Narrow-Band-Interference Suppression in Multiuser CDMA Systems

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***Abstract*— This paper handles the simultaneous suppression of narrow-band and multiaccess interference in code-division multiple-access (CDMA) direct-sequence spread-spectrum (DSSS) systems. The basic structure we refer to is reminiscent of the decorrelating detector, but here the design strategy relies on the concept of combating jointly the two interference sources—precisely, a decision as to the bit transmitted by each user is made based on the projection of the observables onto the orthogonal complement to the subspace spanned by the other users' signatures and the narrow-band interference. We focus on several different implementations of such a strategy, assuming a different degree of prior knowledge as to the narrow-band interference. An important side result of the proposed approach is that, in general, complete suppression of data-like interference may be achieved through periodically time-varying processing. An adaptive version of such a receiver is also presented, wherein the projection direction is estimated based on suitable estimates of the covariance properties of the observables. The value of this method is also assessed by studying the rate of convergence of the estimated direction to the true projection direction.**

I. Introduction

MULTIUSER detection represents a powerful tool to cope with the problem of multiple-access interference in code-division multiple-access (CDMA) systems—it relies on the concept that the interference arising from other users should not be handled as a disturbance to be suppressed, but rather as an additional information-bearing signal, which can be exploited to improve performance [1]. The optimum multiuser detector cannot be used in real situations since its complexity increases exponentially with the user's number. In [2], however, a suboptimum detector, the decorrelating detector, of reduced complexity, is introduced and assessed, showing that it achieves near-optimum performance. The suitability of such a detector is also confirmed by the fact that it has the same near–far resistance as the optimum detector [3], [4] with the additional advantage of not requiring *a priori* knowledge of the signal amplitudes.

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Unfortunately, however, CDMA systems usually turn out to operate also in the presence of external interference, namely, of interfering signals which share the same frequency range as the CDMA systems but originate from different sources. Unlike the multiaccess interference, these interferers exhibit a structure which is significantly different from that of the signal to be decoded and, in fact, they are often referred to as "narrow-band interference" to emphasize the fact that they may arise from data sources whose bit rate is typically slower than the chip rate of the CDMA system. Even though the spread-spectrum nature of the user's signals ensures some protection against narrow-band interferers, especially for large processing gains, a noticeable performance degradation can be observed in situations where the interfering signal is much stronger than the useful ones [5]. This poses the problem of envisaging and assessing *robust* detectors, which ensure reliable transmission in the presence of both narrow-band and multiaccess interference. Significant contributions in this sense are reported in [6], where a new single-user detector is introduced, exploiting frequency-domain analysis and data excision for interference nullification purposes, and in [7], where interference suppression is accomplished by preprocessing the received signal through a tapped-delay-line linear filter. In [8], moreover, an adaptive implementation of this system is presented also, relying on a stochastic-gradient adaptation strategy to cope with time-varying environments. More recently, new suppression methods have been introduced in [9] and [10]. In the former study, in particular, narrow-band interference is modeled as an autoregressive process and estimated through a tapped-delay-line filter, whose tap coefficients may be adjusted to account for time-varying environments. In the latter, instead, suppression is achieved by building a decorrelating detector around an estimate of the narrow-band interferer. Finally, in [11] and [12] an adaptive system is introduced and assessed, wherein interference suppression is achieved by adaptive implementation of a minimum-mean-square-error detector.

In this paper we first introduce and assess a new multiuser detector for synchronous CDMA systems operating in the presence of external interference. The strategy it relies on is to represent the received signal through an orthonormal basis wherein the useful signals are spread quite uniformly along all directions, while narrow-band interference is concentrated in a low-dimensional subspace; thus, interference elimination can be achieved by simple projection operations onto suitable subspaces. We discuss the influence of the processing domain, and we give general design criteria to cope with several commonly encountered types of narrow-band interferers.

An adaptive version of this detector is presented as well, showing that it reduces and—in some cases—nullifies the amount of prior knowledge required as to the structure of the narrow-band interferer.

II. Multiuser Detection and Interference Suppression

Let us consider a synchronous CDMA system, wherein K users simultaneously and synchronously transmit an *uncoded* binary phase-shift keying (BPSK) signal. Assuming that each user is assigned a different pseudonoise (PN) sequence, which directly modulates the source signal, the complex envelope of the received waveform is written as

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{K-1} A_k e^{j\phi_k} b_k(n) s_k(t - nT_b) + i(t) + n(t) \quad (1)$$

where $A_k e^{j\phi_k}$ is a possibly random complex gain, accounting for the channel effect, $[b_0(n), \cdots, b_{K-1}(n)]^T = \mathbf{b}(n)$ is the stream of the binary digits transmitted by the K users in the signaling interval $(nT_b, (n+1)T_b)$, $s_k(t)$, $k = 0, \cdots, K-1$ are the *signatures* of the transmitted waveforms. Since we are dealing with *direct-sequence spread-spectrum* (DSSS) signals, we have

$$s_k(t) = \sum_{m=0}^{N-1} c_{km} u_{T_c}(t - mT_c) \quad (2)$$

where $\mathbf{c}_k = [c_{k0}, \cdots, c_{kN-1}]^T$ represents the PN assigned to the kth user, T_c is the chip interval, N is the processing gain, and

$$u_T(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{elsewhere.} \end{cases} \quad (3)$$

As to the terms $i(t)$ and $n(t)$ in (1), the former represents the narrow-band interference, if present, while the latter is the noise term, that we model as a sample function from a complex zero-mean white Gaussian process, with power spectral density (PSD) $2\mathcal{N}_0$.

For the interference, we adopt a very general model, which subsumes several special cases of relevant practical interest. Precisely, we assume that $i(t)$ is the data signal emitted by an external source whose bit rate $1/T_I$ is Q times slower than the chip rate of the CDMA system, implying $T_I = QT_c$. If this secondary source emits independent symbols, then

$$i(t) = A_I e^{j\theta_I} e^{j2\pi f_I t} \sum_{n=-\infty}^{\infty} b_I(n) u_{T_I}(t - \tau_I - nT_I) \quad (4)$$

where f_I is the frequency offset of the interference, τ_I is an unknown delay, $b_I(n)$ represents a sequence of independent and identically distributed binary variables, taking on values in the set $\{-1, 1\}$, and $A_I e^{j\theta_I}$ represents a possibly random gain, accounting for the channel effect. Thus, the energy of the interfering signal in each signaling interval is

$$\mathcal{E}_I = \overline{A_I^2} T_b \quad (5)$$

and the signal-to-interference power ratio (SIR) for the kth user is

$$\text{SIR}_k = \frac{\overline{A_k^2}}{\overline{A_I^2}} \quad (6)$$

wherein the bar denotes statistical expectation. Notice that the above model, proposed in [5], is quite general, and subsumes also the case where the interference spectrum is discrete as the “degenerate” case that $b_I(n)$ is a possibly random constant.

A. Detector Structure

Let us focus on the interval $(0, T_b)$. At the receiver end, a decision is to be made as to the vector $\mathbf{b}(0)$ actually transmitted, based on the observable waveform (1). To this end, it is customary to project $r(t)$ along the unit vectors of an orthonormal basis of the N-dimensional subspace, $\mathcal{H}$, say, spanned by all of the possible signatures with processing gain N as observed in the interval $(0, T_b)$, so that the classification problem can be cast in the form

$$\mathbf{r} = \sum_{k=0}^{K-1} A_k e^{j\phi_k} b_k(0) \mathbf{s}_k + \mathbf{i} + \mathbf{n} \quad (7)$$

where $\mathbf{r}$, $\mathbf{s}_k$, $\mathbf{i}$, and $\mathbf{n}$ represent the N-dimensional vectors of the projections on $\mathcal{H}$ of $r(t)$, $s_k(t)$, $i(t)$, and $n(t)$, respectively. For the moment, we leave the basis unspecified, deferring to subsequent sections a discussion on its choice. We just stress here that, since the useful signals are spread spectrum, while the interference is narrow-band, the former have a much lower degree of coherence than the latter—as a consequence, spread-spectrum signals usually have significantly nonzero components along unit vectors of most orthonormal basis, while the latter may be “concentrated,” so as to yield only a small number of significant nonzero projections.

A possible multiuser detection structure for the above classification problem is as outlined in Fig. 1, wherein a decision is made according to the rule

$$\hat{b}_k(0) = \text{sgn}(\Re\{e^{-j\phi_k} \mathbf{D}_k^\dagger \mathbf{r}\}), \qquad k = 0, \cdots, K-1. \quad (8)$$

In (8), $\text{sgn}(\cdot)$ is the signum function, $\Re$ denotes real part, $\mathbf{D}_k$'s are K N-dimensional vectors, and † denotes conjugate transpose. Notice that the above decision rule can be cast in the more compact form

$$\hat{\mathbf{b}}(0) = \text{sgn}(\Re\{\mathbf{C}^\dagger \mathbf{r}\}) \quad (9)$$

with $\mathbf{C}$ a $N \times K$ matrix, whose kth column contains the vector $\mathbf{D}_k e^{j\phi_k}$. A possible design criterion at this point is to select $\mathbf{C}$ so as to maximize the system near–far resistance, which would lead to a decorrelating detector. Obviously, this would cope with cancellation of multiaccess interference, but would ensure no protection against external interference—if both interference sources are to be accounted for, a robust design criterion is in order instead. This is, in fact, the object of subsequent sections.

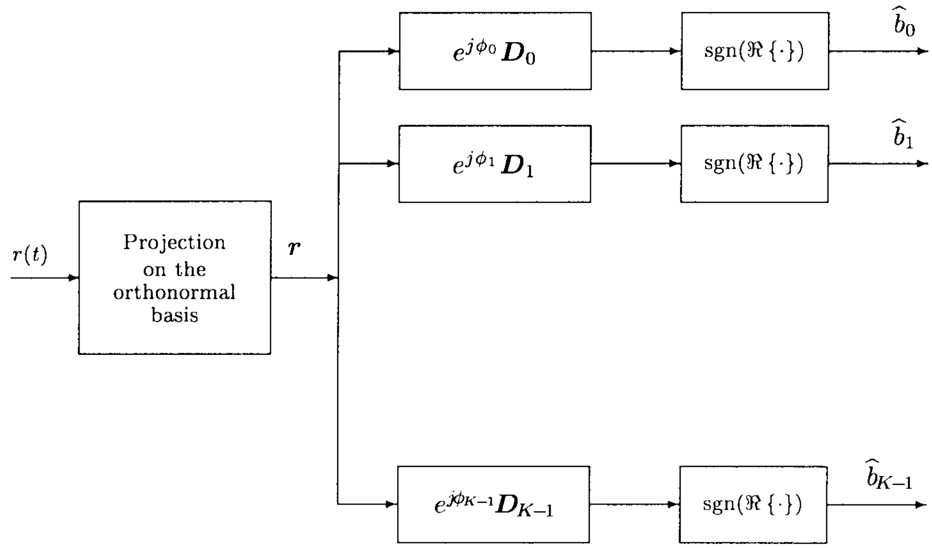

Fig. 1. General structure of the multiuser detector.

B. System Design

We recall here that our final goal is to get rid of both the multiaccess and the external interference. To this end, let us first assume that the vector $\mathbf{i}$, representing the projection of $i(t)$ onto the signatures subspace, is itself contained in an L-dimensional subspace of $\mathcal{H}$, $\mathcal{S}_L$, say, with $L < N$. Notice, however, that in order to strictly confine the interference to such a subspace, we ought to suitably choose the expansion basis, which would in turn require some knowledge on $i(t)$–in most applications, $i(t)$ is only approximately known, whence we can just ensure that the most significant part of the projection of $i(t)$ on $\mathcal{H}$ is contained in $\mathcal{S}_L$. Let us now define an orthonormal basis $\mathbf{e}_1, \cdots, \mathbf{e}_L$ for $\mathcal{S}_L$. Interference suppression can be obviously accomplished if the vectors $\mathbf{D}_k$ fulfill the conditions

$$\mathbf{D}_k^{\dagger}\mathbf{e}_i = 0, \quad k = 0, \cdots, K-1, \qquad i = 1, \cdots, L. \tag{10}$$

As to the multiaccess interference, this can be perfectly nullified if we force the conditions

$$\mathbf{D}_k^{\dagger}\mathbf{s}_i = 0, \qquad i \neq k. \tag{11}$$

The above constraints can be given a more compact form. In particular, defining the $N \times (L+K-1)$ arrays

$$\mathbf{\Psi}_k = (\mathbf{s}_0\mathbf{s}_1 \cdots \mathbf{s}_{k-1}\mathbf{s}_{k+1} \cdots \mathbf{s}_{K-1}\mathbf{e}_1 \cdots \mathbf{e}_L), \quad k = 0, \cdots, K-1 \tag{12}$$

the above conditions can be recast in the form

$$\mathbf{D}_k^{\dagger}\mathbf{\Psi}_k = \mathbf{0}, \qquad k = 0, \cdots, K-1. \tag{13}$$

Notice that these conditions represent $L+K-1$ constraints for each vector $\mathbf{D}_k$, for a total of $K(L+K-1)$ constraints.

Obviously, inasmuch as $L+K-1 < N$, there remain additional degrees of freedom, which can be exploited for system optimization purposes. To illustrate further, let us assume that the fulfillment of (10) yields a complete nullification of the external interference (or, more realistically, that the residual interference can be neglected). Since (11) ensures complete elimination of the multiaccess interference, the estimate of the bit transmitted by the kth user is

$$\hat{b}_k(0) = \mathrm{sgn}(\Re\{A_k b_k(0)\mathbf{D}_k^{\dagger}\mathbf{s}_k + \mathbf{D}_k^{\dagger}\mathbf{n}\}) \tag{14}$$

whence the corresponding error probability is

$$P_k(e) = Q\left(\frac{\Re\{A_k\mathbf{D}_k^{\dagger}\mathbf{s}_k\}}{\sqrt{\mathcal{N}_0\|\mathbf{D}_k\|^2}}\right) \tag{15}$$

with $Q(\cdot)$ the complementary cumulative distribution function (CCDF) of a standard Gaussian variate. Notice that the above bit-error rate (BER) is minimum if $\mathbf{D}_k$ is chosen so as to maximize the argument of the Q-function, with the constraints (10) and (11), or, equivalently, with the constraint (13). Notice also that, if some signal fading is present, the error probability is obtained by averaging (15) over the probability density function (pdf) of A_k—as a consequence, maximizing the quantity $\Re\{\mathbf{D}_k^{\dagger}\mathbf{s}_k\}/\|\mathbf{D}_k\|$ with the said constraints represents the constrained minimum-BER solution for any fading law.

Summing up, we are to solve the following constrained optimization problem:

$$\begin{cases} \mathbf{D}_k = \arg\max\left\{\frac{\Re\{\mathbf{D}_k^{\dagger}\mathbf{s}_k\}}{\|\mathbf{D}_k\|}\right\} \\ \mathbf{D}_k^{\dagger}\mathbf{\Psi}_k = \mathbf{0} \end{cases} \tag{16}$$

Such a problem admits a unique solution, except for a multiplicative factor (whose modulus affects the norm of the $\mathbf{D}_k$'s), which does not actually influence the BER value. We assume for the moment that the columns of the matrix $\mathbf{\Psi}_k$ are linearly independent. This is tantamount to assuming that the L additional constraints represent as many linearly independent vectors of the subspace of the narrow-band interference, and that these directions cannot be expressed as linear combinations of the other users signatures. Under these circumstances, the matrix $\mathbf{\Psi}_k$ defines a $(K-1+L)$-dimensional subspace, $\mathcal{H}_k$ say, of $\mathcal{H}$; let us denote by $\mathcal{H}_k^{\perp}$ the orthogonal complement of this subspace, and by $\mathbf{s}_k^{\perp}$ the projection of $\mathbf{s}_k$ onto $\mathcal{H}_k^{\perp}$.

Obviously, we have

$$\mathbf{s}_k = \mathbf{s}_k^{\|} + \mathbf{s}_k^{\perp} \tag{17}$$

with $\mathbf{s}_k^{\|} \in \mathcal{H}_k$, implying $\mathbf{s}_k^{\|\dagger}\mathbf{s}_k^{\perp} = 0$. The constraints (13) thus imply

$$\mathbf{D}_k \in \mathcal{H}_k^{\perp}. \tag{18}$$

Exploiting the above relationship, we obtain

$$\begin{aligned}\frac{\Re\{\mathbf{D}_k^{\dagger}\mathbf{s}_k\}}{\|\mathbf{D}_k\|} &= \frac{\Re\{\mathbf{D}_k^{\dagger}(\mathbf{s}_k^{\|} + \mathbf{s}_k^{\perp})\}}{\|\mathbf{D}_k\|} = \frac{\Re\{\mathbf{D}_k^{\dagger}\mathbf{s}_k^{\perp}\}}{\|\mathbf{D}_k\|}\\ &\le \frac{|\mathbf{D}_k^{\dagger}\mathbf{s}_k^{\perp}|}{\|\mathbf{D}_k\|} \le \|\mathbf{s}_k^{\perp}\|.\end{aligned} \tag{19}$$

Moreover, since

$$\left.\frac{\Re\{\mathbf{D}_k^{\dagger}\mathbf{s}_k\}}{\|\mathbf{D}_k\|}\right|_{\mathbf{D}_k=\gamma_k\mathbf{s}_k^{\perp}} = \|\mathbf{s}_k^{\perp}\| \tag{20}$$

and $\mathbf{s}_k^{\perp\dagger}\mathbf{\Psi}_k = \mathbf{0}$, then the solution to the constrained maximization problem is

$$\mathbf{D}_k = \gamma_k\mathbf{s}_k^{\perp}. \tag{21}$$

Since γ_k does not affect the BER value, it is assumed unity in the following.

In order to give an explicit form to the relationship (21), we notice that by definition

$$\mathbf{s}_k^{\|} = \mathbf{\Psi}_k\boldsymbol{\alpha} \tag{22}$$

with $\boldsymbol{\alpha}$ the complex $(L+K-1)$-dimensional vector of the components of $\mathbf{s}_k^{\|}$ on the columns of $\mathbf{\Psi}_k$. As a consequence, substituting this relationship into (17), we obtain $\mathbf{D}_k = \mathbf{s}_k - \mathbf{\Psi}_k\boldsymbol{\alpha}$, which, based on (13), yields

$$\mathbf{\Psi}_k^{\dagger}(\mathbf{s}_k - \mathbf{\Psi}_k\boldsymbol{\alpha}) = \mathbf{0} \tag{23}$$

whence

$$\boldsymbol{\alpha} = (\mathbf{\Psi}_k^{\dagger}\mathbf{\Psi}_k)^{-1}\mathbf{\Psi}_k^{\dagger}\mathbf{s}_k \tag{24}$$

and finally

$$\mathbf{D}_k = (\mathbf{I}_N - \mathbf{\Psi}_k(\mathbf{\Psi}_k^{\dagger}\mathbf{\Psi}_k)^{-1}\mathbf{\Psi}_k^{\dagger})\mathbf{s}_k \tag{25}$$

with $\mathbf{I}_N$ the $N \times N$ identity matrix. The above derivation assumes existence of the matrix inverse $(\mathbf{\Psi}_k^{\dagger}\mathbf{\Psi}_k)^{-1}$, namely, linear independence of the constraints. If this is not the case, it is understood that, once the subspaces of the multiaccess and narrow-band interference are determined, only a set of linearly independent directions describing their union will be inserted in $\mathbf{\Psi}_k$.

Notice that the proposed receiver generalizes the decorrelating detector, to which it reduces as no interference is accounted for at the design stage. To illustrate further, assume that $\mathbf{i} = \mathbf{0}$, so that $\mathbf{\Psi}_k$ is the $N \times (K-1)$ matrix whose columns contain all of the signatures except the kth; in this case, a decision is made for the kth user by projecting the observables onto the orthogonal complement to the subspace spanned by the signatures of the $K-1$ interfering users, which is just the operation of the decorrelating detector [13]. From this observation, it is intuitively understood that introducing L additional constraints on each vector $\mathbf{D}_k$ results into a detection loss, when external interference is not actually present at the input. Likewise, we expect that the new detector suffers some loss in terms of near–far resistance, as compared to the decorrelating (or, equivalently, to the optimum) multiuser detector, the amount of such a loss depending on the number L of additional constraints to be added for each user, and, ultimately, on the bandwidth of the interferers.

III. Selection of the Processing Domain

So far, the receiver (25) appears as a generalization to the multiuser case of the receiver proposed in [10]. There are some relevant differences, though, that should be underlined. Precisely, while in [10] the processing domain is dictated by the expansion basis defined by the useful signal and the data-like interference, here we consider an arbitrary basis of the signal space; this makes our receiver more flexible with respect to the type of interference to be suppressed, and introduces several degrees of freedom, which can be exploited to come up with an adaptive procedure.

We maintain here that a key point is the representation of the observables in a basis wherein narrow-band interference has significant projections onto few directions, while the useful signals, which have much lower degree of coherence, are spread quite uniformly along all unit vectors; thus, it is understood that the interference cancellation capabilities strongly depend on the processing domain.

A. Frequency Domain Implementation

Let us assume, at first, that the receiver has only an approximate knowledge of the frequency offset f_I and of the signaling interval T_I of the narrow-band interferer. Since the receiver only knows that the useful signals occupy a bandwidth W in the order of $1/T_c$, centered at a given carrier frequency f_0, while the interference occupies a bandwidth in the order of W/Q, centered at the carrier frequency $f_I + f_0$, the discrimination between user signals and interference may be done in the frequency domain (FD), which corresponds to choosing the orthonormal basis

$$\psi_\ell(t) = \frac{1}{\sqrt{NT_c}}\sum_{m=0}^{N-1} u_{T_c}(t - mT_c)\exp\left(j\frac{2\pi m\ell}{N}\right),\quad \ell = 0, \cdots, N-1. \tag{26}$$

The projection operation may be done easily by evaluating an N-points discrete Fourier transform (DFT) of the sampled output of the chip-matched filter. Due to the narrow-band nature of $i(t)$, its projection on the Fourier basis has a limited number of significant components, in the order of N/Q, concentrated around the Pth frequency bin, with $P = Nf_IT_c$, if Nf_IT_c is an integer number and f_I is nonnegative.

Once the observables are represented in the frequency domain, the orthogonalization procedure to the subspace $\mathcal{S}_L$ just amounts to nullifying several projections, i.e. several frequency bins wherein narrow-band interference is supposedly present. The subspace $\mathcal{S}_L$ is in fact defined by the set of orthonormal vectors $\mathbf{e}_1, \cdots, \mathbf{e}_L$, with[1]

$$\mathbf{e}_1 = \left[\underbrace{0, \cdots, 0}_{P-\frac{L-1}{2}-1}, 1, \underbrace{0, \cdots, 0}_{N-P+\frac{L-1}{2}}\right] \tag{27}$$

while $\mathbf{e}_i$ can be obtained by $\mathbf{e}_1$ through $(i-1)$ circular shifts to the right—the fulfillment of the constraints (10) implies that the vectors $\mathbf{D}_k$ have L zero entries surrounding and including the Pth entry. Notice also that, if Nf_IT_c is not an integer, the above considerations still apply, with P the integer nearest to Nf_IT_c. The number L of frequency bins to be nullified is not uniquely determined; in fact, the inevitable spectral leakage produces a spreading of the narrow-band interference on all frequencies, except in the special case that the interferer is a periodical signal whose harmonics are integer multiples of $1/NT_c$.

Thus, upon suitable choice of L with $L \geq \lfloor N/Q \rfloor$, we can just ensure that the most significant part of the projection of $i(t)$ on the orthonormal basis (26) is contained in S_L; L should be chosen on an intuitive basis as a result of a trial-and-error procedure.

B. SVD Domain Implementation

The said leakage of the narrow-band interference can be avoided if some additional prior knowledge as to the interference is assumed. To be more definite, let us first assume that the covariance matrix of the projection vector of the narrow-band interferer onto the signal space is one and the same for any signaling interval of the CDMA system, *and* that the structure of such a matrix is known to the receiver. The former assumption, which was also made in [10] and, more recently, in [11], amounts to assuming that the ratio T_b/T_I is an integer. The latter, instead, amounts to assuming knowledge of f_I, T_I, and τ_I, and is as restrictive as that made in [8] and [9], wherein the interference covariance is assigned a particular form.

Let us assume, with no loss of generality, that the narrow-band interferer vector $\mathbf{i}$ is the set of N samples taken at the output of a chip-matched filter with sampling rate $1/T_c$, corresponding to the basis $\psi_i(t) = \frac{1}{\sqrt{T_c}} u_{T_c}(t - iT_c)$, $i = 0, \cdots, N-1$. Letting

$$E[\mathbf{i}\mathbf{i}^\dagger] = \mathbf{M}_{\mathbf{ii}} = A_I^2 \mathbf{\Sigma}_{\mathbf{ii}}$$

we thus assume knowledge of the matrix $\mathbf{\Sigma}_{\mathbf{ii}}$. The rank of this matrix is obviously given by the number of independent bits from the narrow-band interferer that fall in the interval $(0, T_b)$, namely N/Q or $N/Q + 1$, whether $\tau_I = 0$ or $\tau_I \neq 0$, respectively. Notice that, due to the "narrow-band" nature of the external interferer, such a rank is in any case much smaller than N. A singular-value decomposition (SVD) of $\mathbf{\Sigma}_{\mathbf{ii}}$ determines the $L = \mathrm{rank}(\mathbf{\Sigma}_{\mathbf{ii}})$ orthogonal directions wherein the vector $\mathbf{i}$ has nonzero projections. The L vectors $\mathbf{e}_i$ to be inserted in (10) and (12) are those determined by such an SVD or some subset thereof, should some of them be linearly dependent on the other users signatures. We explicitly notice that this procedure amounts to changing the signal representation, and adopting a new basis wherein the fulfillment of the L additional constrains (10) just amounts to the nullification of as many projections of the received signal.

[1] To fix the ideas L is assumed to be an odd integer number.

Different arguments apply for the case that T_b/T_I is not an integer. In fact, in this case the projection direction, $\mathbf{D}_k$, is not one and the same for any signaling interval: we thus relax the assumption that we are focusing on the interval $(0, T_b)$, and we focus on a generic interval $(\ell T_b, (\ell+1)T_b)$. If T_b/T_I is not an integer, in fact, the narrow-band interferer is *asynchronous* with respect to the CDMA signaling interval. To fix the ideas, let us expand the signal received in $(\ell T_b, (\ell+1)T_b)$ on the orthonormal basis

$$\psi_{i,\ell}(t) = \psi_i(t - \ell T_b) = \frac{1}{\sqrt{T_c}} u_{T_c}(t - \ell T_b - iT_c), \quad i = 0, \cdots, N-1 \tag{28}$$

and let us define with a slight, but helpful, notational abuse

$$\mathbf{r}(\ell) = \sum_{k=0}^{K-1} A_k e^{j\phi_k} b_k(\ell) \mathbf{s}_k + \mathbf{i}(\ell) + \mathbf{n}. \tag{29}$$

The vector $\mathbf{r}$ of the previous derivation is thus now the vector $\mathbf{r}(0)$. From (29), it is seen that the subspace of the users signals is the same as in the interval $(0, T_b)$. The subspace containing the narrow-band interference vector $\mathbf{i}(\ell)$ varies *periodically* with ℓ. The period is obviously dictated by T_b and T_I since, in fact, as ℓT_b is an integer multiple of T_I, the situation in the interval $(\ell T_b; (\ell+1)T_b)$ reproduces that in the interval $(0, T_b)$, the situation in the interval $((\ell+1)T_b; (\ell+2)T_b)$ reproduces that in the interval $(T_b, 2T_b)$, and so on. As a consequence, in each interval narrow-band interference is contained in one out of m different subspaces, with m the smallest integer such that mT_b is an integer multiple of T_I. The covariance matrix of the interference projection onto the basis (28) is itself a periodical function of the signaling interval. In fact, since

$$\begin{aligned} A_I^2 \mathbf{\Sigma}_{\mathbf{ii}}(\ell) = E[\mathbf{i}(\ell)\mathbf{i}^\dagger(\ell)] &= A_I^2 \mathbf{\Sigma}_{\mathbf{ii}}(\ell + m) \\ &= E[\mathbf{i}(\ell+m)\mathbf{i}^\dagger(\ell+m)] \end{aligned} \tag{30}$$

the sequence of the covariance matrices of the narrow-band interference in successive signaling intervals of the CDMA system is a periodically time-varying one with period m. For noninteger N/Q their ranks are given by

$$\mathrm{rank}(\mathbf{\Sigma}_{\mathbf{ii}}(\ell)) = \begin{cases} \lfloor N/Q \rfloor + 1, & \text{if } \tau_I(\ell) = 0 \text{ or } \tau_I(\ell) \geq (N \bmod Q) T_c \\ \lfloor N/Q \rfloor + 2, & \text{if } \tau_I(\ell) < (N \bmod Q) T_c \end{cases}$$

Here $\tau_I(\ell)$ is the delay measured with respect to ℓT_b, given by

$$\tau_I(\ell) = T_I - \left(\ell T_b - \tau_I - \left\lfloor \frac{\ell T_b - \tau_I}{T_I} \right\rfloor T_I\right)$$

implying $\tau_I = \tau_I(0)$—the ranks are thus in any case small with respect to N.

Assuming that these matrices are known to the receiver, the proposed detector can be easily modified to account for

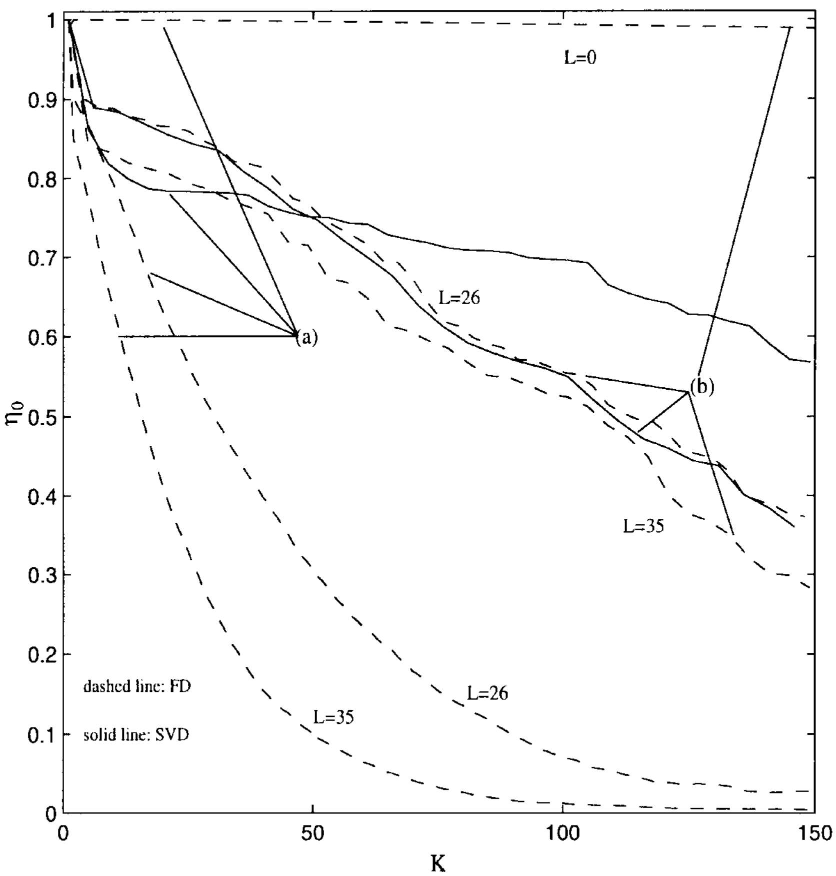

Fig. 2. Near–far resistance for the 0th user of the proposed receivers: $N = 255$, (a) = m-sequences and (b) = "quasi-incoherent" sequences, as spreading codes.

noninteger T_b/T_I. In fact, a decision as to the ℓth bit of the user k can be made according to the rule

$$\hat{b}_k(\ell) = \text{sgn}(\Re\{e^{-j\phi_k}\mathbf{D}_k^\dagger(\ell)\mathbf{r}(\ell)\}), \qquad k = 0, \cdots, K-1 \tag{31}$$

where the sequences $\mathbf{D}_k(\ell)$ are periodical in ℓ with period m, and can be easily determined as outlined for the case of stationary narrow-band interference. In fact, the matrices $\mathbf{\Sigma_{ii}}(\ell)$ can be decomposed via SVD, and the optimum coefficients $\mathbf{D}_k(\ell)$ can be computed as the projection of the signature of the kth user onto the orthogonal complement to the subspace spanned by the multiaccess and the narrow-band interference in that interval. This obviously results into a slightly more complex structure, but actually just amounts to evaluating once and for all the m different coefficients sets, and to switch from $\{\mathbf{D}_k[\ell \bmod m]\}_{k=0}^{K-1}$ to $\{\mathbf{D}_k[(\ell+1) \bmod m]\}_{k=0}^{K-1}$ as the signaling interval changes from the ℓth to the $(\ell+1)$th.

Summing up, the SVD-based method allows complete interference elimination. This entails a simple time-invariant projection if T_b/T_I is an integer, which implies that the subspace of the narrow-band interferer is one and the same, independent of the signaling interval. The price to be paid to generalize the system operation to the case that T_b/T_I is *not* an integer is that the projector is not one and the same for any signaling interval, but is periodically time-varying, with period dictated by T_b and T_I.

We stress here that the newly proposed method is slightly more general than that proposed in [10]. In fact, apart the removal of the constraint that T_b/T_I be an integer, introducing a redundancy in the signal representation, namely resorting to N-dimensional spaces, eventually leads, through the SVD approach, to a processing domain which "adapts itself" to the interference to be removed. As a matter of fact, the system operation depends on the structure of the interference covariance matrix, and not of the particular model that is assumed for $i(t)$. This is of great advantage in designing adaptive suppression procedures.

IV. Performance Assessment

In order to demonstrate the adequacy of the above receiver in real situations, it is necessary to assess its performance under several instances, and precisely:

—as no narrow-band interference is actually present at the input, showing that the detector is still near–far resistant, even though such an interference was accounted for at the design stage;

—as an external narrow-band interference corrupts the received signal, so as to show that the proposed approach yields overall robustness.

A. Near–Far Resistance

With reference to the first point, we assume $\mathbf{i} = 0$. To begin with, we notice that the error probability for a single-user channel, as no narrow-band interference is present, is written as

$$P_k^S(e) = Q\left(\sqrt{\frac{A_k^2 ||\mathbf{s}_k||^2}{\mathcal{N}_0}}\right). \tag{32}$$

By direct comparison with (15), we find the following expression for the near–far resistance, η_k, of the kth user (see also [13])

$$\eta_k = \frac{||\mathbf{s}_k^\perp||^2}{||\mathbf{s}_k||^2} = \frac{||(\mathbf{I}_N - \mathbf{\Psi}_k(\mathbf{\Psi}_k^\dagger \mathbf{\Psi}_k)^{-1}\mathbf{\Psi}_k^\dagger)\mathbf{s}_k||^2}{||\mathbf{s}_k||^2}. \tag{33}$$

In this relationship, the possible dependence on the signal interval of $\mathbf{s}_k^\perp$ induced by noninteger values of T_b/T_I has not been explicitly indicated, in order not to burden the notations. Relationship (33) highlights that the system near–far resistance depends on the norm of the projection of the user signature onto the orthogonal complement to the subspace of the narrow-band and the multiaccess interference. It is thus expected that η_k depends not only on the number of constraints but also on the processing domain. To investigate further, we refer to Fig. 2, wherein the near–far resistance of the zeroth user in the interval $(0, T_b)$ is represented versus the user number K, assuming $N = 255$ and that the system is designed to cancel a data-like signal with $T_I = 10T_c$, $f_I = 0$, and delay $\tau_I = 0$. Since $T_b/T_I = 25.5$, the repetition period of the interference covariance matrix is $m = 2$. Thus, both $\mathbf{\Sigma_{ii}}(0)$ and $\mathbf{\Sigma_{ii}}(1)$ admit $L = 26$ principal directions.

The curves refer to the case of quasi-orthogonal spreading codes (namely, m-sequences) and "quasi-incoherent" spreading codes—we generated "quasi-incoherent" sequences by modulo-2 sum of two m sequences, as suggested in [15], since no more than 16 Kasami sequences of length 255 are available. Under the same instances, the near–far resistance of the FD implementation is represented for several values of the number L of nullified frequency bins. The figure highlights two interesting points. First, quasi-incoherent sequences result in much better performance, under FD implementation, than m sequences, and in fact using the former codes with $L = 26$ nullified frequency bins ensures practically the same performance as the SVD-based method. A possible justification of this behavior is that, for FD implementation, one at first nullifies some frequency bins and then performs the orthogonalization to the residual multiaccess interference; thus, the additional loss induced by this orthogonalization is small only if the user signatures are still nearly orthogonal, after the said nullification. This in turn requires that the original spreading codes be almost incoherent with one another, rather than merely orthogonal, and this explains why "quasi-incoherent" sequences achieve better performance. With SVD processing, instead, the constraints are keyed to the structure of the narrow-band interferer: the orthogonalization procedure may thus take advantage of the quasi-orthogonality between the narrow-band interferers and the CDMA signals, which in fact vary on completely different time scales. Thus, we expect that, with SVD processing, the parameter to be kept under control is not the whole correlation between the signatures, but just its value at zero lag—a confirmation of this fact is that with SVD processing m-sequences achieve better performance than quasi-incoherent sequences, at least for conveniently large K.

In Fig. 3 the near–far resistance of the zeroth user is represented versus the number L of nullified frequency bins with FD implementation, for $K = 2$, 6, and 10, $N = 255$, and Kasami sequences as spreading codes. As expected, η_0 monotonically decreases with the number L of constraints, but the system remains near–far resistant even for L close enough to its theoretical upper bound (which is obviously tied to the dimension N of the signal subspace *and* to the user number K, since $L < N - (K - 1)$).

B. Performance in the Presence of Interference

In principle, (10) should ensure that the receiver is completely insensitive to the interference and that the error probability is as given in (15). This is true if the projection of the interfering signal is strictly contained in a subspace known to the receiver and the expansion basis is suitably chosen—this happens if SVD processing is adopted, but not with FD processing, which may produce significant *spillover* of the interfering signals onto all of the frequencies of the CDMA system. The nominal performance (15) is no longer guaranteed—a sensitivity analysis is thus in order, aimed at investigating the robustness of the proposed procedure.

From a quantitative point of view, we have that the performance in the presence of an interfering signal is written as

$$P_k(e) = E_{\mathbf{i}}\left[Q\left(\frac{A_k||\mathbf{s}_k^\perp||^2 + \Re\{e^{-j\phi_k}\mathbf{s}_k^{\perp\dagger}\mathbf{i}\}}{||\mathbf{s}_k^\perp||\sqrt{\mathcal{N}_0}}\right)\right]. \tag{34}$$

A viable solution to evaluate this expression is to resort to Monte-Carlo methods to compute the statistical average in (34).

As a case study, we consider the case that the interfering signal is a data-like signal as in (4)—the results are shown in Figs. 4 and 5, referring to the case that the spreading codes are Kasami sequences with processing gain $N = 255$ and that the users have equal powers, related to the interference power through the SIR (6). The bandwidth ratio is 0.1 for both figures, implying that $\lfloor N/Q \rfloor = 25$, while $f_I = 0$ for the former figure and $f_I = 0.5/T_c$ for the latter one. For comparison purposes, Figs. 4 and 5 show the performance under two situations:

1) as the structure of the interference covariance matrix is known to such a level of precision that the SVD-based method may be applied; again, the considered interval is $(0, T_b)$;
2) as FD processing is adopted.

In both cases, it is seen that the newly proposed detector largely outperforms the decorrelating detector, whose performance is represented by the curves indexed as "$L = 0$". In particular, for the case of known covariance matrix, interference suppression is almost perfect, and the corresponding performance is extremely close to that of an "optimum" decorrelating detector (ODEC) operating with no narrow-band interference—the small detection loss is thus only due to the presence of L additional constraints in the SVD-based

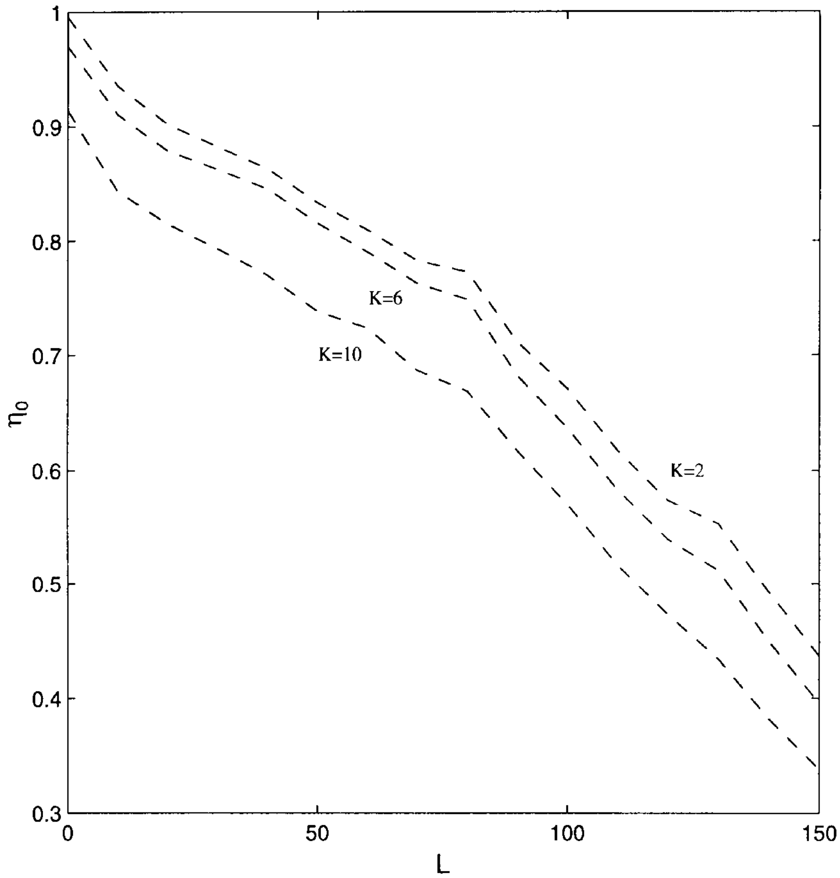

Fig. 3. Near–far resistance of the frequency domain implementation of the proposed receiver versus the number L of excided frequency bins—Kasami sequences as spreading codes, $N = 255$.

detector, which produces some additional noise enhancement. The FD processing, instead, yields some additional loss. Notice, however, that $f_I = 0$ is a more adverse situation than $f_I = 0.5/T_c$ in that, in the latter case, the matched filter, which is a low-pass one, suppresses some interference. Notice again that, unlike the SVD-based method, wherein the number of constraints is uniquely determined, for the case of FD processing the number L should be set on the basis of a trial-and-error procedure.

As a general trend, under FD processing, L should be chosen as a compromise between the conflicting requirements of achieving satisfactory interference suppression, while not incurring in unnecessary waste of information. Extensive computer simulations have, in fact, shown that performance improves as far as L increases up to a limiting value (about 45 for $f_I = 0$ and about 25 for $f_I = 0.5/T_c$); beyond this value, larger values of L result in worse performance.

Based on the results shown in this section, it is seen that the SVD-based method achieves better performance than frequency-domain processing but requires much more prior knowledge as to the interference. Of course, especially for noninteger values of T_b/T_I, FD corresponds to a much lower degree of prior knowledge as to the interference and, also, to a smaller detector complexity, since it allows time-invariant processing. On the other hand, leaving aside the issue of the complexity increase, it is understood that the best strategy is to devise a procedure which achieves the same performance as the SVD-based method, but requires lesser prior knowledge as to the narrow-band interference—the solution to this problem is to resort to an *adaptive* procedure, whose description and assessment forms the object of the next section.

V. Adaptive Interference Suppression

Now we introduce an iterative interference identification procedure, which allows us to relax the assumption of perfect knowledge of the correlation properties of the narrow-band interference. In particular, it is shown that even in the most general case of noninteger T_b/T_I, what is needed is only the interference signaling interval.

To begin with, let us assume that $\mathbf{\Sigma_{ii}}(\ell) = \mathbf{\Sigma_{ii}}(0)$, whereby the problem is obviously the determination of the unique subspace of the narrow-band interference. Obviously, if reliable estimates of the amplitudes of the K users were available, one could easily estimate such a subspace. In fact, the covariance matrix of the observables is

$$\mathbf{M_{rr}} = \sum_{k=0}^{K-1} A_k^2 \mathbf{s}_k \mathbf{s}_k^\dagger + A_I^2 \mathbf{\Sigma_{ii}} + 2\mathcal{N}_0 \mathbf{I}_N. \tag{35}$$

On the other hand, a reliable estimate $\hat{\mathbf{M}}_{\mathbf{rr}}$, say, of the matrix $\mathbf{M_{rr}}$ can be achieved as the sample covariance matrix of the received vector in a number of previous signaling intervals [14]; thus, knowledge of the A_k's allows one to estimate the

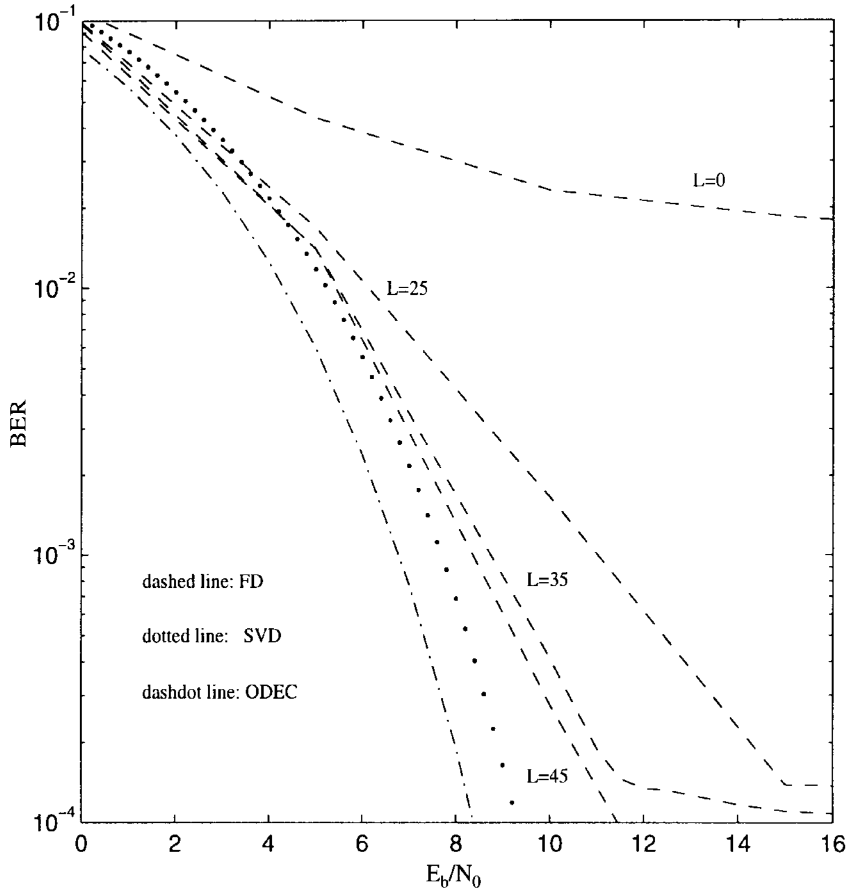

Fig. 4. Performance of the proposed receivers in the presence of multiaccess and narrow-band data-like interference—Kasami sequences as spreading codes, $N = 255$, $K = 10$, $T_I = 10T_c$, $f_I = 0$, SIR $= -20$ dB.

interference covariance matrix as

$$\hat{\mathbf{M}}_{\mathbf{ii}} = \hat{\mathbf{M}}_{\mathbf{rr}} - 2\mathcal{N}_0\mathbf{I}_N - \sum_{k=0}^{K-1} A_k^2 \mathbf{s}_k \mathbf{s}_k^\dagger$$

whose SVD allows to adaptively determine the interference subspace and, ultimately, the constraints to be inserted in (25). Notice that with this method one can also avoid useless waste of information as the narrow-band interference is very weak or absent—in this case, in fact, all of the eigenvalues are nearly zero, and the proposed detector reduces to a plain decorrelating detector.

A major criticism that can be raised against this procedure is that it requires prior knowledge or perfect estimates of the signal amplitudes A_k—it is, in fact, well known that one of the most attractive characteristics of the projection receivers is that they depend on the signatures but not on the amplitudes of the signals to be decoded. We now introduce a new method which does not require such a knowledge.

To fix the ideas, let us assume that we are to decode the user "0." Let us denote by $\mathbf{V}$ the projector of the received signal $\mathbf{r}$ onto the subspace spanned by the $(K-1)$ interfering users. Based on (25), the matrix $\mathbf{V}$ is

$$\mathbf{V} = \tilde{\mathbf{\Psi}}_k(\tilde{\mathbf{\Psi}}_k^\dagger \tilde{\mathbf{\Psi}}_k)^{-1}\tilde{\mathbf{\Psi}}_k^\dagger$$

wherein $\tilde{\mathbf{\Psi}}_k$ is an $N\times(K-1)$ matrix containing on its columns the $(K-1)$ signatures $\mathbf{s}_1, \cdots, \mathbf{s}_{K-1}$. Let us define the new observables

$$\mathbf{z} = (\mathbf{I}_N - \mathbf{V})\mathbf{r}.$$

Obviously, $\mathbf{z}$ represents the projection of $\mathbf{r}$ onto the orthogonal complement to the subspace spanned by the multiaccess interference. The covariance matrix of $\mathbf{z}$ is written as

$$\begin{aligned}\mathbf{M}_{\mathbf{zz}} &= (\mathbf{I}_N - \mathbf{V})\mathbf{M}_{\mathbf{rr}}(\mathbf{I}_N - \mathbf{V})^\dagger \\ &= A_0^2\mathbf{s}_0'\mathbf{s}_0'^\dagger + E[\mathbf{i}'\mathbf{i}'^\dagger] + 2\mathcal{N}_0(\mathbf{I}_N - \mathbf{V}) \\ &= A_0^2\mathbf{s}_0'\mathbf{s}_0'^\dagger + A_I^2\Sigma_{\mathbf{i}'\mathbf{i}'} + 2\mathcal{N}_0(\mathbf{I}_N - \mathbf{V})\end{aligned}$$

wherein $\mathbf{s}_0' = (\mathbf{I}_N - \mathbf{V})\mathbf{s}_0$ represents the projection of the signature of the user "0" onto the orthogonal complement to the subspace of the multiaccess interference, and $\mathbf{i}' = (\mathbf{I}_N - \mathbf{V})\mathbf{i}$ the projection of $\mathbf{i}$ onto the same subspace. Notice that $\mathbf{z}$ does not contain contribution from multiaccess interference; moreover, it contains only that part of the narrow-band interference which is external to the subspace of the other users signatures. Now the problem is to devise, among the directions of the subspace of $\mathbf{z}$, the directions orthogonal to $\mathbf{i}'$. To this end, let us consider the SVD of $\mathbf{M}^{(0)} = \mathbf{M}_{\mathbf{zz}} - 2\mathcal{N}_0(\mathbf{I}_N - \mathbf{V})$, i.e.,

$$\mathbf{M}^{(0)} = \sum_{i=1}^{p} \lambda_i \mathbf{u}_i \mathbf{u}_i^\dagger$$

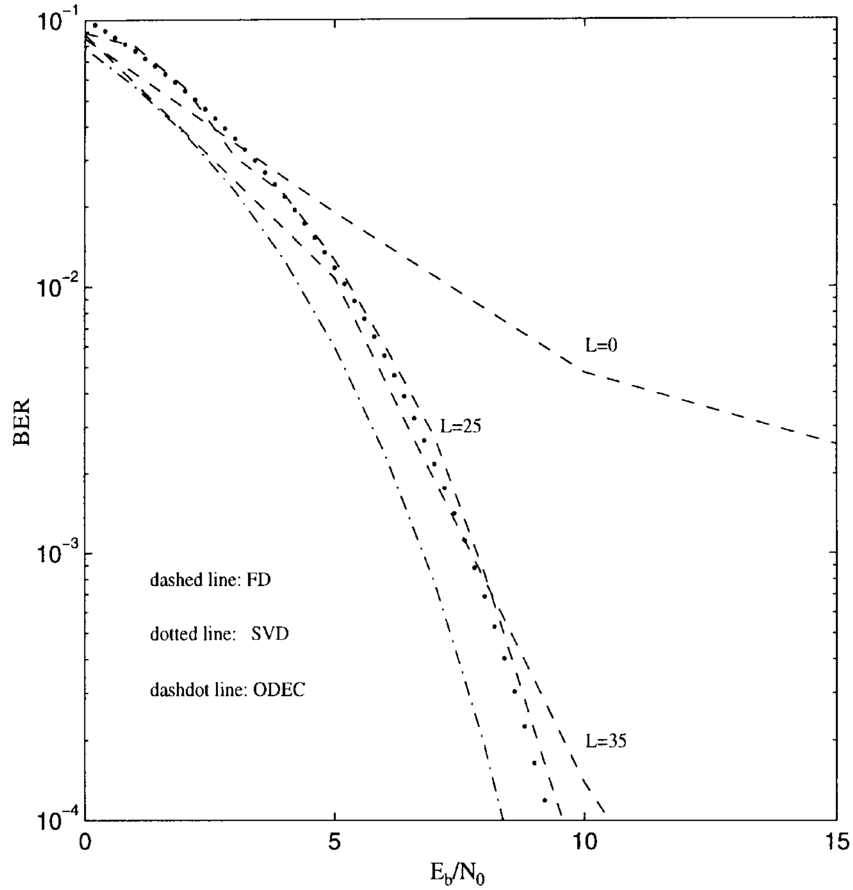

Fig. 5. Performance of the proposed receivers in the presence of multiaccess and narrow-band data-like interference: Kasami sequences as spreading codes, $N = 255$, $K = 10$, $T_I = 10T_c$, $f_I = 0.5/T_c$, $\mathrm{SIR} = -20$ dB.

with p the rank of the matrix: from now on we assume that $\lambda_p \leq \cdots \leq \lambda_1$. First notice that, if $p = 1$, this is to be interpreted as an evidence that the residual narrow-band interference is either absent or too weak to be detected, and the eigenvector corresponding to the nonzero eigenvalue is parallel to $\mathbf{s}_0'$. Thus the system is to assume $\hat{\mathbf{D}}_0 = \mathbf{s}_0'$, and operates as a mere decorrelating detector.

Let us now focus on the case that the rank is $p > 1$. Consider the matrix

$$\mathbf{M}^{(1)} = \mathbf{M}^{(0)} - \beta_1 \mathbf{s}_0' \mathbf{s}_0'^{\dagger} = \left(A_0^2 - \beta_1\right) \mathbf{s}_0' \mathbf{s}_0'^{\dagger} + A_I^2 \Sigma_{\mathbf{i}'\mathbf{i}'} \quad (36)$$

wherein the constant β_1 is chosen so as to maintain nonnegative definiteness of $\mathbf{M}^{(1)}$. Notice that, if β_1 could be set at A_0^2, then the matrix $\mathbf{M}^{(1)}$ would determine the sought subspace of $\mathbf{i}'$. Since we are not assuming prior knowledge of A_0^2, it is readily seen that the subspace of the matrix $\mathbf{M}^{(1)}$ contains in general the direction of $\mathbf{s}_0'$. Let us consider the matrices

$$\mathbf{M}^{(k)} = \mathbf{M}^{(0)} - \beta_k \mathbf{s}_0' \mathbf{s}_0'^{\dagger} \quad (37)$$

wherein again the β_k's are chosen so as to ensure nonnegative definiteness. Since, again

$$\mathbf{s}_0' = \mathbf{s}_0^{\perp} + \mathbf{s}_0'^{\|}$$

wherein $\mathbf{s}_0'^{\|}$ represents the component of $\mathbf{s}_0'$ in the subspace of $\mathbf{i}'$ and $\mathbf{s}_0^{\perp}$ the component of $\mathbf{s}_0'$ in the orthogonal complement of this subspace ($\mathbf{s}_0^{\perp}$ is thus the sought direction), (37) is rewritten as

$$\begin{aligned}\mathbf{M}^{(k)} = &\left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0^{\perp} \mathbf{s}_0^{\perp\dagger}\right) + A_I^2 \Sigma_{\mathbf{i}'\mathbf{i}'} + \left(A_0^2 - \beta_k\right) \\ &\times\left(\mathbf{s}_0^{\perp} \mathbf{s}_0'^{\|\dagger} + \mathbf{s}_0'^{\|} \mathbf{s}_0^{\perp\dagger}\right) + \left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0'^{\|} \mathbf{s}_0'^{\|\dagger}\right). \quad (38)\end{aligned}$$

If the sequence $\{\beta_k\}_{1 \leq k}$ converges to A_0^2, the matrix

$$\mathbf{G}^{(k)} = \left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0^{\perp} \mathbf{s}_0^{\perp\dagger}\right) + A_I^2 \Sigma_{\mathbf{i}'\mathbf{i}'} + \left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0'^{\|} \mathbf{s}_0'^{\|\dagger}\right)$$

for values of k sufficiently high, i.e., $|A_0^2 - \beta_k|$ sufficiently close to zero, is easily seen to admit as eigenvectors corresponding to the nonzeros eigenvalues the set of eigenvectors of the matrix $\mathbf{\Sigma}_{\mathbf{i}'\mathbf{i}'}$ *and* the sought direction $\mathbf{s}_0^{\perp}$ as the eigenvector corresponding to the minimum nonzero eigenvalue. Its SVD is

$$\mathbf{G}^{(k)} = \mathbf{Q}_k \mathbf{\Lambda}_k \mathbf{Q}_k$$

where $\mathbf{Q}_k$ is the matrix whose columns are the eigenvectors of $\mathbf{G}^{(k)}$ and $\mathbf{\Lambda}_k$ the diagonal matrix of its eigenvalues. The matrix $\mathbf{M}^{(k)}$ can thus be rewritten as

$$\begin{aligned}\mathbf{M}^{(k)} &= \mathbf{G}^{(k)} + \left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0^{\perp} \mathbf{s}_0'^{\|\dagger} + \mathbf{s}_0'^{\|} \mathbf{s}_0^{\perp\dagger}\right) \\ &= \mathbf{Q}_k \mathbf{\Lambda}_k \mathbf{Q}_k + \left(A_0^2 - \beta_k\right)\left(\mathbf{s}_0^{\perp} \mathbf{s}_0'^{\|\dagger} + \mathbf{s}_0'^{\|} \mathbf{s}_0^{\perp\dagger}\right). \quad (39)\end{aligned}$$

If k is thus chosen sufficiently high so as to ensure that $0 < |A_0^2 - \beta_k| \leq \epsilon$, with ϵ in turn sufficiently close to zero, the second summand in (39) becomes vanishingly small and it can be shown that the eigenvectors of $\mathbf{M}^{(k)}$ approach those of $\mathbf{G}^{(k)}$ [16]; thus, the eigenvector of $\mathbf{M}^{(k)}$ corresponding to the minimum nonzero eigenvalue approximates $\mathbf{s}_0^{\perp}$. As a consequence, for sufficiently high values of k, the SVD of the matrix $\mathbf{M}^{(k)}$ devises a matrix $\mathbf{Q}_k$ whose first $p-1$ columns represent the subspace of $\mathbf{i}'$. In principle, one could thus solve (25) with the constraints $\mathbf{e}_i$ parallel to the first $p-1$ columns of $\mathbf{Q}_k$. Actually, this is not necessary since the sought vector $\mathbf{D}_0$ is parallel to the pth column of $\mathbf{Q}_k$.

Before passing to the assessment of the convergence properties of the proposed procedure, a final issue should be addressed regarding the choice of the coefficients β_k. In principle, the only constraint that they should fulfill is that the term $(A_0^2 - \beta_k)$ is nonnegative and decreasing. In a brute force approach, this can be controlled by testing the signs of all of the eigenvalues at each step—should one or more eigenvalues become negative, the value of β_k should be decreased. However, a more refined technique could rely on controlling the sign of the minimum nonzero eigenvalue of the matrix $\mathbf{M}^{(k)}$.

For sake of example, let us first consider the very favorable situation where the SIR is vanishingly small. In this situation, the $p-1$ directions corresponding to the largest eigenvalues of the matrix $\mathbf{M}^{(0)}$, $\{\mathbf{u}_i\}_{i=1}^{p-1}$, say, will span a subspace almost coincident with that of $\mathbf{i}'$, and the pth eigenvector is very close to the sought direction $\mathbf{s}_0^{\perp}$. A perfect estimate of A_0^2 is $\frac{1}{|\mathbf{s}_0'^{\dagger}\mathbf{u}_p|^2}\mathbf{u}_p^{\dagger}\mathbf{M}^{(0)}\mathbf{u}_p$, and

$$\mathbf{M}^{(1)} = \mathbf{M}^{(0)} - \frac{1}{|\mathbf{s}_0'^{\dagger}\mathbf{u}_p|^2}\mathbf{u}_p^{\dagger}\mathbf{M}^{(0)}\mathbf{u}_p\mathbf{s}_0'\mathbf{s}_0'^{\dagger}$$

admits rank $p-1$: the sought direction is thus parallel to $\mathbf{u}_p$. Thus, in order to accelerate convergence, the value of β_k should be chosen, at each step, proportional to the eigenvalue corresponding to the eigenvector, $\mathbf{u}_*^{(k)}$ say, whose projection along $\mathbf{s}_0$ has maximum absolute real part.

Summing up, the proposed algorithm admits the following steps, which we illustrate with reference to the user "0" and should be obviously repeated for each user:

1) choose an arbitrary representation basis for the observables, e.g. the basis $\{u_{T_c}(t - iT_c - lT_b)\}_{i=0}^{N-1}$ and represent the observables in this basis;
2) in the chosen basis, determine the matrix $(\mathbf{I}_N - \mathbf{V})$;
3) estimate $\mathbf{M_{rr}}$ resorting to a suitable number of previous signaling intervals, in keeping with the procedure outlined in [14];
4) Evaluate $\hat{\mathbf{M}}^{(0)} = (\mathbf{I}_N - \mathbf{V})(\hat{\mathbf{M}}_{\mathbf{rr}} - 2\mathcal{N}_0\mathbf{I}_N)(\mathbf{I}_N - \mathbf{V})^{\dagger}$ and $\mathbf{s}_0' = (\mathbf{I}_N - \mathbf{V})\mathbf{s}_0$.
5) Set $\beta_0 = 0$ and $k = 0$; compute the rank p of $\mathbf{M}^{(k)}$.
   if such a rank is unity,
      adopt the decorrelating detector and stop;
6) compute the SVD of $\mathbf{M}^{(k)}$; determine the eigenvector $\mathbf{u}_*^{(k)}$ whose projection along $\mathbf{s}_0$ has real part with maximum absolute value; check the sign of the minimum nonzero eigenvalue of the $\mathbf{M}^{(k)}$:
   if this eigenvalue is positive and sufficiently close to zero (i.e. is smaller than a suitable threshold, ϵ_*), set $\hat{\mathbf{D}}_0 = \mathbf{u}_p^{(k)}$ and stop;
   else
   If this eigenvalue is positive,
      set $\beta_{k+1} = \beta_k + \frac{1}{|\mathbf{s}_0'^{\dagger}\mathbf{u}_*^{(k)}|^2}\mathbf{u}_*^{(k)\dagger}\mathbf{M}^{(k)}\mathbf{u}_*^{(k)}$; compute $\mathbf{M}^{(k+1)} = \mathbf{M}^{(0)} - \beta_{k+1}\mathbf{s}_0'\mathbf{s}_0'^{\dagger}$; set $k = k+1$ and return to 6;
   else
      set $\beta_k = \beta_{k-1} + a\frac{1}{|\mathbf{s}_0'^{\dagger}\mathbf{u}_*^{(k-1)}|^2}\mathbf{u}_*^{(k-1)\dagger}\mathbf{M}^{(k-1)}\mathbf{u}_*^{(k-1)}$, with $0 < a < 1$; set $\mathbf{M}^{(k)} = \mathbf{M}^{(0)} - \beta_k\mathbf{s}_0'\mathbf{s}_0'^{\dagger}$ and return to 6.

As a final remark, notice that the estimates of the projection directions $\hat{\mathbf{D}}_k, k = 0, \cdots, K-1$ could be directly obtained through

$$\hat{\mathbf{D}}_k = \mathrm{pinv}(\hat{\mathbf{M}}_{\mathbf{rr}} - 2\mathcal{N}_0\mathbf{I}_N)\mathbf{s}_k \tag{40}$$

wherein $\mathrm{pinv}(\cdot)$ defines the pseudoinverse matrix. Unlike (40), however, the proposed procedure, albeit somewhat lengthy, immediately lends itself to be generalized to account for possible prior uncertainty as to the noise level $2\mathcal{N}_0$. Additionally, the proposed algorithm allows tracking of the interference subspace, which may be useful if only partial orthogonalization is pursued [17].

Let us now consider the extension of the above algorithm to the case of noninteger T_b/T_I. Since, based on (29), we have

$$E[\mathbf{r}(n)\mathbf{r}^{\dagger}(n)] = \sum_{k=0}^{K-1} A_k^2\mathbf{s}_k\mathbf{s}_k^{\dagger} + A_I^2\mathbf{\Sigma}_{\mathbf{ii}}(n) + 2\mathcal{N}_0\mathbf{I}_N \tag{41}$$

a viable solution could be to apply the above procedure m times. Precisely, if reliable estimates of the covariance matrices $E[\mathbf{r}(n)\mathbf{r}^{\dagger}(n)]$, $n = 0, \cdots, m-1$ were available, the estimates $\hat{\mathbf{D}}_k(\ell)$, $k = 0, \cdots, K-1$, $\ell = 0, \cdots, m-1$ of the optimum vectors could be obtained through the above algorithm. To estimate the matrices (41), however, it is necessary to have prior knowledge of T_I, but no information is needed as to f_I and τ_I. In fact, if we assume that a number $N_P = mN_m$, say, of signaling intervals preceeding the interval $(0, T_b)$ are employed for initialization purposes, the sample covariance matrix

$$\frac{1}{N_m}\sum_{i=0}^{N_m-1}\mathbf{r}(\ell + im - N_P)\mathbf{r}^{\dagger}(\ell + im - N_P) \tag{42}$$

converges, in the mean-square sense, to $E[\mathbf{r}(\ell)\mathbf{r}^{\dagger}(\ell)]$ for increasingly high N_m, and can be subsequently used to estimate the optimum coefficients set $\hat{\mathbf{D}}_k(\ell)$, $k = 0, \cdots, K-1$.

The convergence properties of this algorithm are demonstrated in Fig. 6 wherein the quantity

$$\rho = \frac{|\mathbf{D}_0^{\dagger}(0)\hat{\mathbf{D}}_0(0)|}{||\mathbf{D}_0(0)||||\hat{\mathbf{D}}_0(0)||}$$

i.e., the normalized correlation between the true solution, as resulting from (25), and the result of the above algorithm

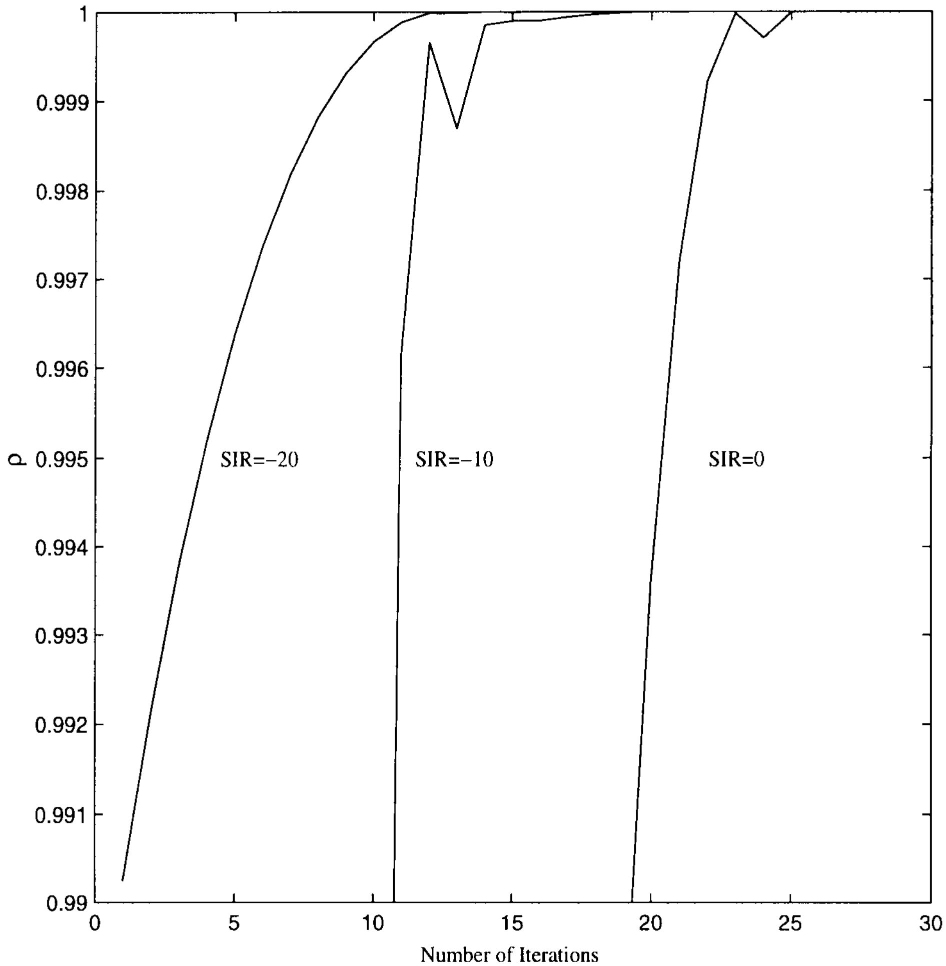

Fig. 6. Convergence of the adaptive system in the presence of multiaccess and narrow-band data-like interference: Kasami sequences as spreading codes, $N = 255$, $K = 10$, $T_I = 10T_c$, $f_I = 0$.

is represented as a function of the number of iterations. The curves are indexed through the SIR, and the interferer has the same parameters as for the previous figures. As a consequence, the covariance matrix of $\mathbf{i}(\ell)$ has period 2, and the covariance matrix $E[\mathbf{r}(0)\mathbf{r}^{\dagger}(0)]$ is estimated through 1000 previous signaling intervals. In all of the reported plots, it has been assumed that convergence is reached when the level of the minimum eigenvalue falls below a given value, which was heuristically set at 10^{-9}. The reported plots show that for all the values of the SIR considered, the algorithm converges to the true solution (the convergence property obviously corresponds to unit normalized correlation)—as expected, the stronger the interference, the faster the convergence. Overall, the number of iterations is less than 30, but, for the far more interesting situation of strong interferers, convergence takes place after less than 20 iterations.

Notice finally that no modification in the proposed algorithm is needed if the narrow-band interference consists of a superposition of a number of data-like signals, all with one and the same bit rate T_I, but with possibly different delays τ_I and frequency offsets f_I. The only limit that should be imposed is that the rank of $\mathbf{\Sigma_{ii}}(\ell)$ should remain, for any ℓ, smaller than $N - K + 1$, to ensure that the SVD is able to isolate the sought directions.

VI. Conclusion

In this paper we have considered the problem of multiuser detection in the presence of narrow-band interference. A new family of detectors has been presented, relying on processing the observables in "transformed domains," wherein the useful signals and the interference may be more easily discriminated—in particular, the former one, which is almost white, does not have dominant components in most orthonormal basis, while the latter may be "concentrated," upon suitable choice of the processing domain, and forced to have a limited number of significant components. Thus, interference suppression may be achieved by projecting the observables onto the orthogonal complement to the subspace spanned by the other users signatures and the narrow-band interference.

The proposed scheme is extremely flexible in that it allows trading some complexity increase for better interference cancellation. In fact, quite satisfactory performance may be obtained by simply resorting to the frequency-domain approach, which requires knowledge of the spectral properties of the interference and allows time-invariant processing. Perfect interference elimination may be achieved by keying the processing domain to the signal to be suppressed, which can, in turn, be done if the covariance properties of the interference are exactly known. In fact, singular-value decomposition of the interference covariance matrix allows complete identification of the subspace wherein narrow-band interference is strictly contained; on the other hand, not only does this strategy assume a larger prior knowledge as to the narrow-band interference but it also entails some additional complexity in that, but for some special cases, the orthogonalization procedure requires periodically time-varying processing.

An adaptive procedure to reduce the amount of prior knowledge required with SVD processing is also introduced; interestingly, this allows identifying the desired projection directions with no prior information as to the delay and frequency offset of the narrow-band interference. In a special case of some practical interest, moreover, no information at all is needed.

The results demonstrate that the proposed approach is very satisfactory. Surprisingly enough, even with frequency-domain implementation and time-invariant processing, the system offers a sufficient protection against narrow-band interferers.

There remain several assumptions to be relaxed, though, in particular that the system is synchronous and that the other users signatures are known to the receiver—the design of completely blind and asynchronous detectors in the presence of narrow-band interference with unknown characteristics is the object of current research.

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