Space-Time Coding for MIMO Radar Detection and Ranging

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***Abstract*—Space-time coding (STC) has been shown to play a key role in the design of MIMO radars with widely spaced antennas: In particular, rank-one coding amounts to using the multiple transmit antennas as power multiplexers, while full-rank coding maximizes the transmit diversity, compromises between the two being possible through rank-deficient coding. In detecting a target at known distance and Doppler frequency, no uniformly optimum transmit policy exists, and diversity maximization turns out to be the way to go only in a (still unspecified) large signal-to-noise ratio region. The aim of this paper is to shed some light on the optimum transmit policy as the radar is to detect a target at an *unknown* location: To this end, at first the Cramér–Rao bounds as a function of the STC matrix are computed, and then waveform design is stated as a constrained optimization problem, where now the constraint concerns also the accuracy in target ranging, encapsulated in the Fisher Information on the range estimate. Results indicate that such accuracy constraints may visibly modify the required transmit policy and lead to rank-deficient STC also in regions where pure detection would require pursuing full transmit diversity.**

***Index Terms*—Diversity-integration tradeoff, MIMO radar, parameter estimation, radar detection, space time codes.**

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) radars with widely spaced antennas were introduced as a means to generate angular diversity and improve detection performance [1], [2]: Under additive white Gaussian noise, transmitting orthogonal waveforms results in using all of the available degrees of freedom to generate as many diversity paths. Already in [2], however, it was remarked that MIMO radar achieves a performance gain over the conventional phased array only in the "large" signal-to-noise ratio (SNR) region, using deviation-based arguments. Space-time coding (STC) for waveform design has been introduced in [3] and [4] to cope with detection under possibly correlated clutter and, more generally, to introduce a further degree of freedom at the transmitter side. Adopting a generalized likelihood ratio test (GLRT) for detecting targets at known delay and Doppler frequency, a number of figures of merit have been introduced and examined for performance optimization purposes, based on the evidence that no uniformly optimum coding strategy exists maximizing the detection probability (P_d) under a false alarm probability $(P_{\rm fa})$ constraint. Such figures of merit as lower Chernoff bound (LCB) to the detection probability and mutual information (MI) between the received signal and the target scattering [3], [4]—the latter first introduced in [5] in the context of radar detection and also proposed in [6] for MIMO radar waveform design—lead to somewhat more tractable optimization problems. In particular, LCB-optimal codes have low rank in the low SNR ratio region, while requiring full-rank transmission only for large SNRs: Since rank-deficient coding amounts to giving up some transmit diversity in favor of stronger signals on the surviving paths, this behavior of MIMO radars has been termed "diversity–integration tradeoff" in [4] and is fully consistent with the results of [2]. As to MI-optimal codes, their rank is always maximum—i.e., full-rank for power-unlimited systems and as large as possible for power-limited systems—as a consequence of the concavity and Schur-concavity of MI [4]: This behavior is consistent with the fact that optimizing MI amounts to neglecting the prior uncertainty as to the target presence, which raises more than one doubt on the suitability of MI as a performance measure when the goal is target detection only.

In this paper, we consider the more realistic scenario wherein both detection and ranging are to be undertaken, so that the MIMO radar task is to jointly detect a target and estimate its delay. Inherent in this scenario is the assumption that the area under control is "sectorized," i.e., that the range cells are processed in groups "small" enough to corroborate the assumption that at most one target may be present in each group: However, it should be underlined that our study is mainly theoretical, and is, in fact, aimed at shedding some light on the best STC strategies (and, hence, transmit policies) in more realistic scenarios where a MIMO radar is required to operate.

In the context above, we first derive a GLRT for detection and ranging, which is obviously a generalization of the one derived in [3] and [4] for detection only. Unfortunately, the task of direct optimization of the detection probability is unmanageable, since no closed-form expression of the pair $(P_{\rm fa}, P_d)$ exists as the delay is unknown, whereby we take the approach of decoupling detection and estimation for waveform optimization purposes: From a mathematical point of view, this amounts to designing the STC as the solution to a constrained maximization of some detection-oriented figures of merit—such as the detection probability and the MI for known target delay—under semi-definite constraints concerning both the transmit/receive energy *and* the Fisher information (FI) for delay estimation—which amounts to setting a lower bound to the maximum achievable accuracy in

Manuscript received December 09, 2009; accepted August 19, 2010. Date of publication September 02, 2010; date of current version November 17, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Gerald Matz. This work was supported in part by the U.S. Office of Naval Research under Grant N00014-08-1-0318.

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Digital Object Identifier 10.1109/TSP.2010.2072923

target ranging. The results are interesting under several points of view. First of all, closed-form expressions of the FI for arbitrary STC and target scattering correlation are derived, along with some interesting symmetry properties thereof (and, hence, of the Cramér–Rao bound of unbiased delay estimators). The additional "accuracy" constraint has a deep impact on the optimum transmit policy. It is in particular interesting the behavior of MI-optimal codes, wherein the constraint on FI destroys the uniform optimality of full-rank coding: More and more stringent constraints on delay estimation accuracy result in lower ranks of both MI-optimal and P_d-optimal codes, or, equivalently, shift to the right the "large SNR" region, wherein full-rank coding is optimum, which also indirectly "rehabilitates" MI as a possible figure of merit in MIMO radar.

The remainder of the paper is organized as follows. The signal model and the problem statement are presented in Section II, while Section III introduces the optimization criteria and contains the derivation of the Fisher information for STC MIMO radars. Code design is given in Section IV, while Section V provides numerical results. Conclusions and hints for future research are finally given in Section VI.

II. System Description

We consider a MIMO radar architecture with M and L widely spaced (isotropic) transmit and receive antennas, respectively. The ith transmit antenna forms its baseband equivalent signal as a linear combination of signals in an orthonormal basis, i.e.,

$$s_i(t) = \sum_{n=1}^{N} a_{\text{in}}\phi_n(t), \quad 0 \le t \le T_s, \quad i = 1 \dots M \tag{1}$$

where $\boldsymbol{a}_i = [a_{i1} \dots a_{iN}]^T \in \mathbb{C}^N$ is the codeword associated with the ith transmit antenna, N is the signal-space dimension, $\{\phi_n(t)\}_{n=1}^N$ is the orthonormal basis of the signal space, and T_s is the duration of the transmitted signal.

The signals received at the L receive antennas may contain or not the echo scattered by a target located at an unknown range, i.e., for $j = 1 \dots L$ we have

$$r_j(t) = \begin{cases} w_j(t), & \text{under } \mathcal{H}_0 \\ \sum_{i=1}^{M} \alpha_{ji}\, s_i(t-\tau) + w_j(t), & \text{under } \mathcal{H}_1 \end{cases} \tag{2}$$

where τ is the unknown time-delay the signals experience because of the path from the transmit antennas to the target *and* the path between the target and the receive antennas; as to $w_j(t)$, it is a (complex) white noise term of power spectral density N_0. In this paper we assume that the noise is Gaussian and white, i.e., we do not consider clutter: In principle, the case of colored Gaussian noise could be however handled through a whitening approach. It is further assumed that the narrowband assumption of [2], [7] is met, so that the target is seen as belonging to the same range cell by all of the receive antennas, or, equivalently, that cell synchronization has been undertaken. The antenna spacing is assumed wide enough as to allow angle diversity [2], which means that target scattering is modeled through LM different coupling coefficients α_{ji} for $j = 1 \dots L$ and $i = 1 \dots M$. Finally, the target, if present, is assumed either to be stationary or to have a known Doppler shift that is compensated for at the receiver [8]. Before proceeding, it is worth underlining that model (2) does not cover all of the possible scenarios a MIMO radar may operate in. In particular, the "narrow-band assumption" advocated in this paper corresponds to assuming that path delays are unresolvable (i.e., the parameter τ is one and the same for all received signals), since the transmitted waveform is in fact "narrow-band." This scenario echoes the context where the MIMO idea has been conceived: In a bandwidth-constrained system, additional degrees of freedom are generated out of a theoretically unlimited resource, the physical space, with no additional frequency occupancy. Other possible scenarios are those wherein the transmitted bandwidth is large enough as to resolve all of the delays "most of the time," and are described by a set $\{\tau_{i,j}\}$, $i = 1, \dots, M$, and $j = 1, \dots, L$, which would result in the possibility of undertaking both ranging and localization (see, e.g., [9]).

Let $\boldsymbol{g}_j = [\alpha_{j,1} \dots \alpha_{j,M}]^T$, $\boldsymbol{A} = (\boldsymbol{a}_1 \dots \boldsymbol{a}_M) \in \mathbb{C}^{N\times M}$ and $\boldsymbol{\phi}(t) = [\phi_1(t) \dots \phi_N(t)]^T$. If we define $\boldsymbol{s}(t) = [s_1(t) \dots s_M(t)]^T$, then

$$\boldsymbol{s}(t) = \boldsymbol{A}^T\boldsymbol{\phi}(t) \tag{3}$$

and (2) can be written in matrix notation as follows.

$$r_j(t) = \begin{cases} w_j(t), & \text{under } \mathcal{H}_0 \\ \boldsymbol{g}_j^T\boldsymbol{A}^T\boldsymbol{\phi}(t-\tau) + w_j(t), & \text{under } \mathcal{H}_1. \end{cases} \tag{4}$$

We refer to the matrix $\boldsymbol{A}$ as the code matrix, and $\delta = \text{rank}(\boldsymbol{A})$ is the number of degrees of freedom used by the selected code, with $1 \le \delta \le \min(N, M)$: δ is of course a major design parameter, which determines the transmit policy.

The transmit energy E_t is fixed and is given by

$$E_t = \text{Tr}\left(\int_{T_s} \boldsymbol{s}(t)\boldsymbol{s}^H(t)dt\right) = \text{Tr}(\boldsymbol{A}^T\boldsymbol{A}^*) = \sum_{i=1}^{M}\lambda_i^2 \tag{5}$$

where $\{\lambda_i\}_{i=1}^M$ is the set of singular values of $\boldsymbol{A}$. The transmission signal-to-noise ratio, denoted by SNR, is given by SNR $= E_t/N_0$.

Denote $\boldsymbol{r}(t) = [r_1(t) \dots r_L(t)]^T$ as the vector of observations, $\boldsymbol{w}(t) = [w_1(t) \dots w_L(t)]^T$ as the overall disturbance and $\boldsymbol{H} = [\boldsymbol{g}_1 \dots \boldsymbol{g}_L]^T$ as the random matrix modeling the target scattering. Then, when a target is present, the received vector is given by

$$\boldsymbol{r}(t) = \boldsymbol{H}\boldsymbol{s}(t-\tau) + \boldsymbol{w}(t) = \boldsymbol{H}\boldsymbol{A}^T\boldsymbol{\phi}(t-\tau) + \boldsymbol{w}(t). \tag{6}$$

Let us assume that $M \le N$[1], and let $\boldsymbol{A}^T$ have a singular value decomposition given by $\boldsymbol{A}^T = \boldsymbol{U\Sigma V}$, with $\boldsymbol{U} \in \mathbb{C}^{M\times M}$ an unitary matrix, $\boldsymbol{\Sigma} \in \mathbb{C}^{M\times M}$ a diagonal matrix with entries $\lambda_1 \ge \lambda_2 \ge \dots \ge \lambda_\delta > \lambda_{\delta+1} = \dots = \lambda_M = 0$, and $\boldsymbol{V} \in \mathbb{C}^{M\times N}$ a matrix with orthonormal rows [11]. Then (6) becomes

$$\begin{aligned} \boldsymbol{r}(t) &= \underbrace{\boldsymbol{HU}}_{\hat{\boldsymbol{H}}}\boldsymbol{\Sigma}\underbrace{\boldsymbol{V\phi}(t-\tau)}_{\boldsymbol{\theta}(t-\tau)} + \boldsymbol{w}(t) \\ &= \hat{\boldsymbol{H}}\boldsymbol{\Sigma}\boldsymbol{\theta}(t-\tau) + \boldsymbol{w}(t) \end{aligned} \tag{7}$$

where $\boldsymbol{\theta}(t-\tau)$ defines a new orthonormal set of functions, and $\hat{\boldsymbol{H}} = \boldsymbol{HU} = [\hat{\boldsymbol{g}}_1 \cdots \hat{\boldsymbol{g}}_L]^T$.

[1]Indeed, the case $M > N$ is readily shown to be equivalent to using only $\min(M, N)$ antennas [10].

In this work, we investigate the problem of waveform design focusing on the construction of the matrix[2] $\boldsymbol{\Sigma}$. Given that we observe $\boldsymbol{r}(t)$, we present two design criteria to find the optimum $\boldsymbol{\Sigma}$ in the scenario where it is necessary to decide whether a target is present in a field under observation and, if it is declared to be present, an estimate of the time-delay associated with its position is required. In particular, we aim at finding the optimum rank δ of the code matrix $\boldsymbol{A}$, i.e., the number of its nonzero eigenvalues, which defines the transmit policy.

III. Performance Parameters for Waveform Design

Optimized waveform design for the coupled problem of target detection and parameter estimation would be in principle feasible by choosing the space-time code matrix $\boldsymbol{A}$ so as to maximize the detection probability subject to constraints on both the false alarm probability and the transmitted power[3]: This would automatically allow taking into account both the detection performance and the delay estimation accuracy.

Unfortunately, the task appears mathematically intractable, hardly leading to closed-form results and ultimately preventing to draw general conclusions. We prefer to take a different, more intuitive approach instead, wherein waveform optimization results from maximization of a detection-oriented figure of merit assuming perfect knowledge of the target delay subject to constraints on the transmitted power *and* to the maximum achievable accuracy in target ranging. At the waveform design and performance assessment stage, we assume that the target scattering matrix is a complex Gaussian matrix with independent and identically distributed circularly symmetric entries.

We start this section presenting the coupled problem of target detection and parameter estimation and its solution. We then present two different criteria for finding the optimized waveforms.

A. Target Detection and Parameter Estimation

The joint target detection and parameter estimation problem under study can be regarded as a composite hypothesis test, wherein the unknown quantities are the vectors $\{\hat{\boldsymbol{g}}_j\}_{j=1}^L$ and the delay τ. Under these conditions, a design strategy is the GLRT, which amounts to maximum-likelihood estimating the unknown parameters based upon the observations and plugging them back into the conditional likelihood ratio [8]. In order to determine such a conditional likelihood, we first represent the received signals in the subspace spanned by the orthonormal signal set $\boldsymbol{\theta}(t-\tau)$: Since the noise is Gaussian and white, the irrelevance theorem ensures that retaining only the projections of the observed waveforms onto such a subspace entails no loss of optimality. As a consequence, the quantities $\boldsymbol{r}_j(\tau)$

$$\boldsymbol{r}_j(\tau) = \int r_j(t)\boldsymbol{\theta}^*(t-\tau)dt, \quad j = 1\dots L \tag{8}$$

representing the projections of the signal received at the jth antenna onto $\boldsymbol{\theta}(t-\tau)$ form a set of sufficient statistic.

The GLRT is thus given by

$$Q_{\mathrm{MAX}} \triangleq \max_{\tau,\hat{\boldsymbol{g}}_1,\dots,\hat{\boldsymbol{g}}_L} \times \log \frac{f(\boldsymbol{r}_1(\tau),\dots,\boldsymbol{r}_L(\tau) \mid \mathcal{H}_1,\tau,\hat{\boldsymbol{g}}_1,\dots,\hat{\boldsymbol{g}}_L)}{f(\boldsymbol{r}_1(\tau),\dots,\boldsymbol{r}_L(\tau) \mid \mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \nu \tag{9}$$

where ν is the detection threshold which is set to achieve a given probability of false alarm. Specializing the test (9) to the case of additive white Gaussian noise we obtain the following theorem.

Theorem 1: The test statistic in (9) is equivalent to

$$Q_{\mathrm{MAX}} = \max_\tau Q(\tau) = \max_\tau \frac{1}{N_0}\sum_{j=1}^L \left\|\boldsymbol{r}_j^\delta(\tau)\right\|^2 \tag{10}$$

where $\boldsymbol{r}_j^\delta(\tau)$ represents the first δ components of $\boldsymbol{r}_j(\tau)$, namely, for $j = 1\dots L$,

$$\boldsymbol{r}_j(\tau) = \begin{pmatrix} \boldsymbol{r}_j^\delta(\tau) \\ \boldsymbol{r}_j^{M-\delta}(\tau) \end{pmatrix}. \tag{11}$$

Proof: To find the conditional likelihood of the observations given $\tau,\hat{\boldsymbol{g}}_1,\dots,\hat{\boldsymbol{g}}_L$, we use the representation basis whose first M signals are $\boldsymbol{\theta}(t-\tau) = [\theta_1(t-\tau)\dots\theta_M(t-\tau)]^T$. Then, the set of vectors $\{\boldsymbol{r}_j(\tau)\}_{j=1}^L$ as defined in (8) is a sufficient statistic. The test is then given by

$$\boldsymbol{r}_j(\tau) = \begin{cases} \boldsymbol{\Sigma}\hat{\boldsymbol{g}}_j + \boldsymbol{w}_j, & \text{under } \mathcal{H}_1 \\ \boldsymbol{w}_j, & \text{under } \mathcal{H}_0. \end{cases} \tag{12}$$

Notice that the log-likelihood ratio is given by

$$\begin{aligned} Q(\tau,\hat{\boldsymbol{g}}_1,\dots,\hat{\boldsymbol{g}}_L) &\triangleq \log\frac{f(\boldsymbol{r}_1(\tau),\dots,\boldsymbol{r}_L(\tau)\,|\,\mathcal{H}_1,\tau,\hat{\boldsymbol{g}}_1,\dots,\hat{\boldsymbol{g}}_L)}{f(\boldsymbol{r}_1(\tau),\dots,\boldsymbol{r}_L(\tau)\,|\,\mathcal{H}_0)} \\ &= -\frac{1}{N_0}\sum_{j=1}^L(\|\boldsymbol{r}_j(\tau)-\boldsymbol{\Sigma}\hat{\boldsymbol{g}}_j\|^2 - \|\boldsymbol{r}_j(\tau)\|^2). \end{aligned} \tag{13}$$

Omitting the dependency on τ, we first maximize over the $\{\tilde{\boldsymbol{g}}_j\}_{j=1}^L$. The estimates $\bar{\boldsymbol{g}}_j$ are found by solving, for $j = 1,\dots,L$,

$$\bar{\boldsymbol{g}}_j = \arg\min_{\tilde{\boldsymbol{g}}_j} \|\boldsymbol{r}_j - \boldsymbol{\Sigma}\tilde{\boldsymbol{g}}_j\|^2. \tag{14}$$

Let $\boldsymbol{\Sigma}_\delta = \mathrm{diag}(\lambda_1,\dots,\lambda_\delta)$ be the diagonal matrix containing the nonzero diagonal elements of $\boldsymbol{\Sigma}$. Decomposition (11) of $\boldsymbol{r}_j$ induces a similar decomposition on $\bar{\boldsymbol{g}}_j$ as $\bar{\boldsymbol{g}}_j = \begin{pmatrix} \bar{\boldsymbol{g}}_j^\delta \\ \bar{\boldsymbol{g}}_j^{M-\delta} \end{pmatrix}$, whereby

$$\bar{\boldsymbol{g}}_j^\delta = \boldsymbol{\Sigma}_\delta^{-1}\boldsymbol{r}_j^\delta. \tag{15}$$

As to $\bar{\boldsymbol{g}}_j^{M-\delta}$, it is obviously irrelevant, since we are not interested in estimating the target scattering, but only in declaring

[2]The reason is that we assume uncorrelated and Gaussian target scattering and white noise, whereby the two unitary matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are not a relevant design parameter, unlike what happens when correlated noise is considered.

[3]A customary constraint that is also forced is one concerning the received average signal-to-clutter ratio, but dealing with both cases would not add much insight to the discussion.

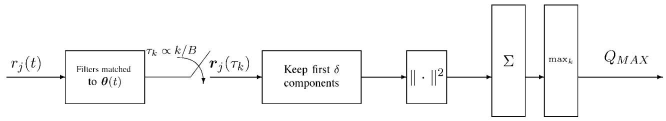

Fig. 1. Scheme of the processing chain implied by GLRT: focusing on the jth receive antenna.

its presence. Based on the above derivations, we immediately obtain

$$\begin{aligned} Q(\tau) &\triangleq \max_{\tilde{\boldsymbol{g}}_1,\ldots,\tilde{\boldsymbol{g}}_L} \log \frac{f(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L \mid \mathcal{H}_1,\tau,\tilde{\boldsymbol{g}}_1,\ldots,\tilde{\boldsymbol{g}}_L)}{f(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L \mid \mathcal{H}_0)} \\ &= -\frac{1}{N_0}\sum_{j=1}^{L}\left(\left\|\boldsymbol{r}_j-\boldsymbol{r}_j^{\delta}\right\|^2-\|\boldsymbol{r}_j\|^2\right) \qquad (16)\\ &= \frac{1}{N_0}\sum_{j=1}^{L}\left\|\boldsymbol{r}_j^{\delta}\right\|^2 \qquad (17)\end{aligned}$$

whereby the GLRT is easily seen to admit the form (10). ■

Before proceeding in our discussion, it is worth pointing out that the test (10) amounts to undertaking, at each receive antenna, customary filtering through a bank of filters keyed to all of the possible delays: For example, Fig. 1 represents a practical implementation of the receiver operation as a signal of bandwidth B is transmitted, so that the accuracy in delay estimation is proportional to $1/B$ and the maximization can be undertaken on a discrete set of delays. The signal picked up by antenna j is filtered through a (noncausal) filter matched to the vector signal $\boldsymbol{\theta}(t)$, and the output is sampled with rate proportional to B. Only the first δ components of the vector are kept and they subsequently undergo square-law detection. Summing across all of the L receive antennas and maximizing over all of the range cells yields finally the test statistic Q_{MAX} whose level is to be compared to the detection threshold to make the final decision.

In the context outlined above, we are now ready to tackle the problem of waveform design, which is discussed next.

B. Criteria for Waveform Design

Denoting $f(\boldsymbol{\Sigma})$ a general detection-oriented figure of merit, we focus our attention on the following problem:

$$\max_{\boldsymbol{\Sigma}} \quad f(\boldsymbol{\Sigma}) \qquad (18)$$

$$\text{given} \begin{cases} \text{power constraint} \\ I_\tau \geq \beta \\ P_{\mathrm{fa}}. \end{cases} \qquad (19)$$

In the above equation, $I_\tau = I_\tau(\boldsymbol{\Sigma})$ is the FI on the delay estimation, whose inverse gives the Cramér–Rao bound of any unbiased estimator of τ, and β is a lower bound for the FI.

1) Detection-Oriented Figures of Merit: In defining $f(\boldsymbol{\Sigma})$, we assume that the delay is perfectly known at the receiver, whereby the test statistic is now simply $Q(\tau)$, no maximization being necessary: From now on, we omit the dependency on the delay whenever we deal with the case of known τ.

We thus consider two possible figures of merit, i.e., the detection probability under a given false alarm constraint, and the mutual information between the set of the observations $\{\boldsymbol{r}_j\}_{j=1}^L$ and the scattering vectors $\{\tilde{\boldsymbol{g}}_j\}_{j=1}^L$.

Under $\mathcal{H}_0$, $\boldsymbol{r}_j = \boldsymbol{w}_j \sim \mathcal{CN}(0, N_0\boldsymbol{I})$. Then Q is a complex chi-squared random variable of δL complex degrees of freedom. Therefore, given a desired probability of false alarm, the detection threshold ν is a function of δ, i.e., $\nu = \nu(\delta)$, and is related to P_{fa} as [12]

$$P_{\mathrm{fa}} = e^{-\nu(\delta)} \sum_{k=0}^{\delta L-1} \frac{\nu(\delta)^k}{k!}. \qquad (20)$$

On the other hand, under $\mathcal{H}_1$, we assume the vectors $\boldsymbol{g}_j$ to be mutually independent zero-mean complex Gaussian random vectors with $\mathbb{E}\{\boldsymbol{g}_j\boldsymbol{g}_j^H\} = \boldsymbol{I}$ $(j = 1,\ldots,L)$ for the analysis, where $\boldsymbol{g}_j$ was defined in (4). Then, $\mathbb{E}\{\hat{\boldsymbol{g}}_j\hat{\boldsymbol{g}}_j^H\} = \boldsymbol{I}$, $\boldsymbol{r}_j \sim \mathcal{CN}(0, \boldsymbol{\Sigma}^2 + N_0\boldsymbol{I})$ and $\boldsymbol{r}_j^\delta \sim \mathcal{CN}(0, \boldsymbol{\Sigma}_\delta^2 + N_0\boldsymbol{I})$, with $\boldsymbol{\Sigma}_\delta = \mathrm{diag}(\lambda_1,\ldots,\lambda_\delta)$ and

$$Q = Q(\delta,\lambda_1,\ldots,\lambda_\delta) = \sum_{j=1}^{L}\sum_{i=1}^{\delta}\left(\frac{\lambda_i^2}{N_0}+1\right)|v_{i,j}|^2 \qquad (21)$$

where $v_{i,j} \sim \mathcal{CN}(0,1)$ is a complex Gaussian random variable. For future reference we thus define

$$f_d(\boldsymbol{\Sigma}) = \Pr\{Q(\delta,\lambda_1,\ldots,\lambda_\delta) > \nu(\delta) \mid \mathcal{H}_1\} \qquad (22)$$

where $\nu(\delta)$ is chosen based on (20). This represents one possible figure of merit to be plugged in (18) in place of $f(\boldsymbol{\Sigma})$.

As an alternative, we also consider the mutual information

$$f_I(\boldsymbol{\Sigma}) = I(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L;\tilde{\boldsymbol{g}}_1,\ldots,\tilde{\boldsymbol{g}}_L) = \sum_{i=1}^{\delta}\log\left(\frac{\lambda_i^2}{N_0}+1\right). \qquad (23)$$

C. Fisher's Information

Let $\hat{\tau} = \hat{\tau}(\boldsymbol{r}(t))$ be an estimate of the time-delay based on the observations $\boldsymbol{r}(t)$, and denote $b(\tau)$ its bias. The estimator variance is bounded through the information inequality [13] as

$$\mathrm{Var}\{\hat{\tau}(\boldsymbol{r}(t))\} \geq \frac{\left[1+\frac{\partial b(\tau)}{\partial\tau}\right]^2}{I_\tau} \qquad (24)$$

where

$$I_\tau = -\mathbb{E}\left\{\frac{\partial^2}{\partial\tau^2}\Lambda(\boldsymbol{r}(t);\tau)\right\} \qquad (25)$$

is the Fisher's information for estimating τ from the observations $\boldsymbol{r}(t)$. In (25), $\Lambda(\boldsymbol{r}(t);\tau)$ denotes the log-likelihood functional of τ, obtained by using the density under $\mathcal{H}_0$ as the "dominating" density, as suggested by Grenander's convergence theorem [13]: We hasten to emphasize here that the result would not change if we used any other density "dominating" the density under $\mathcal{H}_1$, the only constraint being that the probability measure under $\mathcal{H}_1$ be *differentiable*—and, hence, absolutely continuous—with respect to the normalizing probability measure.

In what follows, we assume that the scattering vectors $\hat{\boldsymbol{g}}_j$ are mutually independent zero-mean complex Gaussian random vectors: To keep the derivation as general as possible, we allow them to possess an arbitrary autocorrelation, i.e., $\boldsymbol{R}_j = \mathbb{E}\{\tilde{\boldsymbol{g}}_j\tilde{\boldsymbol{g}}_j^H\}$. Moreover, we introduce the short-hand notation $H\{\boldsymbol{B}\} = (\boldsymbol{B}+\boldsymbol{B}^H)/2$ for the Hermitian part of an arbitrary complex matrix $\boldsymbol{B}$. The FI for time-delay estimation is thus given by the following theorem.

Theorem 2: The Fisher's Information for the time-delay estimate is given by

$$I_\tau = -\frac{2}{N_0^2}\sum_{j=1}^{L}\mathrm{Tr}\left\{\boldsymbol{\Sigma}\left(\boldsymbol{R}_j^{-1}+\frac{\boldsymbol{\Sigma}\boldsymbol{\Sigma}}{N_0}\right)^{-1}\boldsymbol{\Sigma}^H \times\left(H\left\{\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_2^H\right\}+\boldsymbol{Z}_1\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_1^H\right)\right\}$$

where

$$\begin{aligned}\boldsymbol{Z}_1 &= \int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\boldsymbol{\theta}^T(t-\tau)dt\\ &= \boldsymbol{U}^H\int\frac{\partial\boldsymbol{\phi}^*(t-\tau)}{\partial\tau}\boldsymbol{\phi}^T(t-\tau)dt\,\boldsymbol{U}\\ \boldsymbol{Z}_2 &= \int\frac{\partial^2\boldsymbol{\theta}^*(t-\tau)}{\partial\tau^2}\boldsymbol{\theta}^T(t-\tau)dt\\ &= \boldsymbol{U}^H\int\frac{\partial^2\boldsymbol{\phi}^*(t-\tau)}{\partial\tau^2}\boldsymbol{\phi}^T(t-\tau)dt\,\boldsymbol{U}.\end{aligned}$$

Proof: Notice first that under $\mathcal{H}_0$, for $j=1,\ldots,L$, we have $\boldsymbol{r}_j\sim\mathcal{CN}(0,N_0\boldsymbol{I})$. On the other hand, given the assumption that $\hat{\boldsymbol{g}}_j\sim\mathcal{CN}(0,\boldsymbol{R}_j)$ are mutually independent, we have that under $\mathcal{H}_1, \boldsymbol{r}_j\sim\mathcal{CN}(0,\boldsymbol{\Sigma}\boldsymbol{R}_j\boldsymbol{\Sigma}^H+N_0\boldsymbol{I})$ and the following.

$$\begin{aligned}&\log\left(\frac{f(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L\,|\,\tau,H_1)}{f(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_L\,|\,H_0)}\right)\\ &= -\sum_{j=1}^{L}\left(\boldsymbol{r}_l^H(\boldsymbol{\Sigma}\boldsymbol{R}_j\boldsymbol{\Sigma}^H+N_0\boldsymbol{I})^{-1}\boldsymbol{r}_l-\frac{1}{N_0}\boldsymbol{r}_j^H\boldsymbol{r}_j\right)\\ &\quad-\sum_{j=1}^{L}\log\left(\frac{\det(\boldsymbol{\Sigma}\boldsymbol{R}_j\boldsymbol{\Sigma}^H+N_0\boldsymbol{I})}{\det(N_0\boldsymbol{I})}\right) \qquad (26)\\ &= \frac{1}{N_0^2}\sum_{j=1}^{L}\boldsymbol{r}_j^H\boldsymbol{\Sigma}\left(\boldsymbol{R}_j^{-1}+\frac{\boldsymbol{\Sigma}^H\boldsymbol{\Sigma}}{N_0}\right)^{-1}\boldsymbol{\Sigma}^H\boldsymbol{r}_j\\ &\quad-\sum_{j=1}^{L}\log\left(\frac{\det(\boldsymbol{\Sigma}\boldsymbol{R}_j\boldsymbol{\Sigma}^H+N_0\boldsymbol{I})}{\det(N_0\boldsymbol{I})}\right) \qquad (27)\\ &= \frac{1}{N_0^2}\sum_{j=1}^{L}\mathrm{Tr}\left\{\boldsymbol{\Sigma}\left(\boldsymbol{R}_j^{-1}+\frac{\boldsymbol{\Sigma}^H\boldsymbol{\Sigma}}{N_0}\right)^{-1}\boldsymbol{\Sigma}^H\boldsymbol{r}_j\boldsymbol{r}_j^H\right\}\\ &\quad-\sum_{j=1}^{L}\log\left(\frac{\det(\boldsymbol{\Sigma}\boldsymbol{R}_j\boldsymbol{\Sigma}^H+N_0\boldsymbol{I})}{\det(N_0\boldsymbol{I})}\right) \qquad (28)\end{aligned}$$

where the matrix inversion lemma and the trace invariance under cyclic permutations have been used in the first and the second step, respectively.

The correlation of $r_j(t)$ is instrumental to deriving the elements of the Fisher information matrix and is given by

$$\mathbb{E}\{r_j(t)r_j^*(u)\}=\boldsymbol{\theta}^T(t-\tau)\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{\theta}^*(u-\tau)+N_0\delta(t-u). \qquad (29)$$

The expressions of $\boldsymbol{r}_j$, $\partial\boldsymbol{r}_j/\partial\tau$, and $\partial^2\boldsymbol{r}_j/\partial\tau^2$ are also needed in order to derive I_τ and are given, respectively, by

$$\boldsymbol{r}_j=\int r_j(t)\boldsymbol{\theta}^*(t-\tau)dt \qquad (30)$$

$$\frac{\partial\boldsymbol{r}_j}{\partial\tau}=\int r_j(t)\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}dt \qquad (31)$$

and

$$\frac{\partial^2\boldsymbol{r}_j}{\partial\tau^2}=\int r_j(t)\frac{\partial^2\boldsymbol{\theta}^*(t-\tau)}{\partial\tau^2}dt. \qquad (32)$$

Then, given the expression of the log-likelihood ratio (26), the FI I_τ is

$$\begin{aligned}I_\tau &= -\mathbb{E}\left[\frac{\partial^2\log\Lambda}{\partial\tau^2}\right]\\ &= -\frac{1}{N_0^2}\sum_{j=1}^{L}\mathrm{Tr}\left\{\boldsymbol{\Sigma}\left(\boldsymbol{R}_j^{-1}+\frac{\boldsymbol{\Sigma}^H\boldsymbol{\Sigma}}{N_0}\right)^{-1}\boldsymbol{\Sigma}^H\,\mathbb{E}\left[\frac{\partial^2\boldsymbol{r}_j\boldsymbol{r}_j^H}{\partial\tau^2}\right]\right\}.\end{aligned}$$

Noticing that $\mathbb{E}[\partial^2\boldsymbol{r}_j\boldsymbol{r}_j^H/\partial\tau^2]=2H\{\mathbb{E}[\boldsymbol{r}_j\partial^2\boldsymbol{r}_j^H/\partial\tau^2]\}+2\mathbb{E}[(\partial\boldsymbol{r}_j/\partial\tau)(\partial\boldsymbol{r}_j^H/\partial\tau)]$, and using (29)–(32), we have

$$\begin{aligned}&\mathbb{E}\left[\frac{\partial^2\boldsymbol{r}_j\boldsymbol{r}_j^H}{\partial\tau^2}\right]\\ &= 2H\left\{\int\int\boldsymbol{\theta}^*(t-\tau)\mathbb{E}[r_j(t)r_j^*(u)]\frac{\partial^2\boldsymbol{\theta}^T(u-\tau)}{\partial\tau^2}dtdu\right\}\\ &\quad+2\int\int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\mathbb{E}[r_j(t)r_j^*(u)]\frac{\partial\boldsymbol{\theta}^T(u-\tau)}{\partial\tau}dtdu\\ &= 2H\left\{\int\int\boldsymbol{\theta}^*(t-\tau)\boldsymbol{\theta}^T(t-\tau)\boldsymbol{\Sigma}^T \times\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{\theta}^*(u-\tau)\frac{\partial^2\boldsymbol{\theta}^T(u-\tau)}{\partial\tau^2}dtdu\right\}\\ &\quad+2N_0H\left\{\int\int\boldsymbol{\theta}^*(t-\tau)\delta(t-u)\frac{\partial^2\boldsymbol{\theta}^T(u-\tau)}{\partial\tau^2}\right\}\\ &\quad+2\int\int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\boldsymbol{\theta}^T(t-\tau) \times\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{\theta}^*(u-\tau)\frac{\partial\boldsymbol{\theta}^T(u-\tau)}{\partial\tau}dtdu\\ &\quad+2N_0\int\int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\delta(t-u)\frac{\partial\boldsymbol{\theta}^T(u-\tau)}{\partial\tau}dtdu\\ &= 2H\left\{\underbrace{\int\boldsymbol{\theta}^*(t-\tau)\boldsymbol{\theta}^T(t-\tau)dt}_{\boldsymbol{I}} \times\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\underbrace{\int\boldsymbol{\theta}^*(u-\tau)\frac{\partial^2\boldsymbol{\theta}^T(u-\tau)}{\partial\tau^2}du}_{\boldsymbol{Z}_2^H}\right\}\\ &\quad+2N_0H\left\{\int\boldsymbol{\theta}^*(t-\tau)\frac{\partial^2\boldsymbol{\theta}^T(t-\tau)}{\partial\tau^2}dt\right\}\\ &\quad+2\underbrace{\int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\boldsymbol{\theta}^T(t-\tau)dt}_{\boldsymbol{Z}_1} \times\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\underbrace{\int\boldsymbol{\theta}^*(u-\tau)\frac{\partial\boldsymbol{\theta}^T(u-\tau)}{\partial\tau}du}_{\boldsymbol{Z}_1^H}\\ &\quad+2N_0\int\frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\frac{\partial\boldsymbol{\theta}^T(t-\tau)}{\partial\tau}dt.\end{aligned}$$

It is seen that the second and fourth term sum to zero and can be simplified by noticing that as $\int \boldsymbol{\theta}^*(t-\tau)\boldsymbol{\theta}^T(t-\tau)dt = \boldsymbol{I}$, then

$$\begin{aligned}\frac{\partial^2 \boldsymbol{I}}{\partial\tau^2} &= 0\\ &= \int \frac{\partial^2 \boldsymbol{\theta}^*(t-\tau)\boldsymbol{\theta}^T(t-\tau)}{\partial\tau^2}dt\\ &= 2H\left\{\int \boldsymbol{\theta}^*(t-\tau)\frac{\partial^2\boldsymbol{\theta}^T(t-\tau)}{\partial\tau^2}dt\right\}\\ &\quad + 2\int \frac{\partial\boldsymbol{\theta}^*(t-\tau)}{\partial\tau}\frac{\partial\boldsymbol{\theta}^T(t-\tau)}{\partial\tau}dt.\end{aligned}$$

Therefore,

$$\mathbb{E}\left[\frac{\partial^2 \boldsymbol{r}_j\boldsymbol{r}_j^H}{\partial\tau^2}\right] = 2H\left\{\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_2^H\right\} + 2\boldsymbol{Z}_1\,\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_1^H$$

and I_τ is given by

$$\begin{aligned}I_\tau = -\frac{2}{N_0^2}\sum_{j=1}^{L}\mathrm{Tr}\Bigg\{&\boldsymbol{\Sigma}\left(\boldsymbol{R}_j^{-1}+\frac{\boldsymbol{\Sigma}^H\boldsymbol{\Sigma}}{N_0}\right)^{-1}\\ &\times \boldsymbol{\Sigma}^H\left(H\left\{\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_2^H\right\}+\boldsymbol{Z}_1\,\boldsymbol{\Sigma}^T\boldsymbol{R}_j\boldsymbol{\Sigma}^*\boldsymbol{Z}_1^H\right)\Bigg\}.\end{aligned}$$

∎

In what follows, we assume that the coupling coefficients α_{ij} are independent and identically distributed as $\alpha_{ij}\sim\mathcal{CN}(0,1)$ and we specialize the Fisher's information for a particular orthonormal set of functions. We thus obtain the following.

Lemma 1: Let $\boldsymbol{R}_j = \boldsymbol{I}$ and

$$\phi_n(t) = p(t-(n-1)T),\quad n=1,\ldots,N \tag{33}$$

where $p(t)$ is any unit-energy waveform with Fourier transform $P(f)$ and whose autocorrelation satisfies the condition

$$R_p(u) = 0,\quad \forall u \ge u_p = \frac{1}{B} \ll T \tag{34}$$

and B is a measure of its bandwidth.

Then

$$I_\tau = 2L\Omega_{rms}^2\sum_{i=1}^{M}\frac{\left(\frac{\lambda_i^2}{N_0}\right)^2}{1+\frac{\lambda_i^2}{N_0}} \tag{35}$$

with

$$(\Omega_{\mathrm{rms}})^2 = (2\pi)^2B_{\mathrm{rms}}^2 = -\frac{\partial^2 R_p(0)}{\partial t^2}. \tag{36}$$

where B_{rms} is the root-mean-square (rms) bandwidth of the pulse $p(t)$.

Proof: Condition (34) implies orthonormality of the signals $\phi_n(t)$, for $n = 1\ldots N$, i.e.,

$$\begin{aligned}\langle\phi_n(t),\phi_m(t)\rangle &= \int\phi_n(t)\phi_m^*(t)dt\\ &= R_p((m-n)T) = \delta_{m,n}.\end{aligned} \tag{37}$$

Likewise, we have

$$\begin{aligned}\left\langle\frac{\partial\phi_n(t)}{\partial t},\phi_m(t)\right\rangle &= \int j2\pi f|P(f)|^2e^{j2\pi f(m-n)T}df\\ &= \left.\frac{\partial R_p(t)}{\partial t}\right|_{t=(m-n)T} = 0\end{aligned} \tag{38}$$

in that at point $t = 0$ $R_p(t)$ has a maximum, while vanishing after $1/B \ll T$. We also have

$$\begin{aligned}&\left\langle\frac{\partial^2\phi_n(t)}{\partial t^2},\phi_m(t)\right\rangle\\ &= -\int(2\pi f)^2|P(f)|^2e^{j2\pi f(m-n)T}df\\ &= -\left.\frac{\partial^2R_p(t)}{\partial t^2}\right|_{t=(m-n)T} = (2\pi)^2B_{\mathrm{rms}}^2\delta_{mn}.\end{aligned} \tag{39}$$

Then

$$\begin{aligned}\boldsymbol{Z}_1 &= \boldsymbol{U}^H\int\frac{\partial\boldsymbol{\phi}^*(t-\tau)}{\partial\tau}\boldsymbol{\phi}^T(t-\tau)dt\,\boldsymbol{U} = 0,\\ \boldsymbol{Z}_2 &= \boldsymbol{U}^H\int\frac{\partial^2\boldsymbol{\phi}^*(t-\tau)}{\partial\tau^2}\boldsymbol{\phi}^T(t-\tau)dt\,\boldsymbol{U}\\ &= -\Omega_{\mathrm{rms}}^2\boldsymbol{U}^H\boldsymbol{U} = -\Omega_{\mathrm{rms}}^2\boldsymbol{I}.\end{aligned}$$

Therefore,

$$\begin{aligned}I_\tau &= \frac{2L\Omega_{\mathrm{rms}}^2}{N_0^2}\mathrm{Tr}\left\{\boldsymbol{\Sigma}\left(\boldsymbol{I}+\frac{\boldsymbol{\Sigma}^H\boldsymbol{\Sigma}}{N_0}\right)^{-1}\boldsymbol{\Sigma}^H\,\boldsymbol{\Sigma}^T\boldsymbol{\Sigma}^*\right\}\\ &= 2L\Omega_{\mathrm{rms}}^2\sum_{i=1}^{M}\frac{\left(\frac{\lambda_i^2}{N_0}\right)^2}{1+\frac{\lambda_i^2}{N_0}}.\end{aligned}$$

∎

The properties of FI and its behavior as a function of the squared singular values are presented in Section IV-B.

IV. Code Design

Based on the above derivations, the STC matrix design problem (18) becomes

$$\max_{\boldsymbol{\Sigma}}\quad f(\boldsymbol{\Sigma}) \tag{40}$$

$$\text{given}\quad\begin{cases}\mathrm{Tr}(\boldsymbol{A}^T\boldsymbol{A}^*) = \sum_{i=1}^{M}\lambda_i^2 = E_t\\ 2L\Omega_{\mathrm{rms}}^2\sum_{i=1}^{M}\frac{\left(\frac{\lambda_i^2}{N_0}\right)^2}{1+\frac{\lambda_i^2}{N_0}} \ge \beta.\end{cases} \tag{41}$$

The constraint on the probability of false alarm needs not be indicated, since it is implicitly contained in the threshold level $\nu(\delta)$, as shown in (20). In the above equation, $f(\boldsymbol{\Sigma})$ may be either $f_d(\boldsymbol{\Sigma}) = P_d$ or $f_I(\boldsymbol{\Sigma})$ as determined by (23).

Before discussing the solution to the above problem, it is worth to introduce here some relevant properties of $f_I(\boldsymbol{\Sigma})$ and $f_d(\boldsymbol{\Sigma})$, as well as of the FI $I_\tau(\boldsymbol{\Sigma})$.

A. Properties of MI and P_d

As to $f_I(\mathbf{\Sigma})$, it is immediately seen to be concave and symmetric, and thus Schur-concave[4] in $\{\lambda_1^2, \ldots, \lambda_M^2\}$, a property that is retained also under a transmit energy constraint [4]. As a consequence, with no accuracy constraint (i.e., for $\beta = 0$), it is maximized by simply transmitting isotropically along the δ available eigenmodes, which amounts to generating δL diversity paths: The global optimum is, in turn, achieved by setting $\delta = M$, i.e., by a full-rank (orthogonal) coding scheme. This is a major shortcoming of MI as a figure of merit: Indeed, it is well known that the detectability of weak targets is endangered by diversity, and in fact energy integration along a single path is definitely the best transmit policy in the low signal-to-noise ratio region.[5] An intuitive justification of this fact is that MI assumes the target to be present, thus neglecting the prior uncertainty as to which of the two hypotheses, $\mathcal{H}_0$ or $\mathcal{H}_1$, is true.

The nice mathematical properties of MI, unfortunately, do not carry over to $f_d(\mathbf{\Sigma})$, which does not exhibit any global concavity or Schur-concavity property. Rather, it has some "local" properties, which can turn out to be useful in solving problem (40) in the "large" and "small" SNR regimes. Concerning the former, we have the following result.

Theorem 3: In the high SNR regime, i.e., $E_t/N_0 \gg 1$, the code-matrix rank δ that maximizes the probability of detection can be found by solving the following nonconvex optimization problem:

$$\max_{\delta\in\{1,\ldots,\min(M,N)\}} \quad e^{-\nu/(\frac{E_t/\delta}{N_0}+1)} \times \sum_{k=0}^{\delta L-1}\left(\nu\Big/\left(\frac{E_t/\delta}{N_0}+1\right)\right)^k \frac{1}{k!},$$
$$\text{s.t.} \quad P_{\rm fa} = e^{-\nu}\sum_{k=0}^{\delta L-1}\frac{\nu^k}{k!}. \tag{42}$$

Proof: See Appendix A. ■

More can be said on the behavior of P_d if we use the notion of diversity gain given in [15], that characterizes the rate of decay to zero of the probability of a miss for increasingly large SNR under a semi-definite constraint on the false-alarm probability, i.e.,

$$d = -\lim_{\text{SNR}\to\infty}\frac{\log[1-P_d(\text{SNR})]}{\log \text{SNR}}. \tag{43}$$

To this end, we first introduce the concept of *asymptotic equality* between two functions $f_1(x)$ and $f_2(x)$, i.e.,

$$f_1(x) \mathrel{\overset{\circ}{=}} f_2(x) \longrightarrow \lim_{x\to\infty}\frac{f_1(x)}{f_2(x)} = 1.$$

Likewise, we may define $\overset{\circ}{\le}$ and $\overset{\circ}{\ge}$ accordingly. Also, two functions $f_1(x)$ and $f_2(x)$ are *exponentially equal* when $\lim_{x\to\infty}(\log f_1(x))/(\log f_2(x)) = 1$, and we use the notation $f_1(x) \doteq f_2(x)$ for exponential equality.

Remark 1: Asymptotical equality is a sufficient condition for exponential equality.

The following result [15] is fundamental when computing the diversity gain.

Lemma 2: For any M independent real Gaussian random variables, $Y_m \sim \mathcal{N}(\rho\cdot\mu_m, \sigma^2)$, $m = 1\ldots M$, where $\rho \in \mathbb{R}_+$ and $\mu_m \sim \mathcal{N}(0, \sigma_m^2)$, and for any given $\gamma \in \mathbb{R}_+$, in the asymptote of large values of ρ, we have

$$\mathbb{E}_{\boldsymbol{\mu}}\left[P\left(\sum_{m=1}^M Y_m^2 < \gamma\right)\right] \doteq \rho^{-M} \tag{44}$$

where $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_M]$.

Then, the diversity gain is given in the following theorem.

Theorem 4: The diversity gain achieved by a system using L receive antennas and a rank δ code matrix is $L\delta$, i.e., $1 - P_d \doteq \text{SNR}^{-L\delta}$.

Proof: As it was stated in the Proof of Theorem 3, in the high SNR regime, the optimum code matrix has its δ eigenvalues set to be $\sqrt{\text{SNR}/\delta}$. Noticing that under $\mathcal{H}_1$ the received vector is $\boldsymbol{r}_j = \mathbf{\Sigma}\tilde{\boldsymbol{g}}_j + \boldsymbol{w}_j$, and given the expression of the test statistic $Q(\tau)$ in (16), we can apply Lemma 2 by noticing that

$$Q = \sum_{j=1}^L\sum_{i=1}^\delta\left|\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{N_0}}w_{j,i}\right|^2 \tag{45}$$
$$= \sum_{j=1}^L\sum_{i=1}^\delta\left(\text{Re}\left\{\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{N_0}}w_{j,i}\right\}\right)^2 + \left(\text{Im}\left\{\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{N_0}}w_{j,i}\right\}\right)^2. \tag{46}$$

Defining $\hat{g}_{j,i}^r \triangleq \Re(\hat{g}_{j,i})$ and $\hat{g}_{j,i}^i \triangleq \Im(\hat{g}_{j,i})$, then

$$\text{Re}\left\{\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{N_0}}w_{j,i}\right\} \sim \mathcal{N}\left(\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i}^r, \frac{1}{N_0}\right) \tag{47}$$
$$\text{Im}\left\{\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{N_0}}w_{j,i}\right\} \sim \mathcal{N}\left(\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i}^i, \frac{1}{N_0}\right). \tag{48}$$

Let $M = 2L\delta$, $\rho = \sqrt{\text{SNR}}$, $\mu_m = g_{i,j}^r, g_{i,j}^i$, $\sigma^2 = 1/N_0$, then

$$P\left(\sum_{j=1}^L\sum_{i=1}^\delta\left|\sqrt{\frac{\text{SNR}}{\delta}}\hat{g}_{j,i} + \frac{1}{\sqrt{2N_0}}w_{j,i}\right|^2 < \nu\right) \doteq \sqrt{\text{SNR}}^{-2L\delta} = \text{SNR}^{-L\delta} \tag{49}$$

which is the desired result. ■

This result states that asymptotically the optimum transmit policy is given by full diversity and isotropic transmission. It is also worth commenting in passing on the case $\delta = 1$, i.e., as rank-one coding is undertaken at the transmitter (e.g., all of the transmit antennas use one and the same uncoded pulse train): The above results confirm that no transmit diversity is achieved

[4]The main concepts of Schur-concavity, Schur-convexity and related majorization theory [14] are outlined in Appendix A.

[5]These results were established in [2] where a comparison between MIMO with widely spaced antennas and beam-forming for a phased array was undertaken, and in [4], where it was demonstrated that a MIMO with widely spaced antennas should be used as an incoherent power multiplexer along a single eigenmode to facilitate the detection of weak targets.

(i.e., only receive diversity is retained). This corresponds to an incoherent energy integration along the unique transmit eigenmode: Notice that the target scintillation due to angular diversity prevents any coherent beam-forming, which indeed would amount to a coherent integration along a unique eigenmode, and would be achievable only if all of the transmit antennas view the target under the same aspect angle (see also [4]).

B. Properties of Fisher Information

Given the expression for the FI in (35), under the conditions of Lemma 1, the following theorem points out an important property of such bound.

Theorem 5: The FI given in (35) is a Schur-convex function of $[\lambda_1^2,\ldots,\lambda_M^2]^T$.

Proof: Let us consider the function $g:\mathbb{R}_+\to\mathbb{R}$ defined as

$$g(\lambda^2)=2L\Omega_{\rm rms}^2\frac{\left(\frac{\lambda^2}{N_0}\right)^2}{1+\frac{\lambda^2}{N_0}}$$

which is a convex function of λ^2. Then, by Proposition 3.C.1 of [14], as $I_\tau=\sum_{i=1}^M g(\lambda_i^2)$ with g convex, then I_τ is Schur-convex of $[\lambda_1^2,\ldots,\lambda_M^2]^T$. ■

Therefore, as the Fisher's information is Schur-convex, its maximization occurs when the code matrix is rank-one, namely all of the waveforms transmitted by the different sensors are collinear. This result may appear somewhat in contradiction with the results given in other studies (e.g., [9]), where diversity has been shown to be always beneficial. Actually, the key point is that in this paper we are dealing with omnidirectional antennas radiating on a limited bandwidth, so that the paths are unresolvable.[6] On the other hand, it is well known that the FI on the delay of a given waveform, once it is embedded in additive white Gaussian noise, is proportional to the product of the rms bandwidth and the energy contrast [8]. Consistently, for a MIMO radar operating under white noise, the FI takes on the form (35), i.e., the product of the transmitted rms bandwidth (dictated by the basic radar pulse) times an "array factor," tied to the MIMO structure and representing the overall energy contrast: Thus, maximization of the FI requires maximization of this factor, which is well known to require rank-one transmission, i.e., all of the energy is concentrated along an arbitrary (due to the noise whiteness) direction of the signal space.

C. Optimal Codes With Accuracy Constraint

We are now ready to show our main results concerning the MI-optimal and P_d-optimal codes. Notice first that the rms bandwidth is fixed once the basic radar pulse is chosen and it does not play any role in the "transmit policy", therefore, it can be omitted in the code design. For MI, we have the following result.

Theorem 6: For ${\rm SNR}\geq\hat\beta/(2M)+\sqrt{\hat\beta^2/(4M^2)+\hat\beta}$, with $\hat\beta=(M\beta)/2L$, the solution to the optimization problem

$$\max_{\boldsymbol\Sigma}\quad f_I(\boldsymbol\Sigma) \tag{50}$$

$$\text{given}\quad\begin{cases}\text{power constraint}\\ 2L\sum_{i=1}^M\frac{\left(\frac{\Lambda_i^2}{N_0}\right)^2}{1+\frac{\Lambda_i^2}{N_0}}\geq\beta\end{cases} \tag{51}$$

is given by full diversity and isotropic transmission, i.e.,

$$\lambda_i^2=\frac{E_t}{M}=\frac{N_0{\rm SNR}}{M},\quad\text{for } i=1\ldots M. \tag{52}$$

Moreover, for ${\rm SNR}\leq\hat\beta/(2M)+\sqrt{\hat\beta^2/(4M^2)+\hat\beta}$, full diversity and isotropic transmission does not satisfy the constraint and therefore, it is not an allowed transmit policy.

Proof: As $f_I(\boldsymbol\Sigma)$ is a Schur-concave function, it is maximized when $\lambda_i^2=(E_t/M)=(N_0{\rm SNR})/M$ for $i=1\ldots M$. To prove the first part of the theorem, we need to find the SNR region where such solution belongs to the set of feasible solutions.

If all the square-eigenvalues are equal, i.e., $\lambda_i^2=(N_0{\rm SNR})/M$ for $i=1\ldots M$, then the bound on the FI is

$$2LM\frac{\left(\frac{\rm SNR}{M}\right)^2}{1+\frac{\rm SNR}{M}}\geq\beta$$

which is satisfied by all SNR that satisfy

$${\rm SNR}^2-\hat\beta\frac{\rm SNR}{M}-\hat\beta\geq 0 \tag{53}$$

with $\hat\beta=(M\lambda)/2L$. The polynomial ${\rm SNR}^2-\hat\beta({\rm SNR}/M)-\hat\beta$ is a convex quadratic function of the SNR with zeros given by

$$\frac{\hat\beta}{2M}\pm\sqrt{\left(\frac{\hat\beta}{2M}\right)^2+\hat\beta}$$

where it is seen that one root is positive and the other one is negative. Then, for all SNR greater than the positive root, i.e., ${\rm SNR}\geq\hat\beta/(2M)+\sqrt{\hat\beta^2/(4M^2)+\hat\beta}$, (53) is satisfied and the solutions $\lambda_i^2=(E_t)/M$ for $i=1\ldots M$ belongs to the set of feasible solutions.

On the other hand, when ${\rm SNR}\leq\hat\beta/(2M)+\sqrt{\hat\beta^2/(4M^2)+\hat\beta}$, full diversity and isotropic transmission does not belong to the set of feasible solutions. ■

The above theorem is of some importance under several points of view, and the following, in particular:

a) it shows how dramatically a constraint on the ranging accuracy modifies the optimal (in the mutual information sense) transmit policy: Full diversity and isotropic coding is indeed optimal only for large SNR and/or extremely loose ranging accuracy (i.e., low β);
b) equation (52) also allows explicit definition of the ranges of SNRs wherein full rank and isotropic is MI-optimal.

If the figure of merit is P_d, the solution to the constrained optimization problem become less explicit, since P_d, due to the presence of the detection threshold $\nu(\delta)$, is not Schur-concave. However, an insight on how, in this case too, the accuracy

[6]This also implies that here only ranging is feasible, since localization would require paths resolvability.

constraint modifies the optimal transmit policy can be obtained through the following result.

Theorem 7: For SNR $\geq \hat{\beta}/(2M) + \sqrt{\hat{\beta}^2/(4M^2) + \hat{\beta}}$, with $\hat{\beta} = (M\beta)/(2L)$, the solution to the optimization problem

$$\max_{\mathbf{\Sigma}} \quad P_d \tag{54}$$

$$\text{given} \quad \begin{cases} \text{power constraint} \\ 2L\sum_{i=1}^{M} \frac{\left(\frac{\lambda_i^2}{N_0}\right)^2}{1+\frac{\lambda_i^2}{N_0}} \geq \beta \\ P_{\rm fa} \end{cases} \tag{55}$$

is equivalent to the following optimization problem:

$$\max_{\mathbf{\Sigma}} \quad P_d \tag{56}$$

$$\text{given} \quad \begin{cases} \text{power constraint} \\ P_{\rm fa}. \end{cases} \tag{57}$$

Proof: The proof reproduces that of Theorem 6, and relies on determining the SNR range where full diversity and isotropic transmission belongs to the set of feasible solutions. However, in this case there is no information on how the optimum solution looks like, but, as the constraint is Schur-convex, setting λ_i^2 for $i = 1 \ldots M$ differently will also satisfy the constraint. ■

Notice that, as Theorem 6, Theorem 7 clearly shows that more and more stringent accuracy constraints (i.e., larger and larger values of β) shift to the right the "large SNR" region, i.e., the region where full-rank orthogonal coding is optimum.

V. Simulation Results

We consider a MIMO radar with $M = 4$ transmit and $L = 1$ receive antennas, operating at a probability of false alarm $P_{\rm fa} = 10^{-4}$. We set the scattering vectors $\tilde{\boldsymbol{g}}_j$ to have $\boldsymbol{R}_j = \mathbb{E}\{\tilde{\boldsymbol{g}}_j\tilde{\boldsymbol{g}}_j^H\} = \boldsymbol{I}$, the noise power to be $N_0 = 1$ and we vary the transmit power.

To test the results we have presented in this paper, we carry out simulations where both P_d and MI are computed for different values of δ and SNR. In order to set a yardstick for detection performance, we first consider the scenario of Theorem 4, which studies the achievable diversity order in detecting a target at a known distance (i.e., with known delay), so that no constraint on the Fisher information is present in the corresponding optimization problem. Fig. 2 shows the probability of a miss, $1 - P_d$, as a function of SNR for different values of δ. The figure clearly shows that, while in the large SNR region higher diversity order yields a better performance, for low to medium SNR rank-deficient coding is preferable. Notice that the crossing point occurs at a detection probability in the interval (0.5,0.6), but also recall that here no receive diversity is present: Should we choose $L > 1$, we would inevitably observe a shift to the right of such a crossing point. As a side comment, we observe that, consistent with the results of Theorem 4, the curve turns out to be asymptotically linear in SNR, with a slope dictated by δ.

We define $\beta^* \triangleq \beta/(2L)$, where β is the accuracy constraint of (18) with $f(\mathbf{\Sigma}) = P_d$, and illustrate how the optimum rank, i.e., the one maximizing the detection probability P_d, varies with β^*. Fig. 3 shows the optimum rank under an accuracy constraint. The figure clearly shows that increasing β^*, i.e., forcing more and more stringent accuracy constraint, the optimum rank at a given SNR decreases. Fig. 4 shows a similar behavior as MI is the figure of merit: Notice, however, that transitions occur for larger values of β^* as compared to the previous case, a further

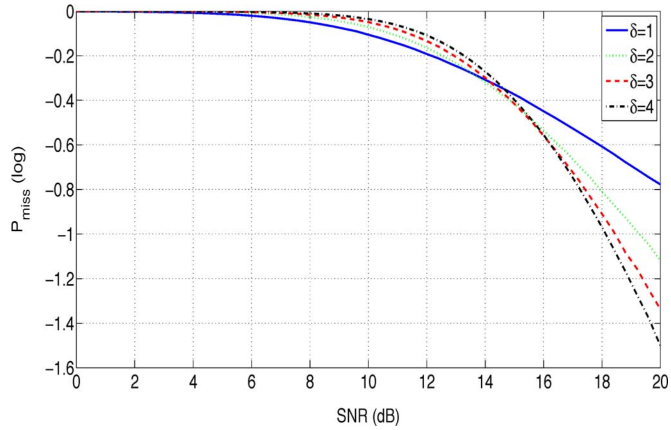

Fig. 2. Probability of target miss detection versus SNR with no constraint on the estimation performance and $P_{\rm fa} = 10^{-4}$.

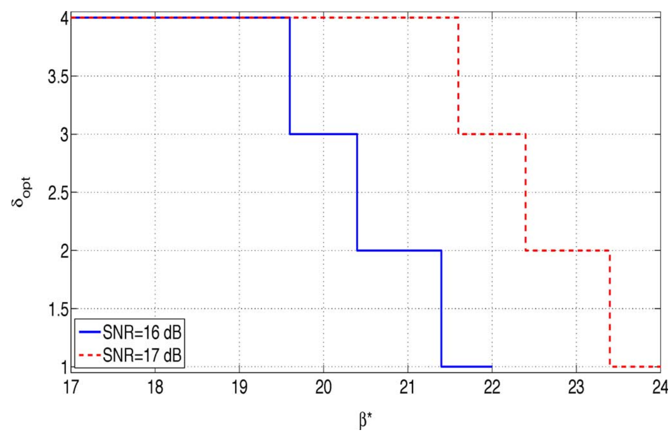

Fig. 3. Best transmit policy as a function of the estimation performance constraint with P_d as the objective function.

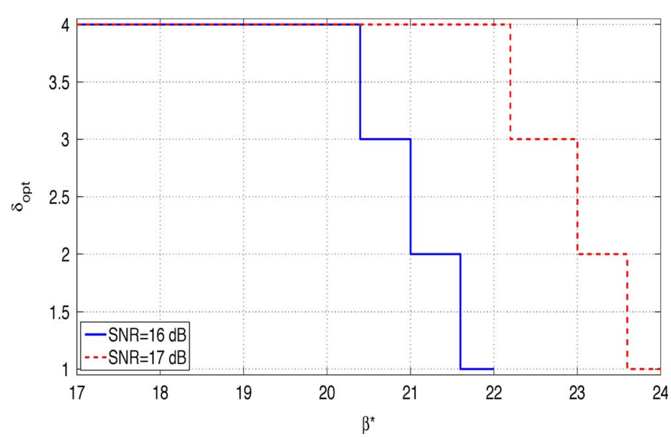

Fig. 4. Best transmit policy as a function of the estimation performance constraint with MI as the objective function.

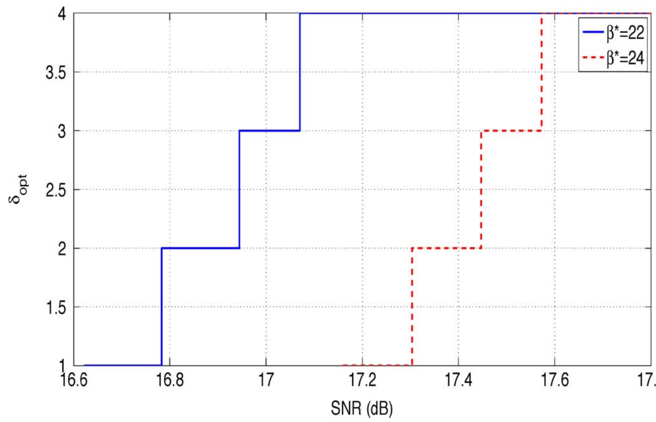

Fig. 5. The optimum policy as a function of SNR with P_d as the objective function.

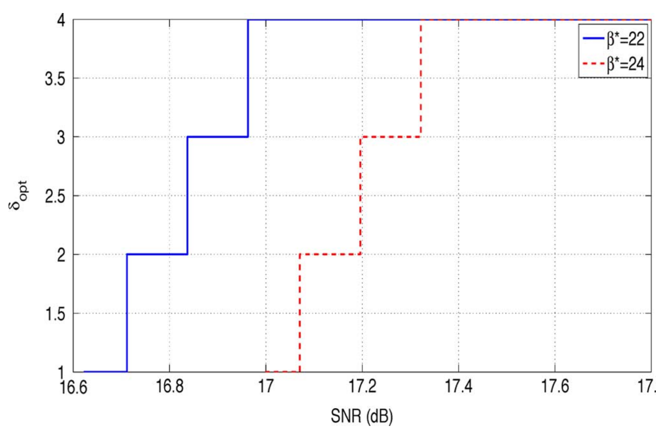

Fig. 6. The optimum policy as a function of SNR with MI as the objective function.

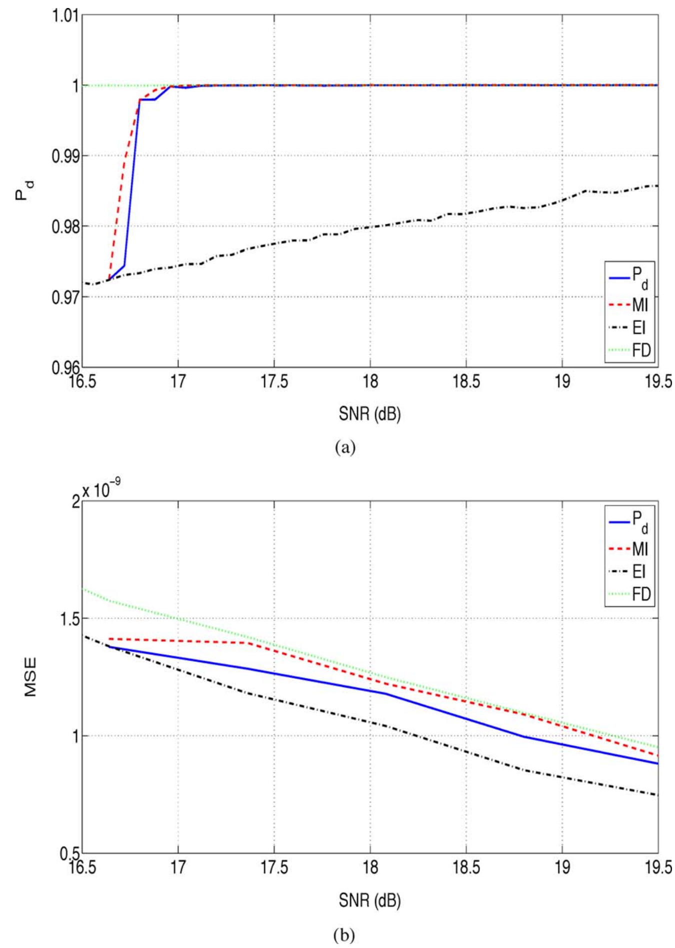

Fig. 7. Probability of detection and the MSE in the coupled scenario. (a) P_d for the complete system when the transmit policy is energy integration (EI), isotropic and full diversity (FD), when using P_d as the criterion for design and when MI is used. (b) The MSE of the parameter estimation when the transmit policy is energy integration (EI), isotropic and full diversity (FD), when using P_d as the criterion for design and when MI is used.

evidence of the fact that MI is not always a suitable figure of merit in the outlined composite hypothesis testing problem.

Figs. 5 and 6 are similar to Figs. 3 and 4, respectively, but they study the optimum rank as a function of the SNR for fixed β^*. As expected, more and more stringent accuracy constraints results in larger and larger values of the SNR wherein transmit diversity—and in particular full diversity—is advantageous over energy integration: Once again, adopting MI as a figure of merit results in a behavior that, albeit similar to the one exhibited by P_d, tends to increase transmit diversity at lower SNR.

Finally, we evaluate the overall system performance when target detection and ranging are to be undertaken. We first solve the optimization problem of (18) in order to find the optimum $\mathbf{\Sigma}$. We set $\beta^* = 22$, so the optimum rank δ is shown in Figs. 5 and 6 for different values of SNR. We then compute the test where both target detection and parameter estimation are performed, i.e., we simulate the GLRT given in (9) whose implementation is shown in Fig. 1. The target is randomly located in one of 100 possible delays uniformly spaced between 0.01 and 1 milliseconds. Fig. 7(a) shows the detection probability as the matrix $\mathbf{\Sigma}$ is chosen when $f(\mathbf{\Sigma})$ in (18) is set to be f_{MI} and f_d, as well as for rank-one and full-rank isotropic transmission. Since we already know that for low SNR rank-deficient coding is preferable, we focus our attention on the large SNR regime. Notice that the said accuracy constraint results in a truly marginal loss—but a loss indeed—of P_d as compared to the one achievable under full diversity. On the other hand, the performance of the delay estimation is evaluated using the mean-squared error (MSE), where the error is normalized by the time difference between two consecutive allowed delays. In Fig. 7(b), the MSE in the range estimation is shown under the different transmit policies considered above. Here, the effect on the accuracy constraint is clearly visible, and the following conclusions can be drawn.

a) Full diversity with isotropic transmission results in significant loss in terms of ranging accuracy, while granting satisfactory detection performance.
b) Rank-one coding is the best policy in terms of ranging accuracy, but results in completely unsatisfactory detection performance as the SNR increases.
c) The proposed strategy of optimizing a figure of merit under a constraint on the Fisher information (or equivalently the Cramér–Rao bound) of the target range appears to close the gap between these two conflicting behaviors.

VI. Conclusion

In this paper, the problem of detection and ranging through a space-time coded MIMO radar has been studied. In particular, the transmit policy has been determined as the solution of a constrained optimization problem, where a given figure of merit is maximized under constraints concerning both the transmit energy and the Fisher information on the unknown parameters. The results confirm the intuition that setting an accuracy constraint has a visible impact on the design of the waveform to be transmitted, and are quite conducive to further investigations, mainly concerning the extension to such scenarios as multiple unknown parameters and non-Gaussian scattering targets.

Appendix

A. Theory of Majorization

This Appendix goes over some useful concepts of the theory of majorization that are used in the paper.

Definition 1 [14]: For any $\boldsymbol{x} = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, let

$$x_{[1]} \geq \cdots \geq x_{[n]} \tag{58}$$

denote the components of $\boldsymbol{x}$ in decreasing order.

Definition 2 [14]: For $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, $\boldsymbol{x}$ is majorized by $\boldsymbol{y}$, i.e., $\boldsymbol{x} \prec \boldsymbol{y}$

$$\text{if} \quad \begin{cases} \sum_{i=1}^{k} x_{[i]} \leq & \sum_{i=1}^{k} y_{[i]}, \quad k = 1, \ldots, n-1 \\ \sum_{i=1}^{n} x_{[i]} = & \sum_{i=1}^{n} y_{[i]}. \end{cases} \tag{59}$$

It can be seen that for $\boldsymbol{a} = [a_1, \ldots, a_n]^T \in \mathbb{R}^n$, then

$$\left[\frac{1}{n}, \ldots, \frac{1}{n}\right]^T \prec \boldsymbol{a} \prec [1, 0, \ldots, 0]^T \tag{60}$$

whenever $a_i \geq 0$ and $\sum_{i=1}^{n} a_i = 1$.

Definition 3 [14]: A real-valued function g defined on a set $\mathbb{R}^n$ is said to be Schur-convex if

$$\boldsymbol{x} \prec \boldsymbol{y} \quad \Rightarrow \quad g(\boldsymbol{x}) \leq g(\boldsymbol{y}). \tag{61}$$

Similarly, g is said to be Schur-concave if

$$\boldsymbol{x} \prec \boldsymbol{y} \quad \Rightarrow \quad g(\boldsymbol{x}) \geq g(\boldsymbol{y}). \tag{62}$$

B. Proof of Theorem 3

Proof: For a fixed δ, the threshold ν is fixed and the probability of detection is given by

$$P_d = 1 - P\left(\frac{Q}{\mathbb{E}(Q)} \leq \frac{\nu(\delta)}{\mathbb{E}(Q)}\right). \tag{63}$$

As $Q' = (Q)/(\mathbb{E}\{Q\})$ is a weighted sum of chi-square random variables, we use the following theorem from [16] to characterize the behavior of $P(Q' \leq \nu(\delta)/\mathbb{E}(Q)$.

Theorem A.1. Probability of Weighted Sum: Let $w_1, \ldots, w_n$ denote i.i.d. random variables, whose pdf is Gamma with arbitrary shape parameter k and common scale parameter $\zeta = 1$.

For nonnegative vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, if $\boldsymbol{\mu}_1 \succeq \boldsymbol{\mu}_2$ and $x \geq 2$, then

$$P\left(\sum_{k=1}^{n} \mu_{1,k}\, w_k \leq x\right) \leq P\left(\sum_{k=1}^{n} \mu_{2,k}\, w_k \leq x\right). \tag{64}$$

On the other hand, if $x \leq 1$ and $\boldsymbol{\mu}_1 \succeq \boldsymbol{\mu}_2$, then

$$P\left(\sum_{k=1}^{n} \mu_{1,k}\, w_k \leq x\right) \geq P\left(\sum_{k=1}^{n} \mu_{2,k}\, w_k \leq x\right). \tag{65}$$

This means that $P(\sum_{k=1}^{n} \mu_k w_k \leq x)$ is Schur-concave for $x \geq 2$ and Schur-convex for $x \leq 1$.

Notice that a complex chi-squared random variable with k complex degrees of freedom is equivalent to a Gamma random variable with scale and shape parameters 1 and k, respectively.

In the high SNR regime, $\nu(\delta)/\mathbb{E}(Q) < 1$ and therefore, $P(Q/\mathbb{E}(Q) \leq \nu/\mathbb{E}(Q))$ is Schur-convex and is minimized by setting all the weights to be equal:

$$Q^{\text{high}} = \left(\frac{E_t/\delta}{N_0} + 1\right) \sum_{l=1}^{L} \sum_{i=1}^{\delta} |v_{i,j}|^2 \tag{66}$$

and therefore,

$$\begin{aligned} P_d^{\text{high}} &= 1 - P(Q^{\text{high}} \leq \nu) \\ &= e^{-\nu/(\frac{E_t/\delta}{N_0}+1)} \sum_{k=0}^{\delta L-1} \left(\nu \Big/ \left(\frac{E_t/\delta}{N_0} + 1\right)\right)^k \frac{1}{k!}. \end{aligned} \tag{67}$$

Unlike what happens for MI, here no global optimum can be envisaged for the rank δ, which must be found by solving the constrained problem given in (42), which is nonconvex, but can be solved numerically with low computational burden. ■

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