this case, the worst-case cost has to be computed over all possible disturbance and target covariance scenarios. Hence, we have

Problem 3.2: For given $\mathcal{E} > 0$, $\sigma_\alpha^2 > 0$ and $\sigma_w^2 \ge N\mathcal{N}_0$, find the code-matrix $\boldsymbol{A}$ which solves

$$\begin{aligned} \min_{A}\max_{\boldsymbol{R}_\alpha,\boldsymbol{R}_w} \quad & f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big) \\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \le \mathcal{E} \\ & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}(\boldsymbol{R}_\alpha) \ge \sigma_\alpha^2 \\ & \boldsymbol{R}_w \in \mathcal{M}_N, \quad \mathrm{tr}(\boldsymbol{R}_w) \le \sigma_w^2 \\ & \lambda_{\min}(\boldsymbol{R}_w) \ge \mathcal{N}_0. \end{aligned}$$

$\mathcal{E}$ represents again the maximum transmit energy; the constraint on $\mathrm{tr}(\boldsymbol{R}_\alpha)$ ensures a minimum level of energy backscattered from the target, while the constraint on $\mathrm{tr}(\boldsymbol{R}_w)$ bounds the disturbance power.

As shown in Section IV-B, if $N < M$ the worst case cost is always $f_1(0)$, independently of the choice of $\boldsymbol{A}$, since there always exists targets lying in the null-space of $\boldsymbol{A}$ that cannot be observed. Again, to ensure observability of all possible targets we must necessarily have $N \geq M$. In this case, a solution is

$$\boldsymbol{A} = \sqrt{\mathcal{E}/M}\boldsymbol{U} \quad (5)$$

where $\boldsymbol{U} \in \mathbb{C}^{N\times M}$ is an arbitrary matrix such that $\boldsymbol{U}^H\boldsymbol{U} = \boldsymbol{I}_M$.

It is interesting to compare solution (5) with solution (4). In both cases, the code rank is equal to M. However, since now $\boldsymbol{R}_w$ is unknown, the transmitter cannot match the left singular vectors of $\boldsymbol{A}$ to the least interfered directions in the signal space. Solution (5) amounts to illuminate the target isotropically. In particular, if only one power amplifier is available, $\boldsymbol{A}$ can be taken equal to $\sqrt{\mathcal{E}/M}[\boldsymbol{I}_M \boldsymbol{O}_{M,N-M}]^T$. This motivates the adoption of the orthogonal waveforms (widely used in MIMO radars) in the absence of target and disturbance information, which here naturally arises as the solution to a min-max problem for a wide class of cost functions. Finally, the min-max cost is

$$f_1\left(\frac{\mathcal{E}\sigma_\alpha^2}{M(\sigma_w^2 - (N-1)\mathcal{N}_0)}\right)$$

and occurs when $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{x}\boldsymbol{x}^H$ and $\boldsymbol{R}_w = \mathcal{N}_0\boldsymbol{I}_N + (\sigma_w^2 - N\mathcal{N}_0)\boldsymbol{U}\boldsymbol{x}\boldsymbol{x}^H\boldsymbol{U}^H$, for some norm-one $\boldsymbol{x} \in \mathbb{C}^M$.

IV. Problems' Solutions

Here we give the solutions to the min-max problems posed in the previous section.

A. *Solution to Problem 3.1*

As to the max-part, we need to solve

$$\begin{aligned}\max_{\boldsymbol{R}_\alpha} \quad & f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big)\\ \text{s.t.} \quad & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}\,(\boldsymbol{R}_\alpha) \geq \sigma_\alpha^2\end{aligned}$$

for fixed $\boldsymbol{A}$. From Schur-convexity of f, we have

$$f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big) \leq f_1\big(\mathrm{tr}(\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\big). \quad (6)$$

Furthermore

$$\begin{aligned}\mathrm{tr}(\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) &\geq \sum_{i=1}^M \lambda_i(\boldsymbol{R}_\alpha)\lambda_{M-i+1}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\\ &\geq \sum_{i=1}^M \lambda_i(\boldsymbol{R}_\alpha)\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\\ &= \mathrm{tr}(\boldsymbol{R}_\alpha)\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\\ &\geq \sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) \quad (7)\end{aligned}$$

where the last inequality follows from the constraint on $\mathrm{tr}(\boldsymbol{R}_\alpha)$ and the first from the following [13, Theorem 9.H.1.h].

Theorem 4.1: If $\boldsymbol{U},\boldsymbol{V} \in \mathcal{M}_n$, then $\mathrm{tr}(\boldsymbol{U}\boldsymbol{V}) \geq \sum_{i=1}^n \lambda_i(\boldsymbol{U})\lambda_{n-i+1}(\boldsymbol{V})$.

Equations (6) and (7) and the fact that f_1 is decreasing implies that

$$f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big) \leq f_1\big(\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\big) \quad (8)$$

and equality holds when $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{v}$ is the norm-one eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}$.

As to the min-part, we need to solve

$$\begin{aligned}\min_{\boldsymbol{A}} \quad & f_1\left(\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\right)\\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \leq \mathcal{E}\end{aligned}$$

and, since f_1 is decreasing, $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})$ is to be maximized. If $N < M$, $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) = 0$ and the worst-case cost is $f_1(0)$ for any choice of $\boldsymbol{A}$. Conversely, if $N \geq M$, the worst-case cost can be made smaller than $f_1(0)$ by optimizing the choice of $\boldsymbol{A}$. To this end, notice that

$$\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) \leq \frac{1}{M}\mathrm{tr}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})$$

and equality holds if $\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A} = c\boldsymbol{I}_M$, for some $c > 0$. Then the code-matrix must be of the form $\boldsymbol{A} = \boldsymbol{U}_w^{(\pi)}(c\boldsymbol{\Lambda}_w^{(\pi)})^{1/2}\boldsymbol{V}^H$, where π is a set of M different integers taken from $\{1,\ldots,N\}$, $\boldsymbol{U}_w^{(\pi)}$ is the submatrix of $\boldsymbol{U}_w$ formed by the columns indexed by π, $\boldsymbol{\Lambda}_w^{(\pi)}$ is the principal submatrix of $\boldsymbol{\Lambda}_w$ formed by the rows and columns indexed by π, and $\boldsymbol{V} \in \mathbb{C}^{M\times M}$ is an arbitrary unitary matrix. At this point, the problem reduces to

$$\begin{aligned}\max_{\pi} \quad & c\\ \text{s.t.} \quad & c\,\mathrm{tr}(\boldsymbol{\Lambda}^{(\pi)}) \leq \mathcal{E}\end{aligned}$$

and then a solution is $\pi = \{N-M+1,\ldots,N\}$, which is that in (4).

B. *Solution to Problem 3.2*

As to the max-part, we need to solve

$$\begin{aligned}\max_{\boldsymbol{R}_\alpha,\boldsymbol{R}_w} \quad & f\big(\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big)\\ \text{s.t.} \quad & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}(\boldsymbol{R}_\alpha) \geq \sigma_\alpha^2\\ & \boldsymbol{R}_w \in \mathcal{M}_N, \quad \mathrm{tr}(\boldsymbol{R}_w) \leq \sigma_w^2\\ & \lambda_{\min}(\boldsymbol{R}_w) \geq \mathcal{N}_0.\end{aligned}$$

for fixed $\boldsymbol{A}$. As in Section IV-A, we find that $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{v}$ is the norm-one eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}$, so that the problem reduces to

$$\begin{aligned}\min_{\boldsymbol{R}_w} \quad & \lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\\ \text{s.t.} \quad & \boldsymbol{R}_w \in \mathcal{M}_N, \quad \mathrm{tr}(\boldsymbol{R}_w) \leq \sigma_w^2\\ & \lambda_{\min}(\boldsymbol{R}_w) \geq \mathcal{N}_0.\end{aligned}$$

Notice now that

$$\begin{aligned}\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) &= \min_{i\in\{1,\ldots,M\}} \lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\\ &= \min_{i\in\{1,\ldots,M\}} \theta_i\lambda_i(\boldsymbol{A}\boldsymbol{A}^H) \quad (9)\end{aligned}$$

for some $\theta_i \in \left[\lambda_{\min}(\boldsymbol{R}_w^{-1}), \lambda_{\max}(\boldsymbol{R}_w^{-1})\right]$, where the last equality results from the following [14, Corollary 4.5.11].

Theorem 4.2: Let $\boldsymbol{M}, \boldsymbol{S} \in \mathbb{C}^{n\times n}$ and let $\boldsymbol{M}$ be Hermitian, then, for each $i = 1, \ldots, n$, there exists $\theta_i \geq 0$ such that $\lambda_n(\boldsymbol{S}\boldsymbol{S}^H) \leq \theta_i \leq \lambda_1(\boldsymbol{S}\boldsymbol{S}^H)$ and $\lambda_i(\boldsymbol{S}\boldsymbol{M}\boldsymbol{S}^H) = \theta_i\lambda_i(\boldsymbol{M})$. Further developing (9), we have

$$\begin{aligned}\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}) &= \min_{i\in\{1,\ldots,M\}} \theta_i\lambda_i(\boldsymbol{A}\boldsymbol{A}^H)\\ &\geq \frac{\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})}{\lambda_{\max}(\boldsymbol{R}_w)}\\ &\geq \frac{\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})}{\sigma_w^2 - (N-1)\mathcal{N}_0} \quad (10)\end{aligned}$$

where the last inequality follows from the constraints on $\boldsymbol{R}_w$. This is the minimum and is attained for $\boldsymbol{R}_w = \mathcal{N}_0\boldsymbol{I}_N + (\sigma_w^2 - N\mathcal{N}_0)\boldsymbol{u}\boldsymbol{u}^H$, where $\boldsymbol{u}$ is the norm-one eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{A}\boldsymbol{A}^H$ and $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{v}$ is the norm-one eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{A}\big(\mathcal{N}_0\boldsymbol{I}_N + (\sigma_w^2 - N\mathcal{N}_0)\boldsymbol{u}\boldsymbol{u}^H\big)^{-1}\boldsymbol{A}^H$.

As to the min-part, we need to solve

$$\begin{aligned}\min_{\boldsymbol{A}} \quad & f_1\left(\frac{\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})}{\sigma_w^2 - (N-1)\mathcal{N}_0}\right)\\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \text{tr}(\boldsymbol{A}^H\boldsymbol{A}) \leq \mathcal{E}\end{aligned}$$

and, since f_1 is decreasing, $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})$ is to be maximized. If $N < M$, $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A}) = 0$ and the worst-case cost is $f_1(0)$ for any choice of $\boldsymbol{A}$. Conversely, if $N \geq M$, the worst-case cost can be made smaller than $f_1(0)$ by optimizing the choice of $\boldsymbol{A}$. In particular, the minimum cost is attained by taking $\boldsymbol{A}$ such that $\boldsymbol{A}^H\boldsymbol{A} = \sqrt{\mathcal{E}/M}\,\boldsymbol{I}_M$, i.e., the solution in (5).

V. Examples of Cost Functions

We now discuss some typically-used cost functions that take the form (3).

A. Signal-to-Disturbance Ratio

Following [7], [15], a meaningful figure of merit for code design is the SDR defined in (2). In general, from [13, Table 3.1], any decreasing function of SDR (e.g., —SDR) is a cost function of the form (3), irrespectively of the distributions of the target scattering and of the disturbance.

B. Linear Minimum Mean-Square Error

Let $\tilde{\boldsymbol{\alpha}}_\ell$ be such that $\mathbb{E}[\tilde{\boldsymbol{\alpha}}_\ell\tilde{\boldsymbol{\alpha}}_\ell^H] = \boldsymbol{I}_M$ and $\boldsymbol{\alpha}_\ell = \boldsymbol{R}_\alpha^{1/2}\tilde{\boldsymbol{\alpha}}_\ell$, $\ell = 1, \ldots, L$. Then the error covariance matrix of a linear minimum mean-square error (LMMSE) estimator of the scattering coefficients $\{\tilde{\boldsymbol{\alpha}}_\ell\}_{\ell=1}^L$ from the received observables $\{\boldsymbol{r}_\ell\}_{\ell=1}^L$ is [16, Theorem 12.1]

$$\boldsymbol{C}_{\text{err}} = \boldsymbol{I}_L \otimes \left(\boldsymbol{I}_M + \boldsymbol{R}_\alpha^{1/2}\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}\boldsymbol{R}_\alpha^{1/2}\right)^{-1}$$

and the LMMSE takes the form

$$\begin{aligned}\text{LMMSE} &= L\sum_{i=1}^{M}\frac{1}{1+\lambda_i(\boldsymbol{R}_\alpha^{1/2}\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}\boldsymbol{R}_\alpha^{1/2})}\\ &= L\sum_{i=1}^{\Delta}\frac{1}{1+\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})}\\ &\quad + L\max\{M-N, 0\}. \quad (11)\end{aligned}$$

This implies that the LMMSE (and, from [13, Table 3.1], any increasing function thereof) is a cost function of the form (3), irrespectively of the distributions of target scattering and disturbance. Notice that if $\{\boldsymbol{\alpha}_\ell\}_{\ell=1}^L$ and $\{\boldsymbol{w}_\ell\}_{\ell=1}^L$ are Gaussian, circularly symmetric, (11) represents also the minimum mean square error and the variance of the maximum *a posteriori* estimator.

C. Mutual Information

Let MI be the mutual information between $\{\boldsymbol{\alpha}_\ell\}_{\ell=1}^L$ and $\{\boldsymbol{r}_\ell\}_{\ell=1}^L$ under H_1, also equal to the mutual information between $\{\tilde{\boldsymbol{\alpha}}_\ell\}_{\ell=1}^L$ and $\{\boldsymbol{r}_\ell\}_{\ell=1}^L$. If the overall disturbance $\{\boldsymbol{w}_\ell\}_{\ell=1}^L$ and the scattering coefficients $\{\boldsymbol{\alpha}_\ell\}_{\ell=1}^L$ are Gaussian, circularly-symmetric, then we can write

$$\text{MI} = L\sum_{i=1}^{\Delta}\log\left(1+\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right).$$

Since MI is Schur concave and increasing, from [13, Table 3.1] any decreasing function of MI (e.g., –MI) is of the form (3).

D. Approximate Miss Probability

If $\{\boldsymbol{w}_\ell\}_{\ell=1}^L$ is Gaussian and circularly-symmetric, the generalized likelihood ratio test for the detection problem (1) takes the form [6]–[8]

$$\sum_{\ell=1}^{L}\|\boldsymbol{P}_\perp\boldsymbol{R}_w^{-1/2}\boldsymbol{r}_\ell\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

where $\boldsymbol{P}_\perp$ is the orthogonal projector onto the range span of $\boldsymbol{R}_w^{-1/2}\boldsymbol{A}$, and η is the detection threshold, to be set based upon the desired false alarm probability, P_{fa}. In this case, we have [6]–[8]

$$P_{\text{fa}} = e^{-\eta}\sum_{k=0}^{L\text{rank}(\boldsymbol{A})-1}\frac{\eta^k}{k!} \quad (12a)$$

$$P_{\text{miss}} = 1 - \mathbb{E}\left[Q_{L\text{rank}(\boldsymbol{A})}\left(\sqrt{2\sum_{\ell=1}^{L}\|\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{\alpha}_\ell\|^2}, \sqrt{2\eta}\right)\right] \quad (12b)$$

where P_{miss} is the miss probability and $Q_m : \mathbb{R}^2 \rightarrow \mathbb{R}$ denotes the generalized Marcum function of order m.

Unfortunately, expression (12b) does not allow drawing general conclusions, whereby we analyze its behavior in the low- and high-SDR regions. First, in order to prevent any target hiding in the null space of $\boldsymbol{A}$, which would lead to $1 - P_{\text{miss}} = P_{\text{fa}}$, the rank of the code-matrix must be M. Now, in the low-SDR region, truncating the series expansion of Q_m [17, Equation (6)] to the first-order term, yields

$$P_{\text{miss}} \approx 1 - P_{\text{fa}} - LN\left(P_{\text{fa}} + \frac{e^{-\eta}\eta^{ML}}{(ML)!}\right)\text{SDR} \quad (13)$$

and therefore the miss probability can be approximated by a function of the form (3), irrespectively of the target scattering distribution. In the high-SDR regime, instead, we can approximate P_{miss} with its Chernoff's bound [6], [8], [9], which, for the case of Gaussian and circularly-symmetric $\{\boldsymbol{\alpha}_i\}_{i=1}^L$, recalling that $\mathrm{rank}(\boldsymbol{A}) = M$, takes the form

$$P_{\text{miss}} \approx \min_{\gamma \geq 0} \Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)$$

where $\Phi(\gamma, \boldsymbol{x}) = e^{\gamma\eta - L\sum_{i=1}^{\Delta}\ln(1+\gamma(1+x_i))}$. While Φ is Schur-convex and decreasing in $\boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$ for γ fixed, in general, $\min_\gamma \Phi$ is not. However, as shown in Appendix A, the min-max design and the minimum over γ can be interchanged and then robustification can be carried out for the function Φ, which is of the form (3).

VI. Spatial Receive Correlation

Consider now the general case $\mathbb{E}[\boldsymbol{\alpha}\boldsymbol{\alpha}^H] = \boldsymbol{Q}_\alpha \otimes \boldsymbol{R}_\alpha$ and $\mathbb{E}[\boldsymbol{w}\boldsymbol{w}^H] = \boldsymbol{Q}_w \otimes \boldsymbol{R}_w$, where $\boldsymbol{Q}_\alpha, \boldsymbol{Q}_w \in \mathcal{M}_L$, $\mathrm{tr}(\boldsymbol{Q}_\alpha) = \mathrm{tr}(\boldsymbol{Q}_w) = L$ and $\lambda_{\min}(\boldsymbol{Q}_w) \geq \varepsilon_0$, with $0 < \varepsilon_0 \leq 1/L$. In this case the received observations can be cast in the LN-dimensional vector

$$\boldsymbol{r} = (\boldsymbol{r}_1^T \cdots \boldsymbol{r}_L^T)^T = \begin{cases} (\boldsymbol{I}_L \otimes \boldsymbol{A})\boldsymbol{\alpha} + \boldsymbol{w}, & \text{under } H_1 \\ \boldsymbol{w}, & \text{under } H_0 \end{cases}$$

and the SDR under H_1 can be defined as

$$\begin{aligned} \mathrm{SDR} &= \mathbb{E}\left[\|(\boldsymbol{Q}_w \otimes \boldsymbol{R}_w)^{-1/2}(\boldsymbol{I}_L \otimes \boldsymbol{A})\boldsymbol{\alpha}\|^2\right] \\ &= \mathrm{tr}\big((\boldsymbol{Q}_w \otimes \boldsymbol{R}_w)^{-1/2}(\boldsymbol{I}_L \otimes \boldsymbol{A}) \\ &\quad \cdot (\boldsymbol{Q}_\alpha \otimes \boldsymbol{R}_\alpha)(\boldsymbol{I}_L \otimes \boldsymbol{A})^H(\boldsymbol{Q}_w \otimes \boldsymbol{R}_w)^{-1/2}\big) \\ &= \mathrm{tr}\big((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \\ &\quad \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big). \end{aligned}$$

The cost function now is $f : \mathbb{R}^{L\Delta} \to \mathbb{R}$ and depends on the eigenvalues of the matrix

$$(\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$$

i.e., on $\boldsymbol{\lambda}\big((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\big)$, which is a $L\Delta$-dimensional vector whose entry $\Delta(\ell-1)+i$ is

$$\lambda_\ell(\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2})\lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})$$

for $\ell = 1, \ldots, L$, $i = 1, \ldots, \Delta$.

At this point, Problem 3.1 can be restated as follows.

Problem 6.1: For a given $\mathcal{E} > 0$ and $\sigma_\alpha^2 > 0$, find the code-matrix $\boldsymbol{A}$ which solves

$$\begin{aligned} \min_{\boldsymbol{A}} \max_{\boldsymbol{R}_\alpha, \boldsymbol{Q}_\alpha} \quad & f\Big(\boldsymbol{\lambda}\Big((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\Big)\Big) \\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \leq \mathcal{E} \\ & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}(\boldsymbol{R}_\alpha) \geq \sigma_\alpha^2 \\ & \boldsymbol{Q}_\alpha \in \mathcal{M}_L, \quad \mathrm{tr}(\boldsymbol{Q}_\alpha) = L. \end{aligned}$$

On the other hand, Problem 3.2 becomes

Problem 6.2: For given $\mathcal{E} > 0, \sigma_\alpha^2 > 0$ and $\sigma_w^2 \geq N\mathcal{N}_0$, find the code-matrix $\boldsymbol{A}$ which solves

$$\begin{aligned} \min_{\boldsymbol{A}} \max_{\boldsymbol{R}_\alpha, \boldsymbol{Q}_\alpha, \boldsymbol{R}_w, \boldsymbol{Q}_w} \quad & f\Big(\boldsymbol{\lambda}\Big((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \\ & \quad \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\Big)\Big) \\ \text{s.t.} \quad & \boldsymbol{A} \in \mathbb{C}^{N\times M}, \quad \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \leq \mathcal{E} \\ & \boldsymbol{R}_\alpha \in \mathcal{M}_M, \quad \mathrm{tr}(\boldsymbol{R}_\alpha) \geq \sigma_\alpha^2 \\ & \boldsymbol{Q}_\alpha \in \mathcal{M}_L, \quad \mathrm{tr}(\boldsymbol{Q}_\alpha) = L \\ & \boldsymbol{R}_w \in \mathcal{M}_N, \quad \mathrm{tr}(\boldsymbol{R}_w) \leq \sigma_w^2 \\ & \boldsymbol{Q}_w \in \mathcal{M}_L, \quad \mathrm{tr}(\boldsymbol{Q}_w) = L \\ & \lambda_{\min}(\boldsymbol{R}_w) \geq \mathcal{N}_0, \quad \lambda_{\min}(\boldsymbol{Q}_w) \geq \varepsilon_0. \end{aligned}$$

As shown in Appendix B, the case $N < M$ is trivial for both problems since the worst case cost is always $f_1(0)$, independently of the choice of $\boldsymbol{A}$. On the other hand, if $N \geq M$, a solution is

$$\boldsymbol{A} = \begin{cases} \sqrt{\mathcal{E}/\mathrm{tr}(\boldsymbol{\Lambda}_w^{(\pi)})}\boldsymbol{U}_w^{(\pi)}\left(\boldsymbol{\Lambda}_w^{(\pi)}\right)^{1/2}\boldsymbol{V}^H, & \text{for Problem 6.1} \\ \sqrt{\mathcal{E}/M}\boldsymbol{U}, & \text{for Problem 6.2} \end{cases}$$

i.e., the same code matrix found in Section III; hence, the presence of a spatial receive correlation in the target scattering does not affect the min-max solution.

Finally, the cost functions given in Section V become

$$\begin{aligned} \mathrm{SDR} &= \sum_{\ell=1}^{L}\sum_{i=1}^{\Delta}\lambda_\ell(\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \\ &\quad \cdot \lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2}) \\ \mathrm{MI} &= \sum_{\ell=1}^{L}\sum_{i=1}^{\Delta}\log\Big(1 + \lambda_\ell(\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \\ &\quad \cdot \lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\Big) \\ \mathrm{LMMSE} &= \sum_{\ell=1}^{L}\sum_{i=1}^{\Delta}\Big(1 + \lambda_\ell(\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \\ &\quad \cdot \lambda_i(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\Big)^{-1} \\ &\quad + L\max\{M-N, 0\} \end{aligned}$$

and are all Schur-convex and decreasing. As to the miss probability, we have

$$P_{\text{miss}} \approx \begin{cases} 1 - P_{\text{fa}} - LN\left(P_{\text{fa}} + \frac{e^{-\eta}\eta^{ML}}{(ML)!}\right)\mathrm{SDR}, & \text{if SDR} \ll 1 \\ \min_{\gamma\geq 0}\Phi\Big(\gamma, \boldsymbol{\lambda}\Big((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) & \\ \quad \otimes(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\Big)\Big), & \text{if SDR} \gg 1 \end{cases}$$

where now $\Phi(\gamma, \boldsymbol{x}) = e^{\gamma\eta - \sum_{\ell=1}^{L}\sum_{i=1}^{\Delta}\ln(1+\gamma(1+x_{\Delta(\ell-1)+i}))}$ and the considerations given in Section V-D retain their validity.

VII. Numerical Results

We consider a MIMO radar system with $M = L = 2$, $N = 4$ and $P_{\text{fa}} = 10^{-4}$. We assume circularly-symmetric Gaussian scattering with $\boldsymbol{Q}_\alpha = \boldsymbol{I}_L$ and $\boldsymbol{R}_\alpha = \boldsymbol{U}_\alpha\boldsymbol{\Lambda}_\alpha\boldsymbol{U}_\alpha^H$,

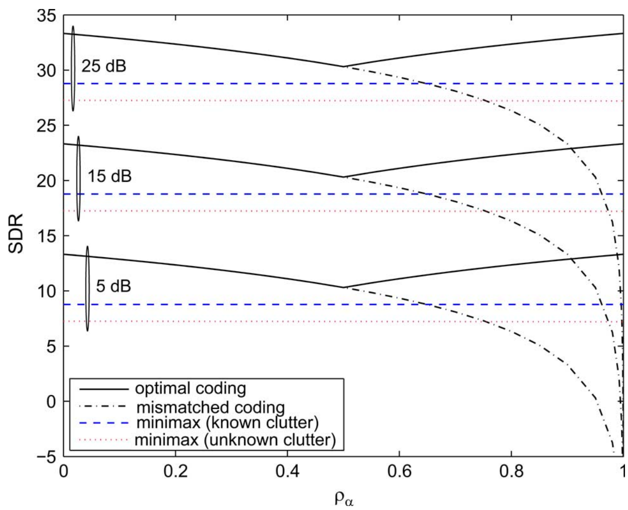

Fig. 2. SDR versus ρ_α for $\gamma = 5$, 15, 25 dB, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{U}_\alpha$.

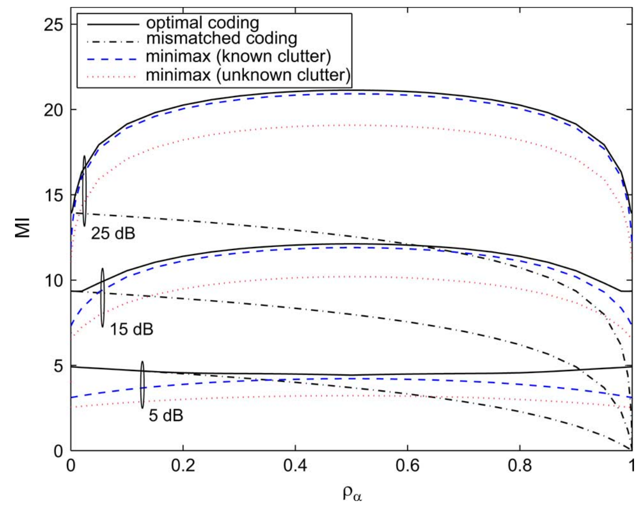

Fig. 4. MI versus ρ_α for $\gamma = 5$, 15, 25 dB, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{U}_\alpha$.

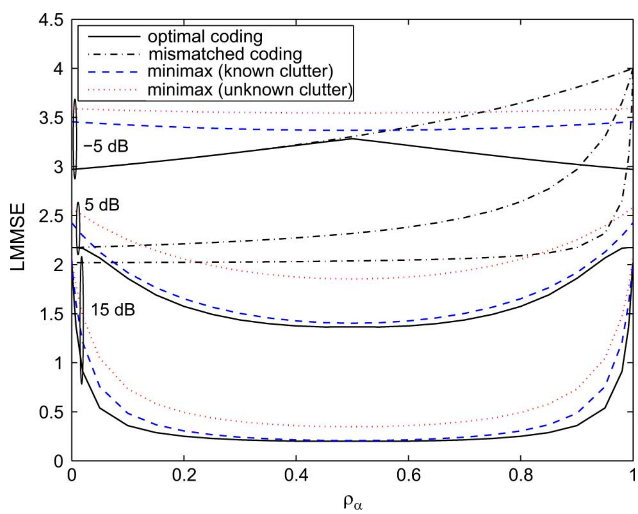

Fig. 3. LMMSE versus ρ_α for $\gamma = -5$, 5, 15 dB, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{U}_\alpha$.

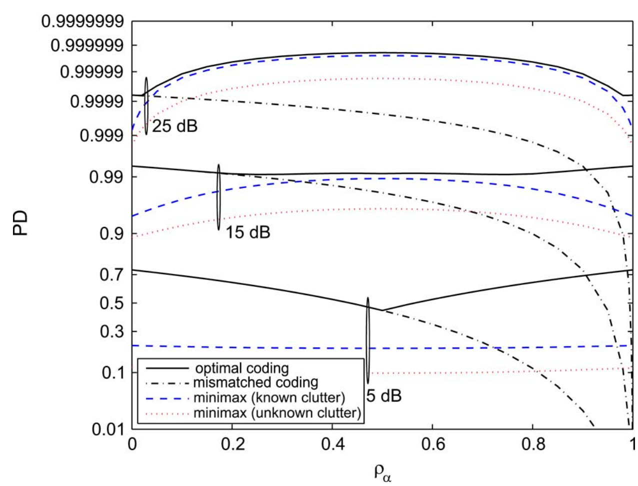

Fig. 5. $P_{\rm d}$ versus ρ_α for $\gamma = 5$, 15, 25 dB, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{U}_\alpha$.

where $\boldsymbol{U}_\alpha \in \mathbb{C}^{2\times 2}$ is unitary, $\boldsymbol{\Lambda}_\alpha = \sigma_\alpha^2 \mathrm{diag}\{\rho_\alpha, 1-\rho_\alpha\}$ and $\rho_\alpha \in [0,1]$. Also, we consider circularly-symmetric Gaussian disturbance with $\boldsymbol{Q}_w = \boldsymbol{I}_L$ and $\boldsymbol{R}_w = \mathcal{N}_0 \boldsymbol{I}_N + \bar{\boldsymbol{R}}_w$, with $\xi = (N\mathcal{N}_0)^{-1}\mathrm{tr}(\bar{\boldsymbol{R}}_w) = 5$ dB.

In Figs. 2–5, we report the performance of the min-max codes versus ρ_α in terms of SDR, LMMSE, MI and $P_{\rm d}$, respectively. Several values of the transmitted energy contrast—defined as $\gamma = \sigma_\alpha^2 \mathcal{E}/(N\mathcal{N}_0)$—are considered. The plots are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{U}_\alpha$. For the sake of comparison, we also include the corresponding performance achievable when: a) both disturbance and target covariance matrices are perfectly known to the transmitter (referred to as *optimal coding*); and b) the transmitter has prior knowledge of the disturbance covariance matrix and of the eigenvectors of $\boldsymbol{\alpha}$, but always assumes $\rho_\alpha = 0$ (referred to as *mismatched coding*).

Several remarks are now in order. Notice first that the optimal STC strategy not only depends on the actual values of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$, but more importantly on the adopted figure of merit. For example, maximizing the SDR requires to adopt a rank-one code-matrix [7], whose right and left singular vectors are matched to the eigenvector of $\boldsymbol{R}_\alpha$ with the largest eigenvalues and to the eigenvector of $\bar{\boldsymbol{R}}_w$ with the smallest eigenvalues, respectively. Instead, optimizing the LMMSE or the MI requires to solve an energy-allocation problem, as shown in [6], [8], and [10]. Finally, $P_{\rm d}$-optimal codes can be found by numerical search, as shown in [18]. Independently of the adopted figure of merit, in the presence of erroneous prior information as to the target covariance matrix the system may experience arbitrarily large losses. In our examples, the performance of the mismatched codes rapidly degrades as $\rho_\alpha \rightarrow 1$, especially at higher values of the transmit energy contrast; the worst-case loss is observed for $\rho_\alpha = 1$, since the target is hidden in the null space of the transmit signal. On the other hand, the min-max strategy presents several advantages: the same solution applies to a large class of cost functions and is independent of the target parameters, simplifying the transmitter design; furthermore, its performance exhibits smooth variations for different values of ρ_α. The min-max codes are full-rank, meaning that they experience

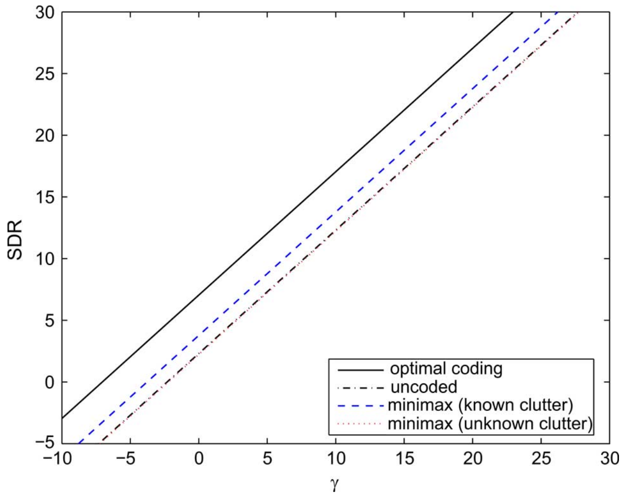

Fig. 6. SDR versus γ, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$.

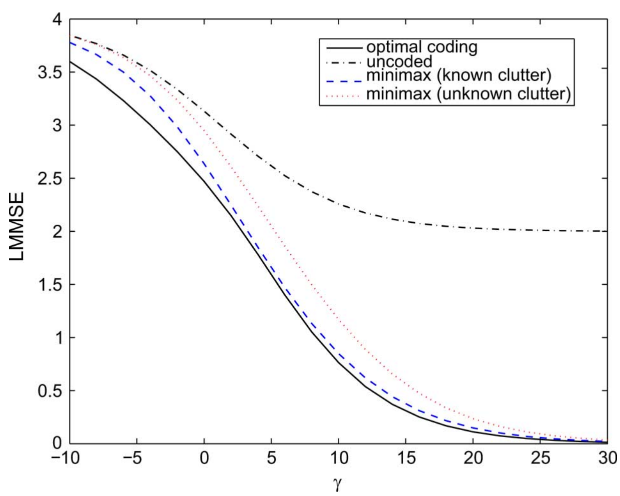

Fig. 7. LMMSE versus γ, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of b$\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$.

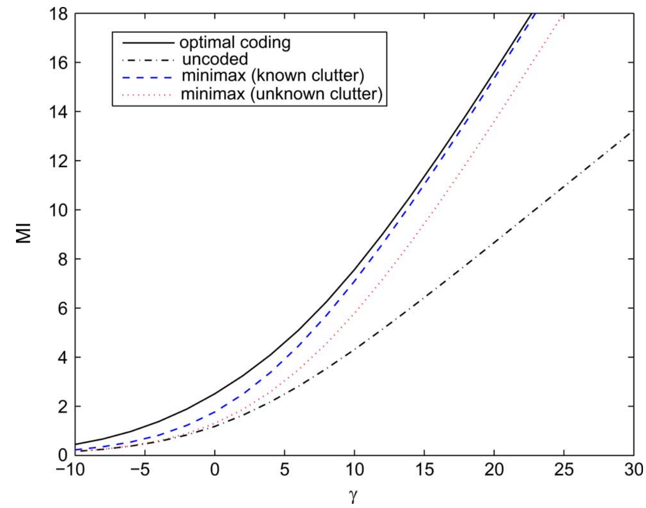

Fig. 8. MI versus γ, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$.

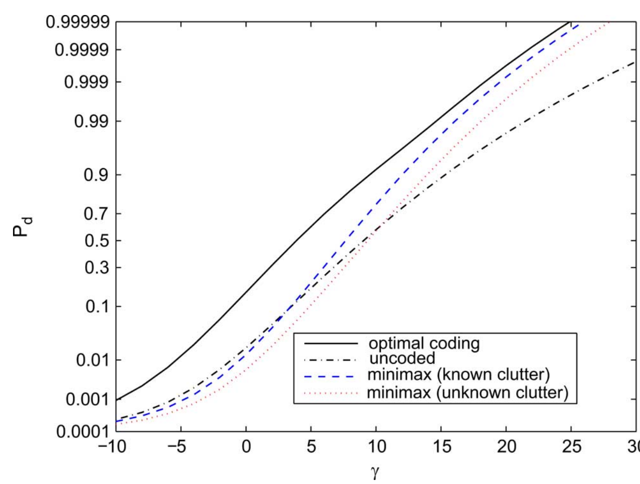

Fig. 9. $P_{\rm d}$ versus γ, Gaussian scattering and disturbance and $\xi = 5$ dB. Curves are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$.

the worst-case loss with respect to the optimal design for small values of γ and/or ill-conditioned $\boldsymbol{R}_\alpha$'s. In this case, indeed, the optimal code is rank-deficient (irrespective of the adopted figure of merit), and the energy should be opportunistically allocated to the most reliable channel mode, in keeping with the diversity-integration tradeoff analysis in [8]. Conversely, the performance of the min-max codes is nearly optimal for large values of the transmit energy contrast and uncorrelated target scattering, in keeping with the asymptotic analysis presented in [9].

In Figs. 6–9, we report the performance of the min-max code design versus γ in terms of SDR, LMMSE, MI, and $P_{\rm d}$, respectively. The plots are obtained by averaging over 1000 random realizations of $\bar{\boldsymbol{R}}_w$ and $\boldsymbol{R}_\alpha$. As a benchmark, we include the corresponding performance achievable under *optimal coding* and that provided by an *uncoded* transmission strategy, wherein $\boldsymbol{A}$ is a scaled all-one matrix. Despite the fact that the min-max codes ensure only maximization of the worst-case SDR, LMMSE and MI, Figs. 6–8 show that they outperform the uncoded transmission even in terms of average SDR, LMMSE and MI. As to the average probability of detection, the uncoded transmission is superior for small values of γ. This is not surprising as, in this region, $P_{\rm d}$ critically depends on the code-matrix rank and better performance is granted by rank-1 coding. Finally, we underline that the prior knowledge of the disturbance covariance matrix can give a significant gain.

VIII. Conclusion

In this work, we have addressed the problem of robust waveform design in the presence of prior uncertainty as to the spatial correlation of the target response for $M \times L$ MIMO radar systems. We have considered a general class of cost functions encompassing several known performance measures such as i) the signal-to-disturbance ratio and the linear minimum mean square error, under very general assumptions on the scattering and the disturbance model and ii) the mutual information and the approximation of the detection probability of the GLRT-detector in the large and small SDR regimes, under Gaussian scattering and disturbance. We have considered both the case that prior information about the second-order statistics of the disturbance are available at the receiver and the case that such information is not accessible. In the former case, the M least interfered directions in the signal space must be selected for transmission and more energy is to be allocated to more interfered modes in order to equalize their SDR's. In the latter, isotropic transmission is the way to go. In any case, the rank of the min-max code-matrix must be equal to M in order to prevent any target miss. All these conclusions hold true independently of the spatial correlation exhibited at the receive sensors. Numerical examples have confirmed the theoretical analysis and have shown that in the presence of erroneous prior information as to the target covariance matrix the system may experience severe losses if no robustification is performed. They have also shown that the performance gap between the min-max design and the optimum one generally reduces as the energy contrast increases and that the min-max strategy almost always outperform uncoded transmissions.

APPENDIX

A. Approximate Miss Probability

Problem 3.1 with the cost function in (13) becomes

$$\min_{\boldsymbol{A}} \max_{\boldsymbol{R}_\alpha} \min_{\gamma} \Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)$$

with the proper constraints. Now, we would like to interchange the order of $\max_{\boldsymbol{R}_\alpha}$ and $\min_\gamma$ invoking the min-max theorem [19, Lemma 36.2]. To this end, we need to show that there exists a saddle point, i.e., a point $(\dot{\gamma}, \dot{\boldsymbol{R}}_\alpha)$ such that

$$\begin{aligned}\Phi&\left(\dot{\gamma}, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\\ &\le \Phi\left(\dot{\gamma}, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\dot{\boldsymbol{R}}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\\ &\le \Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\dot{\boldsymbol{R}}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\end{aligned}$$

for all γ and $\mathbf{R}_\alpha$. However, for each fixed γ, Φ is a function of the form (3) and then, from Section IV-A,

$$\begin{aligned}\Phi&\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\\ &\qquad\le \Phi\left(\gamma, \sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}), 0, \ldots, 0\right)\end{aligned}$$

for all $\boldsymbol{R}_\alpha$ and equality holds if $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{v}$ is the norm-one eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}$. Moreover, since $\Phi(\gamma, \boldsymbol{x})$ admits a minimum over γ for each fixed $\boldsymbol{x}$, it results that

$$\begin{aligned}\dot{\gamma} &= \arg\min_{\gamma} \Phi\left(\gamma, \sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}), 0, \ldots, 0\right)\\ \dot{\boldsymbol{R}}_\alpha &= \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H\end{aligned}$$

is a saddle point and then, from the min-max theorem,

$$\begin{aligned}\min_{\boldsymbol{A}} \max_{\boldsymbol{R}_\alpha} \min_{\gamma} &\Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\\ &= \min_{\boldsymbol{A}} \min_{\gamma} \max_{\boldsymbol{R}_\alpha} \Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\\ &= \min_{\gamma} \min_{\boldsymbol{A}} \max_{\boldsymbol{R}_\alpha} \Phi\left(\gamma, \boldsymbol{\lambda}(\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right).\end{aligned}$$

The proof for Problem III-B.1 is identical.

B. Solution to Problems 6.1 and 6.2

Consider first Problem 6.1. As to the max-part, we have

$$\begin{aligned}f&\left(\boldsymbol{\lambda}\left((\boldsymbol{Q}_w^{-1/2}\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1/2}) \otimes (\boldsymbol{R}_w^{-1/2}\boldsymbol{A}\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1/2})\right)\right)\\ &\le f_1\left(\mathrm{tr}(\boldsymbol{Q}_\alpha\boldsymbol{Q}_w^{-1})\mathrm{tr}(\boldsymbol{R}_\alpha\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})\right)\\ &\le f_1\left(\frac{L\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})}{\lambda_{\max}(\boldsymbol{Q}_w)}\right)\end{aligned} \qquad (14)$$

and equality holds for $\boldsymbol{Q}_\alpha = L\boldsymbol{u}\boldsymbol{u}^H$ and $\boldsymbol{R}_\alpha = \sigma_\alpha^2\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{u}$ is the norm-one eigenvector of $\boldsymbol{Q}_w$ corresponding to its maximum eigenvalue, and $\boldsymbol{v}$ is the norm-one eigenvector of $\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A}$ corresponding to its maximum eigenvalue. At this point, since f_1 is decreasing, $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})$ is to be maximized over $\mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \le \mathcal{E}$ and this problem has been solved in Section IV-A.

Consider now Problem 6.2. Developing (14) and exploiting (10), we have

$$\begin{aligned}f_1&\left(\frac{L\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{R}_w^{-1}\boldsymbol{A})}{\lambda_{\max}(\boldsymbol{Q}_w)}\right)\\ &\qquad\le f_1\left(\frac{L\sigma_\alpha^2\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})}{\big(L-(L-1)\epsilon_0\big)\big(\sigma_w^2-(N-1)\mathcal{N}_0\big)}\right)\end{aligned}$$

and equality holds when $\boldsymbol{Q}_w = \epsilon_0\boldsymbol{I}_L + (L - L\epsilon_0)\boldsymbol{u}\boldsymbol{u}^H$, for some norm-one $\boldsymbol{u} \in \mathbb{C}^L$ and $\boldsymbol{R}_w = \mathcal{N}_0\boldsymbol{I}_N + (\sigma_w^2 - N\mathcal{N}_0)\boldsymbol{v}\boldsymbol{v}^H$, where $\boldsymbol{v}$ is the norm-one eigenvector of $\boldsymbol{A}\boldsymbol{A}^H$ corresponding to its minimum eigenvalue. At this point $\lambda_{\min}(\boldsymbol{A}^H\boldsymbol{A})$ is to be maximized over $\mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \le \mathcal{E}$ and this problem has been solved in Section IV-B.

REFERENCES

[1] E. Fishler, A. Haimovich, R. S. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "MIMO radar: An idea whose time has come," in *Proc. IEEE Int. Radar Conf.*, Philadelphia, PA, Apr. 2004, pp. 71–78.

[2] E. Fishler, A. Haimovich, R. S. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "Performance of MIMO radar systems: Advantages of angular diversity," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov. 2004, vol. 1, pp. 305–309.

[3] E. Fishler, A. Haimovich, R. S. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "Spatial diversity in radars—models and detection performance," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 823–838, Mar. 2006.

[4] A. Haimovich, R. S. Blum, and L. Cimini, "MIMO radars with widely separated antennas," *IEEE Signal Process. Mag.*, vol. 25, no. 1, pp. 116–129, Jan. 2008.

[5] A. De Maio and M. Lops, "Design principles of MIMO radar detectors," presented at the IEEE Waveform Diversity Design Conf., Lihue, HI, Jan. 2006.

[6] A. De Maio and M. Lops, "Design principles of MIMO radar detectors," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 3, pp. 886–898, Jul. 2007.

[7] A. De Maio and M. Lops, *Space-Time Coding in MIMO Radar, MIMO Radar Signal Processing*, Jian-Li and P. Stoica, Eds. New York: Wiley, 2009.

[8] A. De Maio, M. Lops, and L. Venturino, "Diversity-integration tradeoffs in MIMO detection," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5051–5061, Oct. 2008.

[9] A. Aubry, M. Lops, A. M. Tulino, and L. Venturino, "On MIMO detection under non-Gaussian target scattering," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 5822–5838, Nov. 2010.

[10] Y. Yang and R. S. Blum, "Mimo radar waveform design based on mutual information and minimum mean-square error estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 1, pp. 330–343, Jan. 2007.

[11] Y. Yang and R. S. Blum, "Minimax robust MIMO radar waveform design," *IEEE J. Sel. Areas Commun.*, vol. 1, no. 1, pp. 147–155, Jun. 2007.

[12] E. Biglieri, R. Calderbank, A. Costantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2007.
[13] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*. New York: Academic, 1979.
[14] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1993.
[15] N. Levanon, *Radar Principles*. New York: Wiley, 1988.
[16] S. M. Kay, *Fundamental of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.
[17] P. E. Cantrell and A. K. Ojha, "Comparison of generalized Q-function algorithms," *IEEE Trans. Inf. Theory*, vol. 33, no. 4, pp. 591–596, Jul. 1987.
[18] L. Venturino, K. Dong, X. Wang, and M. Lops, "A stochastic-gradient method to design optimal space-time codes for MIMO radar detection," presented at the IEEE Eur. Wireless Conf., Lucca, Italy, Apr. 2010.
[19] R. T. Rockafellar, *Convex Analysis*. Princeton, NJ: Princeton Univ. Press, 1972.

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